Triangle factors in pseudorandom graphs

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An (n, d, λ)-graph is an n vertex, d-regular graph with second eigenvalue in absolute value λ. When λ is small compared to d, such graphs have *pseudo-random* properties. A fundamental question in the study of pseudorandom graphs is to find conditions on the parameters that guarantee the existence of a certain subgraph. A celebrated construction due to Alon gives a triangle-free (n, d, λ)-graph with $d = \Theta(n^{2/3})$ and $\lambda = \Theta(d^2/n)$. This construction is optimal as having $\lambda = o(d^2/n)$ guarantees the existence of a triangle in an (n, d, λ)-graph. Krivelevich, Sudakov and Szabó (2004) conjectured that if $n \in 3\mathbb{N}$ and $\lambda = o(d^2/n)$ then an (n, d, λ)-graph G in fact contains a *triangle factor*: vertex disjoint triangles covering the whole vertex set.

In this talk, we discuss a solution to the conjecture of Krivelevich, Sudakov and Szabó. The result can be seen as a clear distinction between pseudorandom graphs and random graphs, showing that essentially the same pseudorandom condition that ensures a triangle in a graph actually guarantees a triangle factor. In fact, even more is true: as a corollary to this result and a result of Han, Kohayakawa, Person and the author, we can conclude that the same condition actually guarantees that such a graph G contains every graph on n vertices with maximum degree at most 2.