Critical Digraphs

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Abstract

The *dichromatic number* $\vec{\chi}(D)$ of a digraph D is the minimum number of colors needed to color the vertices of D such that each color class induces an acyclic subdigraph of D; such a coloring is called an *acyclic coloring* of D. This coloring concept is practical for many reasons: on one hand, every proper coloring of an undirected graph is also an acyclic coloring of the digraph $D(G)$ that results from G by replacing each edge of G with a pair of opposite arcs (and vice versa). On the other hand, many classic coloring results can be transferred to acyclic digraph coloring.

A digraph D is *k-critical* if $\vec{\chi}(D) = k$ but $\vec{\chi}(D') < k$ for each proper subdigraph D' of D. Introduced by Dirac in the 1950's, critical graphs have developed into a very powerful tool for proving results in coloring theory and lots of effort has been dedicated to analyze properties of critical graphs. However, not much is known about critical digraphs yet.

In the talk, we examine methods for creating infinite families of critical digraphs; the *Dirac-join* and the *Hajós-join*. The major part of the lecture is dedicated to the proof that a digraph D has dichromatic number at least k if and only if it contains a subdigraph that can be obtained from bidirected complete graphs on k vertices by Hajós-joins and identifying non-adjacent vertices.