Reachability in arborescence packings

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An r*-arborescence* B is an orientation of a tree in which all arcs are directed away from a given root r and the arborescence is said to *span* $V(B)$. Given a digraph D, an r-arborescence B that is a subgraph of D is said to be spanning if it spans $V(D)$. In 1973, Edmonds characterized digraphs admitting a packing of k spanning r-arborescences for a fixed root r and some integer k. It can readily be seen that this theorem can be generalized to allow fixed but distinct roots for the arborescences.

In case that some vertex cannot be reached from a given root, the only information we obtain from Edmonds theorem is that the desired packing does not exist. For this reason, in 2009, Kamiyama, Katoh and Takizawa introduced the concept of reachability arborescences. An r-arborescence is called a *reachability* r-arborescence if it spans all the vertices reachable from r in D. They characterize digraphs that, for a given root multiset R, have a packing of arborescences $\{B_r : r \in R\}$ such that B_r is a reachability r-arborescence for all $r \in R$.

A new, shorter proof for the theorem of Kamiyama, Katoh and Takizawa will be presented. Being of inductive nature, it uses a stronger form of Edmonds' theorem and is self-contained otherwise. Further, several ways of generalizing these concepts will be mentioned. Firstly, the condition on the arborescences to be spanning or reachability arborescences can be relaxed to more general conditions called *matroid-based* packing and *matroid-reachability-based* packing. Further, the objects of consideration can be generalized from digraphs to mixed graphs, dypergraphs and mixed hypegraphs. All the proofs considered provide efficient algorithms for finding the desired arborescences.

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