

Uniform Turán density

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In the early 1980s, Erdős and Sós initiated the study of the classical Turán problem with a uniformity condition: the uniform Turán density of a hypergraph H is the infimum over all d for which any sufficiently large hypergraph with the property that all its linear-size subhypergraphs have density at least d contains H . In particular, they raise the questions of determining the uniform Turán densities of $K_4^{(3)-}$ and $K_4^{(3)}$. The former question was solved only recently in [Israel J. Math. 211 (2016), 349–366] and [J. Eur. Math. Soc. 97 (2018), 77–97], while the latter still remains open for almost 40 years. In addition to $K_4^{(3)-}$, the only 3-uniform hypergraphs whose uniform Turán density is known are those with zero uniform Turán density classified by Reiher, Rödl and Schacht [J. London Math. Soc. 97 (2018), 77–97] and a specific family with uniform Turán density equal to $1/27$.

In this talk, we give an introduction to the concept of uniform Turán densities, present a way to obtain lower bounds using color schemes, and give a glimpse of the proof for determining the uniform Turán density of the tight 3-uniform cycle $C_\ell^{(3)}$, $\ell \geq 5$.