

Triangle-free graphs, independent set structure, and the hard-core model

Ross Kang (Radboud University)

The structure of triangle-free graphs has played an influential role in the development of combinatorial mathematics. Both of the fundamental results of Mantel (1907) and of Ramsey (1930) yield global structure from the local property of having no edges in any neighbourhood. A few years ago, I began to systematically explore this classic and well-studied area. This has led to novel questions and developments, especially with respect to independent sets and colourings in graphs. In the talk I will mainly give an overview of the history as well as discuss the focus of current/recent activities.

If time permits, I will present a powerful, general framework for producing global structure from local structure. This generalises and strengthens a vast swathe of the most important results in the area, including those of Ajtai, Komlós, Szemerédi (1981), Shearer (1983/1996), Johansson (1996), Alon (1996), Alon, Krivelevich, Sudakov (1999), Molloy (2019), Bernshteyn (2019), and Achlioptas, Iliopoulos, Sinclair (2019). The framework is built around a technique inspired by statistical physics –namely, a local analysis of the hard-core model– as well as the suitable application of the Lovász local lemma.

This is joint work with Ewan Davies, Rémi de Joannis de Verclos, François Pirot, Jean-Sébastien Sereni.