On sufficient conditions for Hamiltonicity in dense graphs

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We study structural conditions in dense graphs that guarantee the existence of vertex-spanning substructures such as Hamilton cycles. It is easy to see that every Hamiltonian graph is connected, has a perfect fractional matching and, excluding the bipartite case, contains an odd cycle. Our main result in turn states that any large enough graph that robustly satisfies these properties must already be Hamiltonian. Moreover, the same holds for embedding powers of cycles and graphs of sublinear bandwidth subject to natural generalisations of connectivity, matchings and odd cycles.

This solves the embedding problem that underlies multiple lines of research on sufficient conditions for Hamiltonicity in dense graphs. As applications, we recover and establish Bandwidth Theorems in a variety of settings including Ore-type degree conditions, Pósa-type degree conditions, deficiency-type conditions, locally dense and inseparable graphs, multipartite graphs as well as robust expanders.