Defective colouring of hypergraphs

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Erdős and Lovász originally proved the Lovász local lemma to show that every $(r+1)$-uniform hypergraph G with maximum degree Δ has chromatic number $O(\Delta^{1/r})$. We prove the following generalisation of this result. A d-defective colouring of a hypergraph is a vertex-colouring in which every vertex is in at most d monochromatic edges (so $d = 0$ is exactly a proper colouring). We prove that every $(r+1)$-uniform hypergraph G with maximum degree Δ has a d-defective colouring using at most $100(\Delta/(d+1))^{1/r}$ colours. This is tight up to the leading constant. Our proof uses a semirandom argument together with a sunflower decomposition trick. In the talk I will discuss the proof as well as some ideas that do not work.

This is joint work with António Girão (Oxford), Alex Scott (Oxford), and David Wood (Monash).