Extremal sets with minimum shadow

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The shadow of a family F of k-subsets of $[n]$ is the family of all $(k-1)$-subsets contained in some element in F. The celebrated Kruskal-Katona theorem states that initial segments in the colex order minimize the size of the shadow for a given size of the family.

Füredi and Griggs established the uniqueness of the colex order for this shadow minimization problem for some cardinalities of the family F. They also provided examples which show that uniqueness fails to hold in general. With this motivations they posed the question to characterize the extremal sets for this problem.

In this talk I will present a characterization which provides insight to this question. As a consequence of this characterization it can be shown that the l-iterated shadow of an extremal set is always an initial segment in the colex order for $l = O(\log \log n)$. This reveals the relative universality of the colex order as a solution of this extremal problem. On the other hand, every family of sets is contained in a structured extremal set, thus providing a wide range of extremal sets.

This is joint work with Lluís Vena.