We are grateful to the program committee for the high quality of the scientific program.

„Welcome“ from the local organizers!

Dear participant, welcome to the DMV annual meeting 2023 - this time in Ilmenau! We hope you will enjoy this conference and your stay in the heart of the Thuringian forest.

We thank d-fine, FIZ Karlsruhe, Maplesoft and Springer for their generous support.

We are indebted to the Technische Univeristät Ilmenau and the local organizing comittee, this conference would not have been possible without their support.

Program Committee

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- Stefan Heyder (Ilmenau)
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- Timo Reis (Ilmenau)
- Manuel Schaller (Ilmenau)
- Carsten Trunk (chair) (Ilmenau)
- Karl Worthmann (Ilmenau)

Sunday, 24 September 2023

18:00 Musical Opening: Organ Concert at St. James church (St. Jakobuskirche)
Kirchplatz 1, 98693 Ilmenau

Monday, 25 September 2023

09:00 Opening

09:45 **EMMY-NOETHER LECTURE**
Sarah Zerbes (ETH Zürich)
Euler systems and the Birch—Swinnerton-Dyer conjecture

10:45 Coffee Break

11:15 **PLENARY**
Christiane Tretter (Universität Bern)
Challenges for non-selfadjoint spectral problems in analysis and computation

12:15 Lunch

13:30 **SESSIONS**

S02_01 Discrete Mathematics

14:00 **SESSIONS**

S03_01 Mathematics in the Course of Time
S09_01 Mathematics of Data Science

MS03_01 Where optimisation and mathematical systems theory meet
MS05_01 Towards a digital infrastructure for mathematical research
MS08_01 Soziale Dimensionen der Mathematik
MS10_01 Mathematical Analysis of Complex Quantum Systems
MS12_01 Random graphs and statistical network analysis

16:00 Coffee Break

16:30 **SESSIONS**

S09_02 Mathematics of Data Science
S11_01 Analysis & Differential Equations

MS02_01 Combinatorial aspects of finite fields
MS03_02 Where optimisation and mathematical systems theory meet
MS05_02 Towards a digital infrastructure for mathematical research
MS06_01 Mathematics and Arts
MS11_01 Decoding the Disciplines - an introduction to the process
MS12_02 Random graphs and statistical network analysis

19:00 Welcome Reception

Tuesday, 26 September 2023

09:00 **SESSIONS**

- S03_02 Mathematics in the Course of Time
- S09_03 Mathematics of Data Science
- S12_01 Dynamical Systems
- MS02_02 Combinatorial aspects of finite fields
- MS03_03 Where optimisation and mathematical systems theory meet
- MS07_01 Mathematics and society – embedding models in discussions of societal challenges
- MS09_01 Generalizations of complex analysis
- MS10_02 Mathematical Analysis of Complex Quantum Systems

11:00 Coffee Break

11:30 **PLENARY**
Afonso S. Bandeira (ETH Zürich)
Matrix Concentration and Free Probability

12:30 Lunch

14:00 **SESSIONS**

- S02_02 Discrete Mathematics
- S04_01 Teaching Mathematics at tertiary educational institutions and elsewhere
- S07_01 Probability, Statistics, and Financial Mathematics
- S09_04 Mathematics of Data Science
- S12_02 Dynamical Systems
- MS08_02 Soziale Dimensionen der Mathematik
- Springer Publishing with Springer Spektrum

16:00 Coffee Break

16:30 General Assembly of DMV (Parkcafé)

19:00 Conference Dinner

Wednesday, 27 September 2023

09:00 **SESSIONS**

S01_01 Geometry and Topology
S05_01 Mathematical Logic
S10_01 Numerical Mathematics and Scientific Computing
S11_02 Analysis & Differential Equations
S12_03 Dynamical Systems
MS01_01 Data-Driven Methods for Dynamical Systems
MS09_02 Generalizations of complex analysis
MS10_03 Mathematical Analysis of Complex Quantum Systems

09:30 **SESSIONS**

S03_03 Mathematics in the Course of Time

11:00 Coffee Break

11:30 **PLENARY**
Katrin Wendland (Trinity College Dublin)
K3 surfaces and their avatars in conformal field theory

12:30 Lunch

14:00 **PLENARY**
Stefanie Petermichl (Julius-Maximilians-Universität Würzburg)
On the Hilbert Transform

15:30 **PLENARY**
Andreas Thom (TU Dresden)
Equations over groups

16:30 Short Break

16:45 **SESSIONS**

S01_02 Geometry and Topology
S05_02 Mathematical Logic
S08_01 Inverse Problems
S09_05 Mathematics of Data Science
S10_02 Numerical Mathematics and Scientific Computing
S11_03 Analysis & Differential Equations
MS04_01 Extremal and Probabilistic Combinatorics
MS06_02 Mathematics and Arts

18:45 Short Break

19:00 **PUBLIC LECTURE**
Alex Kontorovich (Rutgers University)
Mirror Mirror on the Wall: the story of reflection groups and fractal sphere-packings

Thursday, 28 September 2023

09:00 **PLENARY**
Julia Böttcher (The London School of Economics and Political Science)
Packing graphs, large and small

10:00 Coffee Break

10:30 **PLENARY**
Mario Ohlberger (University of Münster)
Model Order Reduction and Learning for PDE Constrained Optimization and Inverse Problems

11:30 Lunch

13:00 **SESSIONS**

S01_03 Geometry and Topology
S05_03 Mathematical Logic
S12_04 Dynamical Systems
MS04_02 Extremal and Probabilistic Combinatorics
MS06_03 Mathematics and Arts

15:00 Coffee Break

15:30 **SESSIONS**

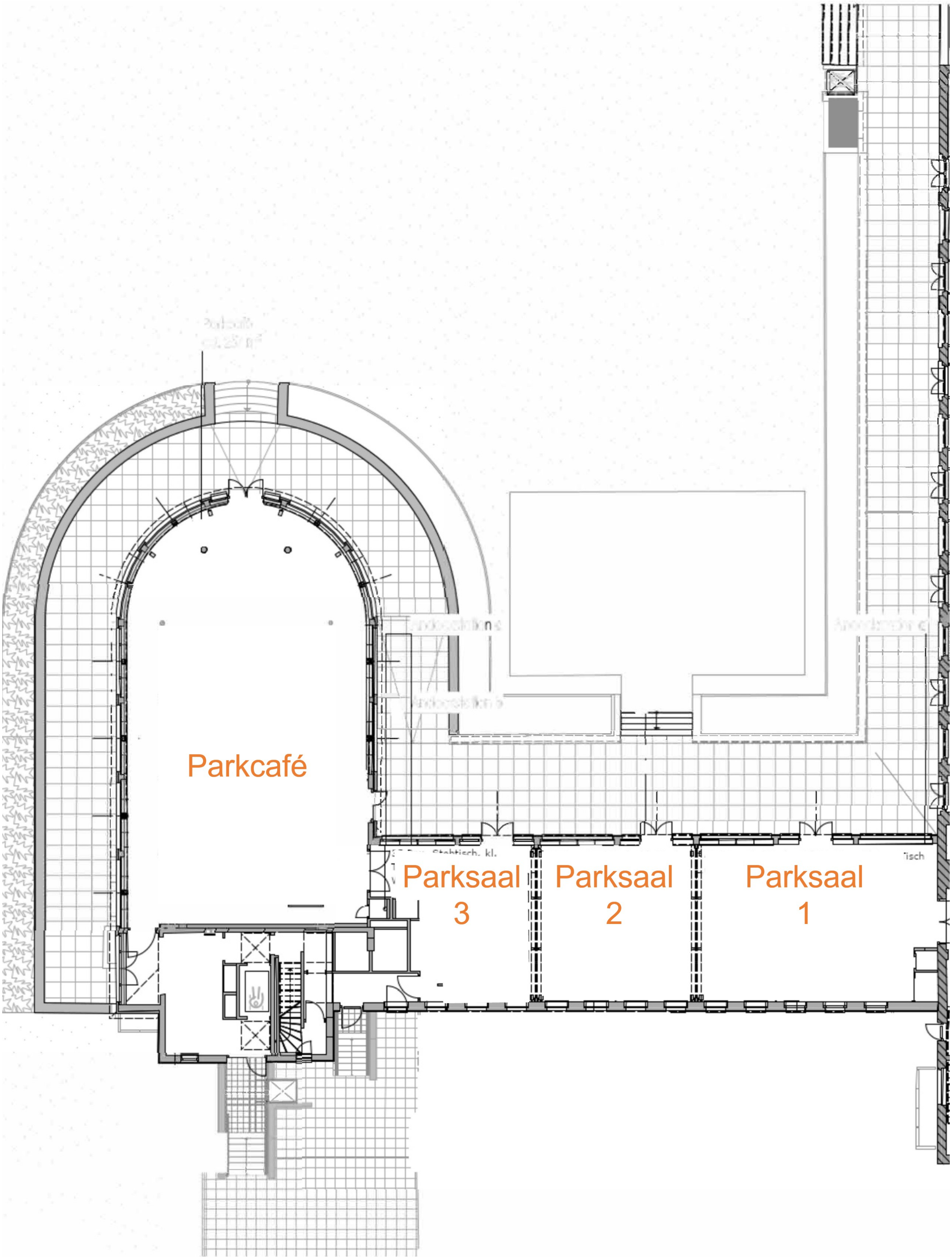
S01_04 Geometry and Topology
S05_04 Mathematical Logic
S08_02 Inverse Problems
MS06_04 Mathematics and Arts

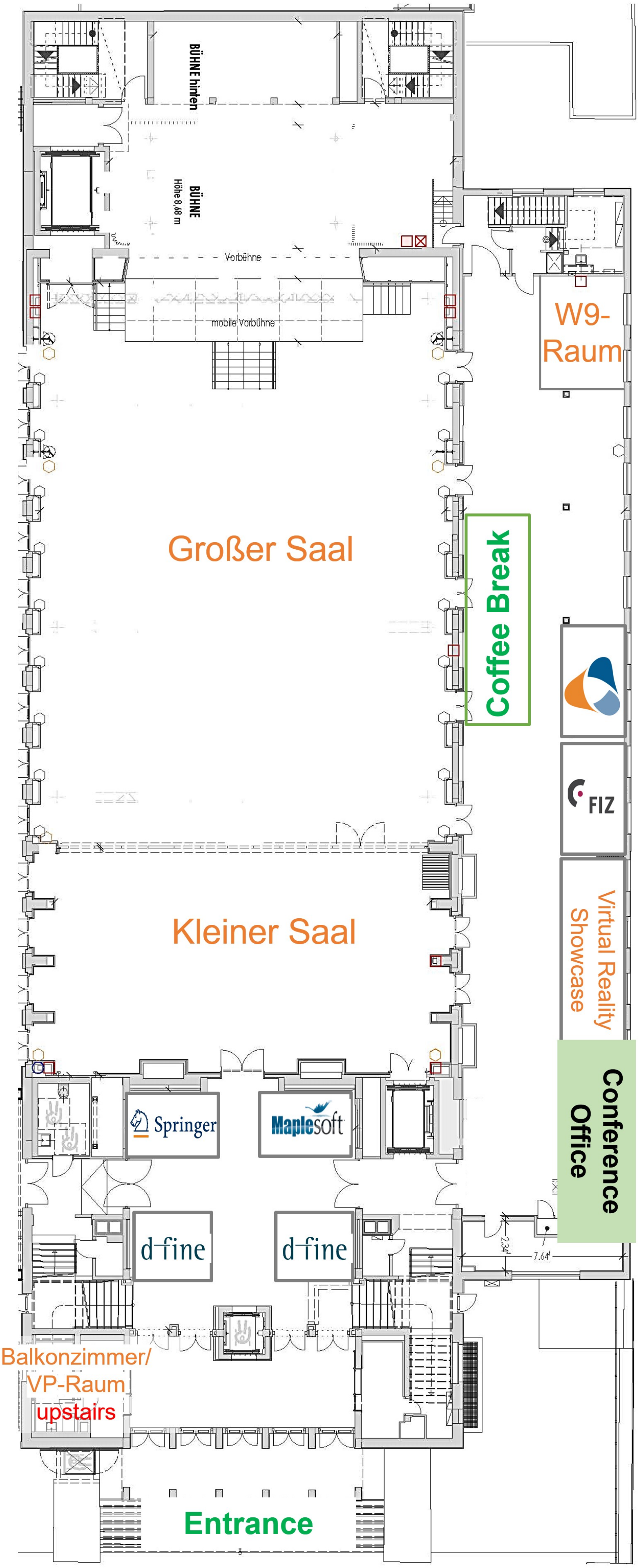
17:30 Closing (Parkcafé)

Allocation of rooms

	Monday 14:00	Monday 16:30	Tuesday 09:00	Tuesday 14:00
Großer Saal	S03	MS06	S03	S12
Kleiner Saal	MS12	MS12	MS10	MS08
Parkcafé	MS10	—	—	Springer
Parksaal 1	S09	S09	S09	S09
Parksaal 2	MS03	MS03	MS03	S02
Parksaal 3	S02	S11	S12	S07
Balkonzimmer	MS08	MS02	MS02	S04
VP-Raum	MS05	MS05	MS09	—
W9-Raum	—	MS11	MS07	—

	Wednesday 09:00	Wednesday 16:45	Thursday 13:00	Thursday 15:30
Großer Saal	S03	MS06	—	—
Kleiner Saal	S12	Lehrertag	MS04	—
Parkcafé	MS10	S09	MS06	MS06
Parksaal 1	S01	S01	S12	S01
Parksaal 2	S05	S11	S01	S08
Parksaal 3	S11	MS04	S05	S05
Balkonzimmer	MS09	S08	—	—
VP-Raum	S10	S05	—	—
W9-Raum	MS01	S10	—	—


Parkcafé
Parksaal 3
Parksaal 2
Parksaal 1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mobile Vorbühne
W9-Raum
Großer Saal
Coffee Break
FIZ
Kleiner Saal
Virtual Reality Showcase
Conference Office
Springer
Maplesoft
d-fine
d-fine
2,34
7,64
Balkonzimmer/
VP-Raum
upstairs
Entrance

PLENARY LECTURES

Monday, 25 September 2023

Sarah Zerbes (ETH Zürich)
Euler systems and the Birch—Swinnerton-Dyer conjecture
Chair: Matthias Kriesell (TU Ilmenau)

Christiane Tretter (Universität Bern)
Challenges for non-selfadjoint spectral problems in analysis and computation
Chair: Heinz Siedentop (LMU)

Tuesday, 26 September 2023

Afonso S. Bandeira (ETH Zürich)
Matrix Concentration and Free Probability
Chair: Thomas Hotz (TU Ilmenau)

Wednesday, 27 September 2023

Katrin Wendland (Trinity College Dublin)
K3 surfaces and their avatars in conformal field theory
Chair: Dorothea Bahns (University of Göttingen)

Stefanie Petermichl (Julius-Maximilians-Universität Würzburg)
On the Hilbert Transform
Chair: Joachim Escher (University of Hannover)

Andreas Thom (TU Dresden)
Equations over groups
Chair: Yury Person (TU Ilmenau)

Alex Kontorovich (Rutgers University)
Mirror Mirror on the Wall: the story of reflection groups and fractal sphere-packings
Chair: Jürgen Richter-Gebert (TU München)

Thursday, 28 September 2023

Julia Böttcher (The London School of Economics and Political Science)
Packing graphs, large and small
Chair: Anusch Taraz (TU Hamburg)

Mario Ohlberger (University of Münster)
Model Order Reduction and Learning for PDE Constrained Optimization and Inverse Problems
Chair: Armin Iske (University of Hamburg)

Euler systems and the Birch—Swinnerton-Dyer conjecture

Sarah Zerbes
ETH Zürich

L-functions are one of the central objects of study in number theory. There are many beautiful theorems and many more open conjectures linking their values to arithmetic problems. The most famous example is the conjecture of Birch and Swinnerton-Dyer, which is one of the Clay Millenium Prize Problems. I will discuss this conjecture and some related open problems, and I will describe some recent progress on these conjectures, using tools called „Euler systems".

Challenges for non-selfadjoint spectral problems in analysis and computation

Christiane Tretter
Universität Bern

Non-selfadjoint spectral problems appear frequently in a wide range of applications. Reliable information about their spectra is therefore crucial, yet extremely difficult to obtain. This talk focuses on tools to master these challenges such as spectral pollution or spectral invisibility. In particular, the concept of essential numerical range for unbounded linear operators is introduced and studied, including possible equivalent characterizations and perturbation results. Compared to the bounded case, new interesting phenomena arise which are illustrated by some striking examples. A key feature of the essential numerical range is that it captures, in a unified and minimal way, spectral pollution which may affect e.g. spectral approximations of PDEs by projection methods or domain truncation methods. As an application, Maxwell's equations with conductivity will be considered.

Matrix Concentration and Free Probability

Afonso S. Bandeira
ETH Zürich

Matrix Concentration inequalities such as Matrix Bernstein inequality have played an important role in many areas of pure and applied mathematics. These inequalities are intimately related to the celebrated noncommutative Khintchine inequality of Lust-Piquard and Pisier. In the middle of the 2010's, Tropp improved the dimensional dependence of this inequality in certain settings by leveraging cancellations due to non-commutativity of the underlying random matrices, giving rise to the question of whether such dependency could be removed.
In this talk we leverage ideas from Free Probability to fully remove the dimensional dependence in a range of instances, yielding optimal bounds in many settings of interest. As a byproduct we develop matrix concentration inequalities that capture non-commutativity (or, to be more precise, „freeness"), improving over Matrix Bernstein in a range of instances. No background knowledge of Free Probability will be assumed in the talk.
Joint work with March Boedihardjo and Ramon van Handel.

K3 surfaces and their avatars in conformal field theory

Katrin Wendland
Trinity College Dublin

K3 surfaces are compact, complex surfaces, which have fascinated algebraic geometers since the mid nineteenth century. Over the past few decades, their appearance in conformal field theory has triggered new interest in these surfaces. The talk showcases this perspective on K3 surfaces and provides an overview of recent developments in the field.

On the Hilbert Transform

Stefanie Petermichl
Julius-Maximilians-Universität Würzburg

The Hilbert transform is a central, classical singular operator in harmonic analysis. It is a frequency filter and gives access to the boundary values of the conjugate function, thus producing orthogonal level sets. It is best understood as an integral operator via a singular kernel or as a multiplier operator on the Fourier transform side. Over twenty years ago my dyadic Hilbert transform model appeared, had many applications, but seemed to come out of nowhere. In this lecture, we consider a slightly modified dyadic model and show that it is an extremely natural extension of the concept of analyticity. As a consequence, we get linear two sided norm bounds if these operators act on functions with values where orthogonality is lost - UMD Banach spaces.

Equations over groups

Andreas Thom
TU Dresden

The study of equations in the language of groups has a long history and many applications. I will explain how topological methods can be applied to solve equations over groups and also mention some recent advances in the study of identities for finite and infinite groups.

Mirror Mirror on the Wall: the story of reflection groups and fractal sphere-packings

Alex Kontorovich
Rutgers University

We will discuss how geometry and arithmetic can interact in unexpected ways to form fractal sphere-packings from reflections in curved mirrors.

Packing graphs, large and small

Julia Böttcher
The London School of Economics and Political Science

Graph packing is an area in discrete mathematics that has seen swift and exciting progress in the last decade, with a number of breakthrough results and the development of an array of important new techniques. In this talk I will introduce this area and its motivation, survey some of the classic and some of the new results, and point out a number of ideas that were crucial for recent advances.

Model Order Reduction and Learning for PDE Constrained Optimization and Inverse Problems

Mario Ohlberger
University of Münster

Model order reduction for parameterized partial differential equations is a very active research area that has seen tremendous development in recent years from both theoretical and application perspectives. A particular promising approach is the reduced basis method that relies on the approximation of the solution manifold of a parameterized system by tailored low dimensional approximation spaces that are spanned from suitably selected particular solutions, called snapshots. With speedups that can reach several orders of magnitude, reduced basis methods enable high fidelity real-time simulations for certain problem classes and dramatically reduce the computational costs in many-query applications. While the „online efficiency“ of these model reduction methods is very convincing for problems with a rapid decay of the Kolmogorov n-width, there are still major drawbacks and limitations. Most importantly, the construction of the reduced system in a so called „offline phase“ is extremely CPU-time and memory consuming for large scale systems. For practical applications, it is thus necessary to derive model reduction techniques that do not rely on a classical offline/online splitting but allow for more flexibility in the usage of computational resources. In this talk we focus on learning based reduction methods in the context of PDE constrained optimization and inverse problems and evaluate their overall efficiency. We discuss learning strategies, such as adaptive enrichment as well as a combination of reduced order models with machine learning approaches in the contest of time dependent problems. Concepts of rigorous certification and convergence will be presented, as well as numerical experiments that demonstrate the efficiency of the proposed approaches.

SECTIONS

S01 Geometry and Topology
Section leaders: Nadine Große (Universität Freiburg), Matthias Ludewig (Regensburg)

S02 Discrete Mathematics
Section leaders: Raman Sanyal (FU Berlin und GU Frankfurt), Anusch Taraz (TU Hamburg), Stefan Weltge (TU München)

S03 Mathematics in the Course of Time
Section leaders: Hans Fischer (KU Eichstätt-Ingolstadt), Achim Ilchmann (TU Ilmenau)

S04 Teaching Mathematics at tertiary educational institutions and elsewhere
Section leaders: Walther Paravicini (Universität Tübingen), Thomas Skill (HS Bochum)

S05 Mathematical Logic
Section leaders: Philipp Luecke (Barcelona), Nadja Valentin (Düsseldorf)

S07 Probability, Statistics, and Financial Mathematics
Section leaders: Hajo Holzmann (Marburg), Anja Sturm (Göttingen)

S08 Inverse Problems
Section leaders: Bernadette Hahn (Stuttgart), Bastian von Harrach-Samet (Frankfurt am Main), Tobias Kluth (Bremen)

S09 Mathematics of Data Science
Section leaders: Dominik Stöger (KU Eichstätt-Ingolstadt), Felix Voigtländer (KU Eichstätt-Ingolstadt)

S10 Numerical Mathematics and Scientific Computing
Section leaders: Dietmar Gallistl (Jena), Daniel Peterseim (Augsburg)

S11 Analysis & Differential Equations
Section leaders: Ralph Chill (TU Dresden), Christian Seifert (TU Hamburg)

S12 Dynamical Systems
Section leaders: Marc Keßeböhmer (Bremen), Sabrina Kombrink (Birmingham), Tony Samuel (Birmingham)

S01 Geometry and Topology

Section leaders: Nadine Große (Universität Freiburg), Matthias Ludewig (Regensburg)

Session S01_01

Wednesday, 27 September 2023 09:00 (Parksaal 1)

09:00 Thomas Schick (Universität Göttingen)
Rigidity of positive scalar curvature under low regularity

10:00 Enno Keßler (Max-Planck-Institut für Mathematik in den Naturwissenschaften)
Super J-holomorphic Curves

10:30 Eugenia Loiudice
Para-Sasakian $\phi-$symmetric spaces

Session S01_02

Wednesday, 27 September 2023 16:45 (Parksaal 1)

16:45 Michael Wiemeler (Universität Münster)
Positively curved manifolds with torus symmetry

17:15 Maximilian Stegemeyer (Universität Leipzig)
String topology and closed geodesics

17:45 Jonathan Glöckle (Universität Regensburg)
Initial data rigidity via Dirac-Witten operators

18:15 Thorsten Hertl (Albert Ludwigs Uni Freiburg)
Moduli Spaces in Riemannian Geometry

Session S01_03

Thursday, 28 September 2023 13:00 (Parksaal 2)

13:00 Anna Siffert
Harmonic maps of ellipsoids

14:00 Rudolf Zeidler (University of Münster)
Metric inequalities with scalar curvature via the Dirac operator

14:30 Elisa Hartmann (Universität Bielefeld)
Coarse sheaf cohomology

Session S01_04

Thursday, 28 September 2023 15:30 (Parksaal 1)

15:30 Peter Hochs (Radboud University)
Equivariant analytic torsion and an equivariant Ruelle dynamical zeta function

16:00 Georg Frenck (Universität Augsburg)
Characteristic numbers of manifold bundles over spheres and positive curvature via block bundles

16:30 Armando Cabrera Pacheco (Universität Tübingen)
Geometric problems in Machine Learning

17:00 Bernhard Stoinski (Private Institute for General Dynamic Logic)
Cook curve meets tape measure

Rigidity of positive scalar curvature under low regularity

Thomas Schick[1], Bernhard Hanke, Simone Cecchini
([1] Universität Göttingen)

The round metric on the sphere is obviously special in many ways. Among those there is the following extremality and rigidity phenomenon proved by Llarull:
If $f\colon (M,g) \to S^n$ is a smooth map of non-zero degree between the connected closed Riemannian spin manifold M and the standard sphere which is 1-Lipschitz then

- either there is a point x in M where the scalar curvature is larger than the one of the sphere (extremality)
- or the map f is a metric isometry (rigidity)

The statement is a metric statement, so it is natural to expect that the smoothness condition on f can be dropped (a question asked by Gromov).
In the talk, we will explain that this is indeed the case (f and the metric g are allow to be of low regularity), and will explain the tools used to get information on low regularity maps and Riemannian metrics.

Super J-holomorphic Curves

Enno Keßler
(Max-Planck-Institut für Mathematik in den Naturwissenschaften)

J-holomorphic curves or pseudoholomorphic curves are maps from Riemann surfaces to almost Kähler manifolds satisfying the Cauchy-Riemann equations. J-holomorphic curves are of great interest because they allow to construct invariants of symplectic manifolds and those invariants are deeply related to topological superstring theory. A crucial step towards Gromov–Witten invariants is the compactification of the moduli space of J-holomorphic curves via stable maps.
In this talk, I want to report on a supergeometric generalization of J-holomorphic curves and stable maps where the domain is a super Riemann surface. Super Riemann surfaces have first appeared in superstring theory as generalizations of Riemann surfaces with an additional anti-commutative variable. Super J-holomorphic curves are solutions to a system of partial differential equations on the underlying Riemann surface coupling the Cauchy-Riemann equation with a Dirac equation for spinors. I will explain how to construct moduli spaces of super J-holomorphic curves, their compactification via super stable maps in genus zero and hint at possible generalizations of Gromov-Witten invariants.

Para-Sasakian $\phi-$symmetric spaces

Eugenia Loiudice

We study the Boothby-Wang fibration of para-Sasakian manifolds and introduce the class of para-Sasakian ϕ-symmetric spaces, canonically fibering over para-Hermitian symmetric spaces. Using this fibration we give a method to explicitly construct simple para-Sasakian ϕ-symmetric spaces. We provide moreover an example of non-semisimple para-Sasakian ϕ-symmetric space.

Positively curved manifolds with torus symmetry

Michael Wiemeler
(Universität Münster)

I will report on recent joint work with Lee Kennard and Burkhard Wilking. In this work we compute the rational cohomology rings of simply connected positively curved manifolds with torus symmetry satisfying various conditions.

String topology and closed geodesics

Maximilian Stegemeyer
(Universität Leipzig)

String topology studies structures such as products, coproducts or related algebraic operations on the homology of the free loop space of a closed manifold. Two of the most prominent examples are the Chas-Sullivan product and the string topology coproduct. It is well-known that the closed geodesics in a Riemannian manifold are precisely the critical points of the energy functional on the free loop space of the respective manifold.
In this talk we show certain connections between properties of the closed geodesics in a manifold and the behavior of the string topology operations on its free loop space. In particular, we will see that in some situations the non-triviality of the string topology coproduct implies a lower bound on the multiplicity of certain closed geodesics. Moreover we will show that the behavior of the closed geodesics in a compact symmetric space can be used to show certain properties of the Chas-Sullivan product as well as the string topology coproduct for symmetric spaces.

Initial data rigidity via Dirac-Witten operators

Jonathan Glöckle
(Universität Regensburg)

Initial data sets (g, k) on a manifold M consist of a Riemannian metric g and a symmetric 2-tensor k. They typically arise in general relativity, when looking at a spacelike hypersurface M of a time-oriented Lorentzian manifold. In this case, g is the induced metric and k the induced second fundamental form. In this talk, we consider the situation where M is a manifold with boundary and (g, k) satisfies the dominant energy condition as well as a certain mean curvature condition along the boundary. Using Dirac-Witten operators we prove a rigidity theorem à la Eichmayr-Galloway-Mendes stating that M must be diffeomorphic to a cylinder $N \times [0, 1]$ and is foliated by MOTS carrying non-trivial parallel spinors for the induced metrics. A special case of this is a rigidity statement for Riemannian metrics of non-negative scalar curvature and with mean convex boundary.

Moduli Spaces in Riemannian Geometry

Thorsten Hertl
(Albert Ludwigs Uni Freiburg)

This talk is a small exposition to moduli spaces of Riemannian metrics satisfying initially fixed conditions. We first introduce different notions of moduli spaces, compare these notions to each other, and discuss their advantages and disadvantages. Then we turn our attention to the moduli spaces of metrics with special curvature conditions and explain how and with which methods these spaces are studied. In the final part we discuss more current results.
This introductory talk will be also accessible to Master students with specialisation in differential geometry or topology.

Harmonic maps of ellipsoids

Anna Siffert

In this talk we provide the existence of harmonic maps between ellipsoids and put this in the context of the vast theory of harmonic maps between Riemannian manifolds. This is joint work with Volker Branding.

Metric inequalities with scalar curvature via the Dirac operator

Rudolf Zeidler
(University of Münster)

In Riemannian comparison geometry, positive lower bounds on sectional and Ricci curvature have concrete consequences such as global upper bounds on volumes and distances. In contrast, the study of positive *scalar curvature* from a geometric perspective is much more subtle. However, Gromov proposed a number of conjectures which-in specific situations-predict concrete metric inequalities under lower scalar curvature bounds reminiscent of classical comparison geometry. One particular example is Gromov's *width conjecture*, which may be viewed as an analogue of Myers' theorem. It states the following:
Let M be a closed connected manifold of dimension $n-1 \neq 4$ such that M does not admit a metric of positive scalar curvature. Let g be a Riemannian metric on $V = M \times [-1,1]$ of scalar curvature bounded below by $n(n-1) = \mathrm{scal}_{\mathbb{S}^n}$. Then

$$\mathrm{width}(V,g) := \mathrm{dist}_g(\partial_- V, \partial_+ V) \leq \frac{2\pi}{n},$$

where the latter expression is the distance between the two boundary components of V with respect to g.
There are essentially two main tools to study scalar curvature. One is based on geometric measure theory and the use of minimal hypersurfaces going back to Schoen and Yau. The other is based on the Dirac operator $\mathcal{D}$ on spin manifolds and the corresponding Schrödinger-Lichnerowicz formula $\mathcal{D}^2 = \nabla^* \nabla + \frac{\mathrm{scal}}{4}$ which together with index theory (in the sense of Atiyah-Singer) yields a powerful tool to study scalar curvature.
In this talk, I will present a selection of work from the last few years treating Gromov's width conjecture and related problems in situations where the Dirac operator method is available.

Coarse sheaf cohomology

Elisa Hartmann
(Universität Bielefeld)

A certain Grothendieck topology assigned to a metric space gives rise to a sheaf cohomology theory which sees the coarse structure of the space. Already constant coefficients produce interesting cohomology groups. In degree 0 they see the number of ends of the space. In this paper a resolution of the constant sheaf via cochains is developed. It serves to be a valuable tool for computing cohomology. In addition coarse homotopy invariance of coarse cohomology with constant coefficients is established. This property can be used to compute cohomology of Riemannian manifolds. The Higson corona of a proper metric space is shown to reflect sheaves and sheaf cohomology. Thus we can use topological tools on compact Hausdorff spaces in our computations. In particular if the asymptotic dimension of a proper metric space is finite then higher cohomology groups vanish. We compute a few examples. As it turns out finite abelian groups are best suited as coefficients on finitely generated groups.

Equivariant analytic torsion and an equivariant Ruelle dynamical zeta function

Peter Hochs
(Radboud University)

Analytic torsion was introduced by Ray and Singer as a way to realise Reidemeister-Franz torsion analytically. (The equality was independently proved by Cheeger and Müller.) The Ruelle dynamical zeta function is a topological way to count closed curves of flows on compact manifolds. The Fried conjecture states that, for a suitable class of flows, the Ruelle dynamical zeta function has a well-defined value at zero, and that the absolute value of this value equals analytic torsion. With Hemanth Saratchandran, we define equivariant versions of analytic torsion and of the Ruelle dynamical zeta function, for proper actions by locally compact groups, with compact quotients. These have some natural fundamental properties, generalising properties of their non-equivariant counterparts. The resulting equivariant version of Fried's conjecture does not hold in general, but it does hold in some classes of examples. This motivates the search for general conditions under which the equivariant Fried conjecture is true.

Characteristic numbers of manifold bundles over spheres and positive curvature via block bundles

Georg Frenck
(Universität Augsburg)

In this talk, I will investigate the following question: Given a closed oriented d-manifold M, an integer $k \geq 1$ and a universal characteristic class $c \in H^{k+d}(BO;\mathbb{Q})$, does there exist a fiber bundle $M \to E \to S^k$ such that $\langle c(E), [E]\rangle \neq 0$?
This question has connections to both geometric topology, since oriented cobordism is classified by these so-called characteristic numbers, as well as Riemannian geometry, since an example of such numbers is given by the $\hat{\mathcal{A}}$-genus, which obstructs the existence of positive scalar curvature in the presence of a spin structure. Employing the classical theory of block bundles and surgery theory, it turns out that one can give a fairly complete answer to the above question, provided that k is small compared to d.

Geometric problems in Machine Learning

Armando Cabrera Pacheco
(Universität Tübingen)

Machine Learning has gained a lot of attention in the last years, mainly motivated by the success of neural networks dealing with highly complex prediction tasks. As an application-driven discipline, the progress is often is made by using empirical methods and many theoretical aspects remain unknown. In this talk, we will review some basic principles of machine learning from a mathematical point of view and reinterpret them in the context of differential geometry. In particular, we will focus on geometric properties of loss functions, which are used to measure the quality of predictions — a fundamental component of a machine learning system.

Cook curve meets tape measure

Bernhard Stoinski[1], Steffen Rust[2]
([1] Private Institute for General Dynamic Logic; [2] HAWK Göttingen)

Why a tree circumference determined from a point cloud can become infinite, and how the resulting Hausdorff dimension D can be compared with the Euclidean or topological dimension D_T
In the measurement of natural objects, such as trees, traditional measurement methods still have a high degree of importance. The use of modern methods such as LiDAR scanners provides increasingly accurate measurement values in the field of surveying. At this point, however, the high accuracy results in differences between D_T, the real measurable world, and D, the calculable world of a point cloud and the associated Hausdorff dimensions. The Hausdorff dimension of the point set resulting from scanning thereby satisfies Szpilrajn inequality:

$$D \geq D_T. \tag{1}$$

In addition to the theoretical consideration of D, the lecture will also show a practical application example. The focus here is on achieving comparability of D and D_T of tree circumferences, with the coastline paradox that occurs in the process.

S02 Discrete Mathematics

Section leaders: Raman Sanyal (FU Berlin und GU Frankfurt), Anusch Taraz (TU Hamburg), Stefan Weltge (TU München)

Session S02_01

Monday, 25 September 2023 13:30 (Parksaal 3)

13:30 Jürgen Richter-Gebert (Technical University of Munich)
The surprising flexibility of configurations

14:00 Sophie Rehberg
Poset Permutahedra

14:30 Paul-Hermann Zieschang (University of Texas)
Hypergroups, Buildings, and Twin Buildings

15:00 Alheydis Geiger (Max Planck Institute for Mathematics in the Sciences)
Positive del Pezzo Geometry

15:30 Christoph Helmberg (TU Chemnitz)
Spectral Properties of the Signless Laplacian of Threshold Graphs

Session S02_02

Tuesday, 26 September 2023 14:00 (Parksaal 2)

14:00 Michela Ascolese (Università degli Studi di Firenze)
Randomized algorithms to generate hypergraphs with given degree sequences

14:30 Francesco Di Braccio (London School of Economics)
Hamilton decompositions of regular tripartite tournaments

15:00 Yannick Mogge (Hamburg University of Technology)
Waiter-Client Games on Randomly Perturbed Graphs

15:30 Dennis Clemens (Technische Universität Hamburg)
Creating spanning trees in positional games

The surprising flexibility of configurations

Leah Wrenn Berman, Gábor Gévay, Jürgen Richter-Gebert, Sergei Tabachnikov
(University of Alaska Fairbanks, Bolyai Institute, University of Szeged, Technical University of Munich, Pennsylvania State University)

We present recently discovered relations between (n_4) incidence configurations and the classical Poncelet Porism. Poncelet's result from 1822 can be interpreted as a flexibility statement on polygons that are simultaneously inscribed into one and circumscribed around another conic. It will be demonstrated how this flexibility can be transformed to motions of (n_4)-configurations. These are incidence configurations where each point is incident to four lines and vice versa. This proves the existence of new non-trival motions for configurations that were known for over 30 years. The proof of this result touches various areas as diverse as geometry of billiards, regular arrangements of circles, discrete differential geometry, algebraic geometry, invariant theory and elliptic functions. We will give a brief overview how these areas come together in that context.
Moreover we provide explicit geometric constructions for specific values of n that allow to control the full motional freedom and we give algebraic characterisations of the underlying Poncelet Polygons. The talk will be illustrated by various interactive geometric visualisations.

Poset Permutahedra

Alexander E Black, Sophie Rehberg, Raman Sanyal

I will report on ongoing work on an interesting class of polytopes: poset permutahedra. Poset permutahedra can be defined as the intersection of the usual permutahedron with an order cone. They also have a representation as certain fiber polytopes, more precisely, as monotone path polytopes of order polytopes. This provides a powerful perspective in order to investigate their rich and interesting combinatorial, geometric and arithmetic properties: Lattice points in poset permutahedra are enumerated by certain generalizations of score sequences on posets, deformations related to submodular functions on the Birkhoff lattice of the poset, the face structure has a combinatorial interpretation in terms of the poset, and we can interpret the volume and describe interesting subdivisions.
This is joint work in progress with Alexander E. Black and Raman Sanyal.

Hypergroups, Buildings, and Twin Buildings

Paul-Hermann Zieschang
(University of Texas)

In [5], buildings (in the sense of [3]) were described as a special class of association schemes, and the association schemes in this class were called Coxeter schemes. Association schemes in general can be viewed as regular representations of hypergroups (in the sense of [2]). In [1], the hypergroups whose regular representations are Coxeter schemes are called Coxeter hypergroups, so that buildings and regular representations of Coxeter hypergroups can be regarded as equivalent mathematical objects. (From an algebraic point of view, Coxeter hypergroups can be treated in a similar way as Coxeter groups.) The present talk is a report on an attempt to find those hypergroups whose regular representations correspond to twin buildings (in the sense of [4]) in exactly the same way as the regular representations of Coxeter hypergroups correspond to buildings.

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[1] C. French, P.-H. Zieschang, Regular actions of Coxeter hypergroups, to appear in Comm. Algebra

[2] F. Marty, Sur une généralisation de la notion de groupe, in Huitième Congres des Mathématiciens, Stockholm 1934, 45-49

[3] J. Tits, Buildings of Spherical Type and Finite BN-Pairs, Springer Lecture Notes in Math. 386, Berlin Heidelberg New York (1974)

[4] J. Tits, Twin buildings and groups of Kac-Moody type, London Math. Soc. Lecture Note Ser. 165, Cambridge University Press (1992)

[5] P.-H. Zieschang, An Algebraic Approach to Association Schemes, Springer Lecture Notes in Math. 1628, Berlin Heidelberg New York (1996)

Positive del Pezzo Geometry

Alheydis Geiger
(Max Planck Institute for Mathematics in the Sciences)

Real, complex, and tropical algebraic geometry join forces in a new branch of mathematical physics called positive geometry. We develop the positive geometry of del Pezzo surfaces and their moduli spaces, viewed as very affine varieties. Their connected components are derived from polyhedral spaces with Weyl group symmetries. We study their canonical forms and scattering amplitudes, and we solve the likelihood equations. This is joint work with Nick Early, Marta Panizzut, Bernd Sturmfels and Claudia Yun.

Spectral Properties of the Signless Laplacian of Threshold Graphs

Christoph Helmberg[1], Guilherme Porto, Guilherme Torres, Vilmar Trevisan
([1] TU Chemnitz)

We show that for threshold graphs the eigenvalues of the signless Laplacian interlace with the node degrees greater or equal to the i-th smallest degree d_{i-1} and less or equal to the $i-1$-st smallest degree d_{i-1} for the unique i satisfying $d_{i-1} \leq n-i \leq d_i$. As an application, we show that the signless Brouwer conjecture holds for threshold graphs, i.e., for threshold graphs the sum of the k largest eigenvalues is bounded by the number of edges plus $k+1$ choose 2.

Randomized algorithms to generate hypergraphs with given degree sequences

Michela Ascolese[1], Matthias Lienau, Matthias Schulte, Anusch Taraz
([1] Università degli Studi di Firenze)

The question whether there is a simple k-hypergraph whose degrees are equal to a given sequence of integers is a well-known reconstruction problem in graph theory, that is motivated from discrete tomography. The problem can be solved in polynomial time in case of graphs, but becomes NP-hard for $k \geq 3$.

In our work we approach this problem by randomized algorithms and show that it is possible to generate the required hypergraphs with positive probability if the integer sequence satisfies certain constraints, improving some previous results.

This is joint work with Matthias Lienau, Matthias Schulte and Anusch Taraz.

Hamilton decompositions of regular tripartite tournaments

Francesco Di Braccio[1], Joanna Lada[1], Viresh Patel[2], Yani Pehova[1], Jozef Skokan[1]
([1] London School of Economics; [2] Queen Mary University of London)

A *regular k-partite tournament* is a regular orientation of the complete balanced k-partite graph. Kuhn and Osthus proved that if $k \geq 4$ any large enough regular k-partite tournament can be decomposed into edge-disjoint Hamilton cycles. Resolving a well-known conjecture by Jackson, Granet showed that this is also true when $k = 2$. Surprisingly, Granet showed that this is false when $k = 3$ by constructing a regular tripartite tournament in which any collection of edge-disjoint Hamilton cycles has to miss at least 3 edges. Our main result which we will talk about is a proof that regular tripartite tournaments are approximately Hamilton decomposable, i.e. that for any $\varepsilon > 0$ and n large enough, any regular tripartite tournament on n vertices contains edge-disjoint Hamilton cycles covering all but at most εn^2 edges. We also discuss a conjectured strengthening of this result according to which Granet's construction is the unique regular tripartite tournament that is not perfectly Hamilton decomposable.

Waiter-Client Games on Randomly Perturbed Graphs

Yannick Mogge
(Hamburg University of Technology)

Waiter-Client games are played on a hypergraph $(X, \mathcal{F})$, where $\mathcal{F} \subseteq 2^X$ denotes the family of winning sets. During each round, Waiter offers a predefined amount (called bias) of elements from the board X, from which Client takes one for himself while the rest go to Waiter. Waiter wins the game if she can force Client to occupy any winning set $F \in \mathcal{F}$.
We consider Waiter-Client games played on randomly perturbed graphs. These graphs consist of the union of a deterministic graph G_α on n vertices with minimum degree at least αn and the binomial random graph $G_{n,p}$. Depending on the bias we determine the order of the threshold probability for winning the Hamiltonicity game and the k-connectivity game on $G_\alpha \cup G_{n,p}$.

This is joint work with Dennis Clemens, Fabian Hamann, and Olaf Parczyk.

References

[1] Dennis Clemens, Fabian Hamann, Yannick Mogge, and Olaf Parczyk, *Maker-Breaker Games on Randomly Perturbed Graphs*, SIAM Journal on Discrete Mathematics 35(4) (2021), 2723-2748.

Creating spanning trees in positional games

Grzegorz Adamski[1], Sylwia Antoniuk[1], Małgorzata Bednarska-Bzdęga[1], Dennis Clemens[2], Fabian Hamann[2], Yannick Mogge[2]
([1] Adam Mickiewicz University; [2] Technische Universität Hamburg)

A prominent question in graph theory is the embedding of trees, ranging from the embedding a fixed tree to universality and packing problems. As such it is no surprise that positional games involving spanning trees have been studied frequently. In this walk, we will discuss recent results on such games played on the complete graph K_n in which one player aims to create a copy of a given spanning tree. The main focus will be on the largest value $\Delta = \Delta(n)$ such that the building player can win as long as the tree in question has maximum degree bounded by Δ. We will also present universality results and compare these with the random graph intuition.

S03 Mathematics in the Course of Time

Section leaders: Hans Fischer (KU Eichstätt-Ingolstadt), Achim Ilchmann (TU Ilmenau)

Session S03_01

Monday, 25 September 2023 14:00 (Großer Saal)

14:00 Renate Tobies
Felix Klein and "Ingenieurmathematik"

15:00 Waltraud Voss
"Eine Hochschule (auch) für Mathematiker ...". Zur Ausbildung von Fachnachwuchs für die Mathematik an Polytechnischer Schule, Polytechnikum und TH Dresden.

15:30 Rainer Kaenders (Rheinische Friedrich-Wilhelms-Universität Bonn)
Alfred Kempes Beweis zur Erzeugung algebraischer Kurven durch Stangenkonstruktionen nach fast 150 Jahren

Session S03_02

Tuesday, 26 September 2023 09:00 (Großer Saal)

09:00 Karlheinz Schüffler (Hochschule Niederrhein)
Die Tonleiter – Trivialität oder Problem?

10:00 Peter Ullrich (Universität Koblenz)
Christoph Gottlieb Schröter (1699–1782) und die „Nothwendigkeit der Mathematik bey gründlicher Erlernung der musikalischen Composition“ (1747)

10:30 Hans Fischer (KU Eichstätt-Ingolstadt)
Critique of Bernoulli's Law of Large Numbers during the 1920s: Keynes and von Mises

Session S03_03

Wednesday, 27 September 2023 09:30 (Großer Saal)

09:30 Alfred Holl
Der Nürnberger Rechenmeisterstreit 1551 – Inhaltliche Aspekte

10:00 Thomas Hotz (TU Ilmenau)
Oval constructions by G.B. Benedetti, 1585

10:30 Achim Ilchmann
The role of mathematics for the beauty of baroque architecture

Felix Klein and "Ingenieurmathematik"

Renate Tobies

The lecture shows how Klein succeeded in establishing technical disciplines at the University of Göttingen and in implementing his ideas about applications of mathematics. This will be demonstrated by means of protocols of some research seminars on applied mathematics organised by Klein. His new results on the theory of the top, construction statics and friction theory (Painlevé-Klein problem) will be presented.

"Eine Hochschule (auch) für Mathematiker ...". Zur Ausbildung von Fachnachwuchs für die Mathematik an Polytechnischer Schule, Polytechnikum und TH Dresden.

Waltraud Voss

Der „Entwurf der Ordnung der Prüfung für das Lehramt an Höheren Schulen im Deutschen Reich“ machte entschiedene Einsprüche der TH Dresden nötig, hatte doch bei ihm offensichtlich „die preußische Sicht“ den Blick auf die Tatsachen verstellt. Alle deutschen Technischen Hochschulen wurden darin bezüglich des Studiums der Mathematik, Physik und Chemie den Universitäten gleichgestellt, für die Geographie hingegen war diese Gleichstellung nur für die preußischen Technischen Hochschulen Aachen und Hannover vorgesehen, und das Studium der Biologie sollte ganz den Universitäten vorbehalten bleiben. Die TH Dresden mit ihrer langen Tradition auf diesen Gebieten und der – häufig genutzten – Möglichkeit des Vollstudiums und der Promotion war „vergessen“ worden. Die Korrektur erfolgte, und die am 30. Januar 1940 veröffentlichte neue „Ordnung“ bestätigte einmal mehr, dass die TH Dresden die einzige technische Hochschule in Deutschland war, an der das gesamte mathematisch-naturwissenschaftliche Fächerspektrum bis zum Abschluss studiert und mit der Promotion gekrönt werden konnte. Dem war eine lange Entwicklung vorausgegangen. Bereits im Organisationsplan der Technischen Bildungsanstalt Dresden (TBA) von 1828 war die „Zweite Abteilung“ unter anderem „zur Unterweisung solcher Schüler bestimmt, die ... als Lehrer und Beamtete einer rein wissenschaftlichen Ausbildung bedurften“. Seit 1848 (!) konnten an der TBA in Mathematik und Physik studierte Semester auf die entsprechenden Fachstudien an der Universität Leipzig angerechnet werden. 1862 wurde mit der Gründung der „Lehrerabteilung D“ neben den bisher bestehenden drei technischen Abteilungen die Ausbildung höherer Lehrer institutionalisiert. An der Spitze der Dresdner Lehrerabteilung stand zunächst der Mathematikordinarius Oskar Schlömilch – und neben ihm Professor Johann Andreas Schubert, noch heute allgemein bekannt als Konstrukteur der ersten deutschen Lokomotive „Saxonia“ und der Göltzschtalbrücke, dieses Wunderwerks der Technik. Schubert vertrat die „Angewandte Mathematik“, die damals (so im Organisationsplan von 1842 ausgewiesen) auch alle mechanisch-technischen Disziplinen erfasste. Motor der Entwicklung war seit 1873 Direktor Gustav Zeuner, der die technischen Disziplinen noch stärker als bisher theoretisch untermauern wollte und im Zusammenhang damit die mathematisch-naturwissenschaftlichen und allgemein-wissenschaftlichen Fächer in einem Maße ausbaute, das bis dahin beispiellos im deutschen technischen Hochschulwesen war. So wurde das Polytechnikum Dresden zu „einer Hochschule (auch) für Mathematiker“. Die Aussicht, auch eigenen Fachnachwuchs heranziehen und neue Strukturen mit aufbauen zu können, ließ hervorragende Mathematiker und Physiker – so Leo Königsberger und August Toepler – dem Ruf nach Dresden folgen. Auf Initiative von Leo Königsberger und Gustav Zeuner wurden 1875 am Polytechnikum Dresden das Mathematische Seminar (in der Ausprägung eines Mathematischen Instituts) und die Sektion für Reine und Angewandte Mathematik der Naturwissenschaftlichen Gesellschaft Isis zu Dresden, als einer Dresdner Mathematischen Gesellschaft, begründet. Das Studium in der Dresdner Lehrerabteilung stand theoretisch auf hohem Niveau, war aber auch stets anwendungsorientiert, so gehörten Darstellende Geometrie, Geodäsie und Technische Mechanik, die Ende des 19. Jahrhunderts dann als „Angewandte Mathematik“ allgemein zum Prüfungsgegenstand für künftige höhere Lehrer der Mathematik wurden, in Dresden von Anfang an zum Studienplan. 1879 wurde eine eigene Staatsprüfungskommission am Dresdner Polytechnikum installiert. Damit war ein wichtiger Schritt zur Rangangleichung an die Universität Leipzig getan, die dann 1912 mit der Erteilung des Promotionsrechts für die Allgemeine Abteilung, in die die Lehrerabteilung 1890 (beim Übergang des Polytechnikums zur TH Dresden) integriert worden war, vollzogen wurde. Eine Reihe namhafter Mathematiker ist aus TBA/Polytechnischer Schule/Polytechnikum/TH Dresden hervorgegangen. Später als die Dresdner „Lehrerabteilung“ entstanden ähnliche Einrichtungen an einigen Hochschulen süddeutscher Länder, 1868 in München, 1869 in Darmstadt, 1876 in Stuttgart, die jedoch nicht alle ununterbrochen fortbestanden. In Dresden wurde auch während einer ernsten „Durststrecke“ am Bestand der Lehrerabteilung festgehalten, so dass deren Tradition bis in die 1940er Jahre ungebrochen war und am längsten zurückreichte. Zu den Studenten, die die Abschlussprüfung (noch keine Staatsprüfung) der Dresdner Lehrerabteilung ablegten, gehören: 1864 Burmester, Louis Ernst Hans, später

Professor der Darstellenden Geometrie (und Kinematik) in Dresden und München, 1865 Albrecht, Theodor, später Professor und Sektionschef am Preußischen Geodätischen Institut zu Potsdam, 1870 Helm, Georg, später Professor der Mathematik und mathematischen Physik an Polytechnikum/TH Dresden. Unter den ersten, die vor der Staatsprüfungskommission des Polytechnikums Dresden die Prüfung für das höhere Schulamt ablegten, waren: Plunder, Paul, später Regierungsrat am Patentamt zu Berlin; Freyberg, Johannes, Privatdozent für Physik am Polytechnikum Dresden, dann Professor in Dortmund; Hennig, Richard, Assistent am Polytechnikum Dresden, dann Professor an der TH Riga; Heymann, Carl Woldemar, Professor an der Gewerbeakademie Chemnitz; Witting, Alexander, Professor am Gymnasium zum Heiligen Kreuz in Dresden, nebenamtlich langjähriger Assistent für Darstellende Geometrie an Polytechnikum/TH Dresden. Die TH Dresden ist diejenige unter den deutschen technische Hochschulen, die die meisten frühen Promovenden der Mathematik hervorgebracht hat, und an der bis 1945 auch die meisten Frauen, nämlich sieben, in Mathematik promoviert wurden. Darunter ist mit Johanna Wiegandt die erste Frau überhaupt, die an einer TH Deutschlands aufgrund einer mathematischen Doktorarbeit promoviert wurde. (Diese Aussagen gehen aus Untersuchungen von Renate Tobies hervor.) Von den Mathematikprofessoren, die an der TH Dresden vor 1945 den Doktorgrad erwarben, seien nur die bei vielen Mathematikern auch heute bekannten genannt: Alwin Walther (Darmstadt), Herbert Seifert (Heidelberg), Alfred Kneschke (Freiberg).

Alfred Kempes Beweis zur Erzeugung algebraischer Kurven durch Stangenkonstruktionen nach fast 150 Jahren

Rainer Kaenders
(Rheinische Friedrich-Wilhelms-Universität Bonn)

Im 17. Jahrhundert vertrat Descartes die Auffassung, dass Stangenkonstruktionen Verallgemeinerungen von Konstruktionen mit Zirkel und Lineal seien, und im 19. Jahrhundert fragte man sich, welche Kurven sich mit Stangenkonstruktionen überhaupt zeichnen ließen. Es stellte sich heraus, dass selbst das Zeichnen einer geraden Linie eine Herausforderung darstellt. Sir Alfred Bray Kempe (1849-1922) bewies schließlich, dass jeder Ausschnitt einer reellen algebraischen Kurve durch eine Stangenkonstruktion gezeichnet werden kann. Schon für eine Parabel ist seine Konstruktion anspruchsvoll, wie wir im Vortrag sehen werden, der auch den allgemeinen Beweis darstellt.

Die Tonleiter – Trivialität oder Problem?

Karlheinz Schüffler
(Hochschule Niederrhein)

Die Tonleiter – Trivialität oder Problem? Mathematischer Weg von der „Harmonia perfecta maxima“ der Antike bis hin zur Temperierungstheorie der modernen Musikwissenschaft – ein Überblick.
Abstract: Der Zusammenhang zwischen Musik und Mathematik zeigt sich nirgendwo deutlicher als in der Architektur der Tonbeziehungen (also der musikalischen Intervalle) und deren Aufbau zu musikalischen Grundstrukturen wie Skalen, ihren Subskalen und Akkorden. Hierbei können aber auch die – a priori als nicht mess- oder definierbar geltenden – ästhetisch-musikalischen Aspekte (Reinheit, Dis- und Konsonanz, Tonarten-Charakteristiken u.v.m.) auf profunde Weise mit einfließen. Wir stellen zunächst die pythagoräisch-babylonische musikalische Welt der „Harmonia maxima perfecta“ in Gestalt ihrer berühmten symmetrischen Medietäten-Proportionenkette $6 : 8 : 9 : 12$ vor, welche über vielfältige Verallgemeinerungen hinaus und nicht zuletzt dank der Hyperbel des Archytas den Kosmos antiker rational-harmonischer Intervalle generiert. Von essentieller Bedeutung – sowohl im antiken als auch im modernen Sinn – ist zudem das Kriterium einer Kommensurabilität oder einer Nicht-Kommensurabilität für musikalische Intervalle. Dieses ist aufs engste mit der Primzahlarithmetik verbunden, und es entscheidet über Endlichkeiten, Periodizitäten und Lösbarkeiten für musikalische Grundaufgaben. Als Anwendung zeigen wir einen modernen Aspekt der Iterations-Dynamik für pythagoräische und andere Intervall-Algebren: Das Theorem von Levy-Poincaré beschreibt speziell die Topologie der Tonverteilungen aller durch reine Quinten und Oktaven erzeugten Intervalle und allgemeiner diejenige einer Iterationsalgebra multipler Erzeuger-Intervalle. Die Temperierungstheorie entwickelt sich nun bereits in der Renaissance (aus den musikalisch geforderten „Mittelton-Skalen“) und erreicht im Bachzeitalter und in der sich anschließenden Klassik ihre nachhaltigsten Ausprägungen, die sich (vor allem in der Orgelwissenschaft) als essentielles Teilgebiet der Musiktheorie etabliert haben. Wir skizzieren die beiden Hauptmethoden der Temperierung – also der Aufgabe, eine chromatische oder heptatonische „Tonleiter“ bei vorgegebenen musikalischen Anforderungen zu konstruieren – nämlich die Quinten-Iterationen mit ihren einfachen oder multiplen Wolfsquinten-Modellen und die Euler´schen Tongitter-Auswahlsysteme mit ihren Mikro-Tonalitäten („Kommata“). Wir zeigen hierbei auch simultan, wie vor allem die ganzzahlige Lineare Algebra als ein profunder Partner musikalischer Prozesse die moderne Mathematik mit der antiken Musik verbindet. Schließlich stellen wir exemplarisch einige signifikante Temperierungssysteme vor. (Der Vortrag enthält eventuell demonstrierende Hörbeispiele).

Christoph Gottlieb Schröter (1699–1782) und die „Nothwendigkeit der Mathematik bey gründlicher Erlernung der musikalischen Composition“ (1747)

Peter Ullrich
(Universität Koblenz)

Schröter verbrachte seine Jugend in Hohnstein, Dresden, Leipzig und Jena. 1726 ging er als Organist nach Minden und 1732 nach Nordhausen, wo er bis zu seinem Tode blieb. Auch war er als Komponist und Musiktheoretiker tätig und entwickelte vor Gottfried Silbermann eine Hammermechanik für das Pianoforte. In seiner im Vortragstitel genannten Schrift von 1747 behandelte er erfolgreich die gleichstufige Stimmung mit einer mathematischen Methode, die die Potenziteration für Matrizen vorwegnimmt.

Critique of Bernoulli's Law of Large Numbers during the 1920s: Keynes and von Mises

Hans Fischer
(KU Eichstätt-Ingolstadt)

In the period from about 1920 to 1930, there were two interpretations of the notion of probability: That of probabilities as „degrees of rational belief“ according to Keynes (also called „logical probabilities“), which included the so-called Laplace probabilities, and the frequentist interpretation in von Mises' succession. In both interpretations it was argued that Bernoulli's law, and even more general laws of large numbers provide only a limited gain in knowledge. The reasons for this criticism, however, were very different. While von Mises at least granted laws of large numbers a significant role with respect to the mode of convergence of relative frequencies to the corresponding probabilities, Keynes even stated about Bernoulli's law that it would hardly entail „logical insight“.

Der Nürnberger Rechenmeisterstreit 1551 – Inhaltliche Aspekte

Alfred Holl

Rechenmeister der frühen Neuzeit gerieten aus den verschiedensten Gründen aneinander – sogar mit gerichtlichen Nachspielen. Die Konkurrenz war groß, und die Städte konnten sich die besten für feste Anstellungen aussuchen. Rein inhaltliche Auseinandersetzungen zwischen Rechenmeistern kamen wohl eher selten vor. Ein gut dokumentiertes Beispiel ist der Nürnberger Rechenmeisterstreit von 1551 zwischen Johann Neudörffer (1497–1563) auf der einen Seite und dem Berufseinsteiger Niklas Werner (1520–1570) sowie seinem älteren Hintermann Hieronymus Rosa (+ 1562) auf der anderen. Wenige Jahre später, Ende 1557, stellte der Nürnberger Rechenmeister und Neudörffer-Schüler Wolff Michel (+ 1568) in seinem 240-seitigen handschriftlichen Tractetlein (Stadtbibliothek Nürnberg Nor. H 323) die sich mathematisch widersprechenden Methoden der Widersacher bei der Lösung spezieller wirtschafts-mathematischer Probleme gegenüber. Den Gegnern von Neudörffer fehlte das nötige kaufmännische Wissen, was zum Teil in sehr amüsanten Methoden mündete. In meinem Beitrag möchte ich Beispiele aus den Bereichen Warentransport, Zins, Zinseszins und Rückdiskontierung besprechen.

Oval constructions by G.B. Benedetti, 1585

Thomas Hotz, Achim Ilchmann
(TU Ilmenau)

Ovals play an important rôle in architecture and the fine arts. Starting with S. Serlio in 1545, constructions of ovals with specific, fixed proportions had been described at the beginning, while the earliest general construction mentioned in the literature on ovals appears to be the one published by A. Bosse in 1655. However, Giovanni Battista Benedetti's collection *Diversarum speculationum mathematicarum et physicarum liber*, already published in 1585, contains a letter in which he presents two general constructions for ovals given their axes, along with proofs of their correctness. In the talk, we explain these constructions as well as their significance. Moreover, we offer possible explanations on how Benedetti found these as well as why they apparently have remained unnoticed by the community until now.

The role of mathematics for the beauty of baroque architecture

Achim Ilchmann

The following problems will be treated: What is an oval, how is it constructed, which role does it play in baroque architecture?
Why is the ellipse a symbol for the mechanistic world view? How are ellipse and oval related?
Which role does the oval play in the design of protestant churches and baroque libraries? As an example we consider the Gotteshilfkirche Waltershausen and the library in Wolfenbüttel.

S04 Teaching Mathematics at tertiary educational institutions and elsewhere

Section leaders: Walther Paravicini (Universität Tübingen), Thomas Skill (HS Bochum)

Session S04_01

Tuesday, 26 September 2023 14:00 (Balkonzimmer)

14:00 Anja Panse[1]; Frank Feudel[2] ([1] Universität Paderborn; [2] Humboldt-Universität zu Berlin)
Guided notes – A means for fostering students' engagement in mathematics lectures without changing the whole mode of teaching

14:30 Alexander Zimmermann (Private Pädagogische Hochschule Burgenland)
Lack of Clarity in the Presentation of important Proofs in Academic Mathematics Textbooks - a Logical and Definitional Case Analysis

15:00 Stephan Scholz (Hochschule Ravensburg-Weingarten)
Electrical Circuit Models for Courses on Simulation and Control of Differential Equations: From Transfer Functions to PDEs

15:30 Steven Jones (Brigham Young University)
Promoting connections between learning calculus in math classes and using calculus in science, engineering, and other disciplines

Guided notes – A means for fostering students' engagement in mathematics lectures without changing the whole mode of teaching

Anja Panse[1], Frank Feudel[2]
([1] Universität Paderborn; [2] Humboldt-Universität zu Berlin)

In traditional mathematics lectures, students often have problems to process the information presented, because they are busy copying the instructor's writings from the blackboard. At worst, they take on a rather passive role just noting down the signs mechanically instead of following the instructor's explanations. One approach to address this problem without changing the whole mode of teaching as in a flipped classroom might be the use of guided notes. These are preprinted lecture notes with blanks at certain positions that the students are required to fill in as the lecture progresses. Such notes provide several possibilities for guiding students' behavior during the lecture with respect to their note-taking and their attention. In particular, the use of guided notes gives instructors room for several activities that can foster students' active engagement with the content during the lecture.
In the talk, we want to present instructors' possibilities for influencing students' work in or for mathematics lectures by means of guided notes, and especially illustrate these with specific examples from a first-semester course named „Introduction to Mathematical Thinking and Working" at the University of Paderborn. In addition, we want to present some results of an accompanying research project in which we investigated how guided notes can support students in the important activity of note-taking. Finally, we want to give a short outlook on a research project investigating at which positions of guided notes students wish to have the blanks.

Lack of Clarity in the Presentation of important Proofs in Academic Mathematics Textbooks - a Logical and Definitional Case Analysis

Alexander Zimmermann
(Private Pädagogische Hochschule Burgenland)

Empirical research is still showing high dropout rates in mathematics and mathematics-related higher education (Heublein, Hutzsch & Schmelzer 2022, Lumpe 2019). According to Lumpe, more than half of first-year students abandon mathematical studies (Lumpe 2019, 178). The transition from school mathematics to academic mathematics seems to cause considerable difficulties for many students, thus affecting not only students of the subject but also trainee teachers. Special programmes have already been developed and introduced, addressing the high dropout rates in the STEM subjects concerned, such as the MINTFIT programme at Hamburg universities and colleges and the plusMINT orientation study at the University of Kassel. Bridge courses and similar are important measures to facilitate the transition from school-based to academic mathematics.

In this talk I will look at an aspect that has not yet received as much attention in university mathematics teaching as it deserves, given its enormous potential. A significant reason, perhaps the main reason, for many difficulties in the introductory phase of studies is that in university mathematics (deductive) proofs are almost in the foreground, while pure arithmetic is relegated to secondary status. In school mathematics, on the other hand, the emphasis is on the latter, while the former plays at best a minor role.
By using a standard proof of a basic mathematical theorem borrowed from analysis textbooks, logical flaws (in presentation) are shown by means of logical and definitional analysis. The proof is analysed step by step, in order to identify leaps, gaps and hidden premises. In view of the claim to prove a theorem in the mathematical sense, i.e. to proceed in a purely deductive way, these shortcomings weigh heavily. These deficiencies turn out to be so serious that the proof given does not prove the theorem, but actually proves a different theorem. This is, to put it mildly, highly problematic from the point of view of higher education didactics. First-year students are particularly affected, since at the beginning of their studies they are confronted not only with new, complex and abstract content, but also - to mention only two other aspects - with new methods, especially proof methods, and a new (semi-)formal language. It is therefore not surprising that the study of mathematics, especially at the beginning, is characterised more by repetition and acceptance than by real understanding and recognition. In any case, this seems to make the entry into both subject and teacher training in mathematics and other STEM subjects unnecessarily difficult. Reason enough to take a closer look at this problem.

References:

Heublein, U., Hutzsch, C., & Schmelzer, R. 2022. *Die Entwicklung der Studienabbruchquoten in Deutschland*. (DZHW-Brief 05|2022). Hannover. https://doi.org/10.34878/2022.05.dzhw_brief

Lumpe, M. 2019. Studienabbruch in den MINT-F chern. Fallstudien an der Universit t Potsdam und m gliche Folgerungen. In: W. Schubarth, S. Mauermeister, F. Schulze-Reichelt & A. Seidel (Eds.): *Alles auf Anfang! Befunde und Perspektiven zum Studieneingang*. Potsdam, p. 177-192.

Electrical Circuit Models for Courses on Simulation and Control of Differential Equations: From Transfer Functions to PDEs

Stephan Scholz, Lothar Berger
(Hochschule Ravensburg-Weingarten)

Electrical circuits consisting of resistors, conductors and inductors can be modeled as simple linear dynamical systems, and so they are common examples in basic control engineering lectures. Intermediate bachelor students in "STEM" degree progams are usually familiar with the modeling of electrical circuits because the fundamental Kirchhoff laws are taught in school or in the beginning of the study program. Therefore, electrical circuits are excellent examples to study the modeling, identification, simulation and control of technical systems in lectures and projects. Furthermore, they can be easily extended step-by-step to describe more complex dynamics, and they can be realized cheaply as hardware with a microcontroller. In this presentation, we introduce the modeling, simulation and control of electrical circuits and discuss their application in courses related to differential equations.

In particular, we explain the stepwise extension of electrical circuits from small to large-scale systems and highlight significant properties like diffusive, oscillatory and time-delayed behavior. Lastly, we present examples from finished and on-going projects and ideas for future case studies.

Promoting connections between learning calculus in math classes and using calculus in science, engineering, and other disciplines

Steven Jones
(Brigham Young University)

It is becoming increasingly clear that there are major disconnects between students' learning of mathematics in math classes and the usage of those same ideas in science, engineering, or other disciplines. Even students who get good grades in math can struggle to know how to use those math concepts outside of their math classes. In this talk I will focus specifically on central calculus concepts as examples to discuss these disconnects: functions/graphs, derivatives, concavity, and integrals. I will explore the meanings and understandings commonly developed within math classes, and compare that to the meanings and understandings often held for these same concepts in science, engineering, or other real-world contexts. I will then discuss how we might address some of these disconnects in our calculus teaching.

S05 Mathematical Logic

Section leaders: Philipp Luecke (Barcelona), Nadja Valentin (Düsseldorf)

Session S05_01

Wednesday, 27 September 2023 09:00 (Parksaal 2)

09:00 Anton Freund (Universität Würzburg)
On the logical strength of better quasi orders

10:00 Juan Aguilera (TU Wien)
Scattered Goedel Logics: Computability and Model Theory

Session S05_02

Wednesday, 27 September 2023 16:45 (VP-Raum)

16:45 Stefan Hoffelner (WWU Münster)
Forcing Separation, Reduction and Uniformization

17:45 Jonathan Osinski (University of Hamburg)
The Large Cardinal Strength of Upwards LST Numbers

Session S05_03

Thursday, 28 September 2023 13:00 (Parksaal 3)

13:00 Manuel Bodirsky (TU Dresden)
Model-theoretic Challenges in Constraint Satisfaction

14:00 Tobias Kaiser (University of Passau)
Periods, Power Series, and Integrated Algebraic Numbers

Session S05_04

Thursday, 28 September 2023 15:30 (Parksaal 3)

15:30 David Bradley-Williams (Czech Academy of Sciences, Prague)
Limits of betweenness relations and their automorphism groups

16:30 Alexander Zimmermann (Private Pädagogische Hochschule Burgenland)
Zum Zusammenhang zwischen dem Komprehensionsprinzip und der Substitution in der Prädikatenlogik zweiter Stufe

On the logical strength of better quasi orders

Anton Freund
(Universität Würzburg)

The well quasi orders are characterized by the condition that each infinite sequence $x_0, x_1, \ldots$ admits $i < j$ with $x_i \leq x_j$. They have good closure properties under finitary but not under infinitary constructions. To secure the latter, Nash-Williams has introduced the more restrictive notion of better quasi order, which then played a crucial role in Laver's proof of Fraïssé's conjecture (that the countable linear orders are well quasi ordered by embeddability). This talk recalls the definition of better quasi order and presents recent results that gauge its logical strength in the framework of reverse mathematics. In particular, it will be shown that infinite iterations of the Turing jump can be obtained from the assumption that the antichain with three elements is a better quasi order, while Marcone has proved that the antichain with two elements does not lead beyond elementary combinatorics.

Scattered Goedel Logics: Computability and Model Theory

Juan Aguilera
(TU Wien)

I will survey some results concerning model-theoretic and computability-theoretic aspects of Gödel logics. Gödel logics were introduced by Gödel in his proof that intuitionistic logic is not finite-valued. They are extensions of intuitionistic logic which take truth values in some closed subset of the unit interval. In order to do this, implication must be made discontinuous. Particularly interesting are Gödel logics whose set of truth values is scattered, and in particular wellordered or converse-wellordered. We will discuss these logics and survey results related to complexity, expressibility, definability, and discuss various ordinals and cardinals associated to these logics.

Forcing Separation, Reduction and Uniformization

Stefan Hoffelner
(WWU Münster)

The Separation property, the Reduction property and the Uniformization property, introduced in the 1920's and 1930's are three classical regularity properties of pointclasses on the reals. The celebrated results of Y. Moschovakis on the one hand and D. Martin, J. Steel and H. Woodin on the other, yield a global description of the behaviour of these regularity properties for projective pointclasses under the assumption of large cardinals. In particular, under PD, for every natural number n, Π^1_{2n+1}-sets and hence Σ^1_{2n+2}-sets do have the Uniformization property (and therefore the weaker Reduction property and the Separation property for the dual pointclass).
Yet the question of universes which display an alternative behaviour of theses regularity properties has remained a complete mystery, mostly due to the absence of forcing techniques to produce such models. Indeed, even the question of the forceability of a universe where the Σ^1_3-Separation property holds was a well-known open problem since 1968.
In my talk, I want to outline some recently obtained techniques, which turn the question of a universe with, say, the Π^1_3-Reduction property into a fixed point problem for certain sets of forcing notions. This fixed point problem can be solved, yielding a specific set of forcing notions which in turn can be used to force the Π^1_n-Reduction property or, with more complicated techniques, the Π^1_n-Uniformization property (for $n > 2$) over fine structural inner models with large cardinals (for n=3, the inner model is just L). For even n, these universes outright contradict the PD-induced pattern, for odd n these universes give new lower bounds in terms of consistency strength.

The Large Cardinal Strength of Upwards LST Numbers

Victoria Gitman[1], Jonathan Osinski[2]
([1] CUNY Graduate Center; [2] University of Hamburg)

Löwenheim-Skolem-Tarski (LST) numbers of strong logics describe analogues of the downward Löwenheim-Skolem Theorem for extensions of first-order logic. Ever since Magidor showed that the LST number of second-order logic is the first supercompact cardinal (see [2]) there has been a rich history of studying connections of large cardinals to LST numbers of several logics.
Motivated by this, Galeotti, Khomskii and Väänänen introduced the notion of *upward Löwenheim-Skolem-Tarski (ULST) numbers* of strong logics which similarly describe analogues of the upward Löwenheim-Skolem Theorem (see [1]). They were able to show that the ULST number of second-order logic is at least n-extendible for any natural number n and conjectured that it is precisely the first extendible.
In this talk we present further results on the large cardinal strength of the ULST-numbers of several logics including second-order logic, sort logic, the well-foundedness quantifier, the equicardinality quantifier and infinitary logics $\mathcal{L}_{\kappa\lambda}$. In particular, we confirm the conjecture by Galeotti, Khomskii and Väänänen. Note that there are two additional independent unpublished proofs of the conjecture by Boney and Osinski and by Hayut.
This is joint work with Victoria Gitman.

References

[1] L. Galeotti, Y. Khomskii and J. Väänänen. Bounded Symbiosis and Upwards Reflection. Preprint, 2021.

[2] M. Magidor. On the role of supercompact and extendible cardinals in logic. *Israel J. Math.*, 28(1-2), 1977, pp. 1-33.

Model-theoretic Challenges in Constraint Satisfaction

Manuel Bodirsky
(TU Dresden)

Homogeneous structures and their reducts can be used to model many computational problems from finite model theory as constraint satisfaction problems (CSPs). In this talk I will give a survey on open model-theoretic problems for such structures that are relevant for obtaining complexity classification results for the corresponding CSPs. In particular, I will discuss finite Ramsey expansions and Thomas' closed supergroup conjecture.

Periods, Power Series, and Integrated Algebraic Numbers

Tobias Kaiser
(University of Passau)

Abstract. Periods are defined as integrals of semialgebraic functions defined over the rationals. Periods form a countable ring not much is known about. Examples are given by taking the antiderivative of a power series which is algebraic over the polynomial ring over the rationals and evaluate it at a rational number. We follow this path and close these algebraic power series under taking iterated antiderivatives and nearby algebraic and geometric operations. We obtain a system of rings of power series whose coefficients form a countable real closed field. Using techniques from o-minimality we are able to show that every period belongs to this field. In the setting of o-minimality we define exponential integrated algebraic numbers and show that exponential periods and the Euler constant are exponential integrated algebraic number. Hence they are a good candiate for a natural number system extending the period ring and containing important mathematical constants.

Limits of betweenness relations and their automorphism groups

David Bradley-Williams [1], John K Truss [2]
([1] Czech Academy of Sciences, Prague; [2] University of Leeds)

Some permutation groups are best represented/constructed as kinds-of-products of or limits of simpler permutation groups. We discuss joint work with John K. Truss (Leeds) in which we constructed a new infinite family of structures called "limits of betweenness relations" as a kind of tree-limit of trees (for appropriate combinatorial meanings of the word "tree"). A crucial part of the construction is an particular instance of categorical generalizations of the Fraïssé construction from Model Theory (which will be sketched in the talk). Further we plan to say how their automorphism groups fit into the landscape of infinite primitive Jordan permutation groups and the structure theory of Jordan groups established by S. Adeleke, D. Macpherson, and P. M. Neumann.

Zum Zusammenhang zwischen dem Komprehensionsprinzip und der Substitution in der Prädikatenlogik zweiter Stufe

Alexander Zimmermann
(Private Pädagogische Hochschule Burgenland)

Auf den ersten Blick ist die Sprache der Mengenlehre enorm leistungsfähig und zugleich äußerst einfach, enthält sie doch bloß eine einzige nichtlogische Konstante, nämlich das Elementschaftszeichen ›$\in$‹ zur Bezeichnung der zweistelligen Elementschaftsrelation. Da die Mengenlehre jedoch i. a. als formale Theorie aufgebaut wird, wird die Frage nach der Bedeutung von ›$\in$‹ in aller Regel völlig vernachlässigt. Dies aber ist hochproblematisch, lässt die mengentheoretische Sprache doch Ausdrücke wie ›$a \in a$‹ und sowohl ›$a \in b$‹ als auch ›$b \in a$‹ zu. Im Laufe der Zeit wurden verschiedene Wege zur Lösung dieses Problems beschritten, etwa durch Erweiterung der Axiomenmenge um das Fundierungsaxiom. Allerdings wird dabei das Problem von der Ebene der Sprache auf die der Beweisbarkeit gehoben. Letztlich zeigt sich, dass keiner der bisherigen Lösungswege frei von sowohl philosophischen als auch logischen Problemen ist.
Wie die obigen Beispiele zeigen, wird bei der Sprache der Mengenlehre die Unterscheidung zwischen Subjekt und Prädikat preisgegeben, also jene Unterscheidung, in deren Rahmen wir denken. So gelangt man unweigerlich zur Frage, ob es eine formale Theorie gibt, die dem entspricht, was man in der Mengenlehre auszudrücken vermag, jedoch unter Wahrung der Unterscheidung zwischen Subjekt und Prädikat. Hierfür bietet sich sofort die Prädikatenlogik zweiter Stufe an, deren Sprache die Subjekt-Prädikat-Struktur hat und nicht auf die mengentheoretische Elementschaftsrelation Bezug nimmt.

Im Vortrag wird exemplarisch untersucht, wie in der Prädikatenlogik zweiter Stufe wichtige mengentheoretische Theoreme formuliert werden können und welche Probleme dabei auftreten. Dies wird anhand eines grundlegenden mengentheoretischen Satzes, nämlich des Cantorschen Theorems, gezeigt. Dabei werden insbesondere auch philosophische Aspekte berücksichtigt.
Zunächst stellt sich die Frage, wie das Pendant des Cantorschen Theorems in der Prädikatenlogik zweiter Stufe aussieht. Anschließend wird geprüft, ob dieses Pendant im Rahmen der Prädikatenlogik zweiter Stufe sowohl syntaktisch ableitbar als auch semantisch allgemeingültig ist. Dazu ist es nötig, auf Basis der eingeführten Sprache der Prädikatenlogik zweiter Stufe sowohl ein syntaktisches als auch ein semantisches System der Prädikatenlogik zweiter Stufe zu entwickeln. Um nicht von den wesentlichen Betrachtungen abzulenken, werden hierfür zwar hinreichend ausdrucksstarke, aber möglichst einfache Systeme definiert.
Als besonders wichtig und zugleich äußerst interessant erweisen sich zweierlei Fragen: Erstens die Frage nach der Stellung des Komprehensionsprinzips im Rahmen der Prädikatenlogik zweiter Stufe und zweitens die nach dem Zusammenhang zwischen diesem Prinzip und der Formulierung der spezifischen Substitutionsregel für die Prädikatenlogik zweiter Stufe. Dazu werden unterschiedliche Substitutionsregeln diskutiert.
Es wird betont, dass sämtliche Untersuchungen unter dem besonderen Gesichtspunkt der formalen Logik und modernen Definitionstheorie erfolgen.

S07 Probability, Statistics, and Financial Mathematics

Section leaders: Hajo Holzmann (Marburg), Anja Sturm (Göttingen)

Session S07_01

Tuesday, 26 September 2023 14:00 (Parksaal 3)

14:00 Moritz Wemheuer (Georg-August-Universität Göttingen)
A Central Limit Theorems for Functions on Sparse Weighted Inhomogeneous Random Graphs

14:30 Max Berger (Philipps-Universität Marburg)
From dense to sparse design: Optimal rates under the supremum norm for estimating the mean function in functional data analysis

15:00 Martin Wendler (Otto-von-Guericke Universität Magdeburg)
Bootstrap for change-point detection in Hilbert-space-valued time series and random fields

15:30 Felix Konrad (Uni Bonn)
Regularly Varying Stochastic Processes

A Central Limit Theorems for Functions on Sparse Weighted Inhomogeneous Random Graphs

Moritz Wemheuer
(Georg-August-Universität Göttingen)

We prove a central limit theorem for functions on edge-and-vertex-weighted sparse inhomogeneous random graphs with rank-one kernels that satisfy a *good local approximation* property. This good local approximation property is satisfied by a number of combinatorial optimisation problems that admit a recursive formulation on the underlying graph.
The result extends recent work by Cao (2021, Ann. Appl. Probab. 31) for edge-weighted Erdős-Rényi random graphs and is based on a „perturbative“ version of Stein's method as well as a careful analysis of the local structure of the inhomogeneous random graph - specifically an explicit coupling to a mixed Poisson branching process.
This is joint work with Anja Sturm (University of Göttingen).

From dense to sparse design: Optimal rates under the supremum norm for estimating the mean function in functional data analysis

Max Berger, Hajo Holzmann, Philipp Hermann
(Philipps-Universität Marburg)

In the setting of functional data analysis, we derive optimal rates of convergence in the supremum norm for estimating the Hölder-smooth mean function of a stochastic processes which is repeatedly and discretely observed at fixed, multivariate, synchronous design points and with additional errors. Similarly to the rates in L_2 obtained by Cai, T. T. and M. Yuan (2011) in *Optimal estimation of the mean function based on discretely sampled functional data: Phase transition*, for sparse design a discretization term dominates, while in the dense case the $\sqrt{n}$ rate can be achieved as if the n processes were continuously observed without errors. However, our analysis differs in several respects from the one by Cai and Yuan. First, we do not assume that the paths of the processes are as smooth as the mean, but still obtain the $\sqrt{n}$ rate of convergence without additional logarithmic factors in the dense setting. Second, we show that in the supremum norm, there is an intermediate regime between the sparse and dense cases dominated by the contribution of the observation errors. Third, and in contrast to the analysis in L_2, interpolation estimators turn out to be sub-optimal in L_∞ in the dense

setting, which explains their poor empirical performance. We also obtain a central limit theorem in the supremum norm and discuss the selection of the bandwidth. Simulations and real data applications illustrate the results.

Bootstrap for change-point detection in Hilbert-space-valued time series and random fields

Martin Wendler[1], Lea Wegner
([1] Otto-von-Guericke Universität Magdeburg)

Our aim is to construct a test for the hypothesis of stationarity against the alternative of a location shift in a sequence or fields of dependent, Hilbert-space-valued random variables. We will also consider robust tests, generalizing the Wilcoxon-Mann-Whitney 2-sampe U-statistics to functional data. Since this class of test statistics does not rely on dimension reduction, the limit distribution provides an infinite-dimensional covariance operator as a parameter, which is difficult to estimate. Because of this, we will discuss how the dependent wild bootstrap can be adapted to random fields and to U-statistics with values in a Hilbert-space.

Regularly Varying Stochastic Processes

Felix Konrad
(Uni Bonn)

The concept of regular variation arises in Extreme Value Theory when classifying the domain of attraction of the Fréchet distribution. In this talk I will present some of the recent developements of the study of regularly varying stochastic processes. There the so-called Tail Measure allows us to describe both the spatial as well as the temporal dependence structure of a multivariate stochastic process X and allows us to derive limit theorems, e.g. for the running maximum, of X. One key ingredient of the study of Tail Measures is the so-called Tail Process, introduced by Basrak & Segers (2009), which is a stochastic process that describes the behaviour of X after an extreme value occuring at time $t = 0$. I will show how the Tail Process alone suffices to reconstruct the Tail Measure, given that the Tail Measure is shift-invariant. It will also be shown how to determine the Tail Process certain types of Markov Processes.
The talk is based on the recent work of Soulier (Extremes, 2022) as well as my bachelor thesis.

S08 Inverse Problems

Section leaders: Bernadette Hahn (Stuttgart), Bastian von Harrach-Samet (Frankfurt am Main), Tobias Kluth (Bremen)

Session S08_01

Wednesday, 27 September 2023 16:45 (Balkonzimmer)

16:45 Christian Gerhards (TU Bergakademie Freiberg)
Inverse Magnetization Problems in Paleomagnetism

17:15 Tilo Arens (Karlsruhe Institute of Technology (KIT))
Monotonicity-based shape reconstruction in a waveguide

17:45 Long Nguyen-Tuan (Bauhaus-Universität Weimar)
Towards Smart Structures - Structural Health Monitoring using Inverse Analysis

18:15 David Erzmann (University of Bremen)
Accelerating 3D Topology Optimization through Sample-Efficient Deep Learning

Session S08_02

Thursday, 28 September 2023 15:30 (Parksaal 2)

15:30 Thorsten Hohage (Georg-August-Universität Göttingen)
passive imaging in helioseismology: uniqueness results and efficient iterative algorithms for correlation data

16:00 Philipp Mickan (Georg-August University of Göttingen)
Logarithmic Stability and Instability Estimates for Random Inverse Source Problems

16:30 Michael Quellmalz (TU Berlin)
Motion detection in diffraction tomography

17:00 Tobias Kluth (University of Bremen)
Solving linear inverse problems with invertible residual networks

Inverse Magnetization Problems in Paleomagnetism

Christian Gerhards
(TU Bergakademie Freiberg)

Igneous rocks like lava and meteorites are capable of capturing and retaining information on the intensity and direction of ambient magnetic fields through thermoremanent magnetization. Thus, they can provide information on the evolution of planetary magnetic fields and past events influencing it. Data is typically available only for the corresponding magnetic field (at planetary scale via satellite bound vector magnetometers, at laboratory scale via high resolution scanning magnetic microscopes), not about the underlying magnetization of interest. This is a fairly well-studied inverse potential field problem that suffers from the typical non-uniqueness and instability. However, the instrumental setup and geophysical circumstances often allow reasonable additional assumptions that ameliorate some of these aspects. Here, we will provide a recent overview on some potential applications and particularities in paleomagnetism.

Monotonicity-based shape reconstruction in a waveguide

Tilo Arens[1], Roland Griesmaier[1], Ruming Zhang[2]
([1] Karlsruhe Institute of Technology (KIT); [2] Technische Universität Berlin)

Consider a closed cylindrical waveguide in which a time-harmonic acoustic wave propagates and is scattered by a compactly supported penetrable scatterer. Such a problem is described by the Helmholtz equation with Neumann boundary conditions on the waveguide boundary and the condition that the scattered field can be written as a modal expansion consisting only of waveguide modes propagating away from the scatterer. The incident fields are represented as conjugate single layer potentials defined on verticle surfaces outside the domain of interest containing the scatterer. With this problem we associate a near field operator that maps the density of the incident field potential to values of the scattered field measured on the same surfaces.
We develop novel monotonicity relations for the eigenvalues of this associated near field operator, and we use them to establish linearized monotonicity tests that characterize the support of the scatterer in terms of the near field observations of the scattered field. In order to prove these characterizations, the existence of loocalized wave functions, which are solutions to the scattering problem in the waveguide that have arbitrarily large norm in some prescribed region, while at the same time having arbitrarily small norm in some other prescribed region, needs to be established. As a byproduct we obtain a uniqueness result for the inverse medium scattering problem in the waveguide with a simple proof.
Some numerical examples are presented to document the potentials and limitations of this approach.

Towards Smart Structures - Structural Health Monitoring using Inverse Analysis

Long Nguyen-Tuan , Tom Lahmer
(Bauhaus-Universität Weimar)

In the concept of new intelligent concrete 2.0, the problem areas within the structure are automatically recognized and fixed during their life span. As a result, the structure becomes more durable and sustainable. Model-based structural health monitoring emerges as a method to detect the damage automatically. The method relies on the comparisons of model responses and those recorded of a structure of interest. If the structural behavior changes due to ageing, cracks, fatigue or other reasons, the misfit between model response and structural behavior increases. Minimizing this misfit by means of solving an inverse problem, the current state of the structure is mapped into the model. By this, we expect that the health conditions of the structure will be identified in time and can be fixed, accordingly.
In this paper, we introduce our recent studies on damage/crack identification using inverse analysis based on coupled multi-physical fields in both static and dynamic settings. In terms of the static model, theory and result of the identification of multi-damage locations is presented. The inverse problem bases on an adjoint method for the coupled hydro-mechanical model using the Extended Finite Element Method (XFEM) and Level Set function. In terms of the dynamic model, the full waveform inverse analysis for ultrasonic wave is utilized to identify the cracks in the concrete structure. Results of the inverse problems may show a scatter due to different sources of uncertainties in model parameters, boundary conditions, measurement data, field of measurements, regularization strategy, ... The uncertainty in the inverse solutions is quantified by their probability distributions according to a sampling process.

Accelerating 3D Topology Optimization through Sample-Efficient Deep Learning

David Erzmann
(University of Bremen)

Recent developments in Deep Learning (DL) show great potential for Topology Optimization (TO). However, this subfield still faces challenges due to a lack of established methods and datasets. We propose a novel approach to address these issues by integrating physics-based preprocessing and equivariant networks into an efficient DL pipeline. We demonstrate significant improvements in sample efficiency and physical correctness of predictions. Finally, we address one of the primary bottlenecks in TO by showcasing the application of neural operators as substitutes for classical Partial Differential Equation (PDE) solvers.

passive imaging in helioseismology: uniqueness results and efficient iterative algorithms for correlation data

Thorsten Hohage
(Georg-August-Universität Göttingen)

Helioseismology is the study of the interior of the Sun given observations of oscillations of the solar surface. Of particular interest are flows in the convection zone (the outer 30% of the Sun), but also density, pressure, and other quantities. Observations consist of the line-of-sight velocities of the solar surface obtained via Doppler shift measurements. High resolution images of oscillations of the solar surface have been recorded continuously at a cadence of 45 seconds by satellite and ground-based instruments since more than 25 years. The main bottleneck today is to extract the desired information from this huge data set. We will report on some recent progress in this direction.
We will discuss both the forward problem to predict observational data if the properties of the interior of the Sun are given, and the inverse problem to reconstruct quantities in the interior from observations on the surface. Solar oscillations are modelled as a random process driven by turbulence, and ideal noise-free data consist of the covariance operator of the surface oscillations. Computations in (local) helioseismology are typically carried out in the frequency domain. The forward problem is then defined in terms of a system of linear second order differential equations. Recently, we established well-posed of this differential equation using T-coercivity arguments and a new Helmholtz-type decomposition.
Under certain assumptions the covariance data are proportional to the imaginary part of the Green's function of the differential equations. Under this assumption and for a simplified scalar model we report on uniqueness results for several types of inverse problems, i.e. that under certain assumptions unknown interior quantities are uniquely determined by the imaginary part of Green's function at the surface.
Numerical reconstructions are challenging since data are high dimensional (they depend of 5 variables, one frequency variable and 2^2 surface variables) and extremely noisy. To tackle these challenges we will discuss a new approach called iterative helioseismic holography and show some numerical results.

Logarithmic Stability and Instability Estimates for Random Inverse Source Problems

Philipp Mickan, Thorsten Hohage
(Georg-August University of Göttingen)

We study the inverse source problem to determine the strength of a random acoustic source from correlation data. More precisely, the data of the inverse problems are correlations of random time-harmonic acoustic waves measured on a surface surrounding a region of random, uncorrelated sources. Such a model is used in experimental aeroacoustics to determine the strength of sound sources [1]. Uniqueness has been previously established [1,2]. In this talk we report on logarithmic stability results and logarithmic convergence rates for the certain regularisation methods applied to the inverse source problem by establishing a variational source condition under Sobolev type smoothness assumption. We also present logarithmic instability estimates using an entropy argument. Furthermore, we will show numerical experiments supporting our theoretical results.

[1] Thorsten Hohage, Hans-Georg Raumer, Carsten Spehr. *Uniqueness of an inverse source problem in experimental aeroacoustics*. Inverse Problems, 36(7):075012, 2020.
[2] Anthony J. Devaney. *The inverse problem for random sources*. Journal of Mathematical Physics, 20(8):1687–1691, 1979.

Motion detection in diffraction tomography

Michael Quellmalz[1], Peter Elbau[2], Otmar Scherzer[2], Gabriele Steidl[1]
([1] TU Berlin; [2] University of Vienna)

We study the mathematical imaging problem of optical diffraction tomography (ODT) for the scenario of a rigid particle rotating in a trap created by acoustic or optical forces. Under the influence of the inhomogeneous forces, the particle carries out a time-dependent smooth, but irregular motion. The rotation axis is not fixed, but continuously undergoes some variations, and the rotation angles are not equally spaced, which is in contrast to standard tomographic reconstruction assumptions. Once the time-dependent motion parameters are known, the particle's scattering potential can be reconstructed based on the Fourier diffraction theorem, considering it is compatible with making the first order Born or Rytov approximation.

The aim of this presentation is twofold: We first need to detect the motion parameters from the tomographic data by detecting common circles in the Fourier-transformed data. This can be seen as analogue to method of common lines from cryogenic electron microscopy (cryo-EM), which is based on the assumption that the assumption that the light travels along straight lines. Then we can reconstruct the scattering potential of the object utilizing non-uniform Fourier methods.

[1] M. Quellmalz, P. Elbau, O. Scherzer, G. Steidl. Motion Detection in Diffraction Tomography by Common Circle Methods. Accepted for The Mathematics of Computation. https://arxiv.org/abs/2209.08086

Solving linear inverse problems with invertible residual networks

Tobias Kluth
(University of Bremen)

Data-driven solution techniques for inverse problems, typically based on specific learning strategies, exhibit remarkable performance in applications like image reconstruction tasks. These learning-based reconstruction strategies often follow a two-step scheme. First, one uses a given dataset to train the reconstruction scheme, which one often parametrizes via a neural network. Second, the reconstruction scheme is applied to a new measurement to obtain a reconstruction. We follow these steps but specifically parametrize the reconstruction scheme with invertible residual networks (iResNets). We demonstrate that the invertibility opens the door to investigations into the influence of the training and the architecture on the resulting reconstruction scheme. In particular, these investigations reveal a formal link to the regularization theory of linear inverse problems. In this context we investigate the effect of different iResNet architectures, loss functions, and prior distributions on the trained network. Moreover, we analytically optimize the parameters of specific classes of architectures in the context of Bayesian inversion, showing the influence of the prior and noise distribution on the solution. The presented work is joint work with C. Arndt, A. Denker, S. Dittmer, N. Heilenkötter, M. Iske, P. Maass, and J. Nickel.

S09 Mathematics of Data Science

Section leaders: Dominik Stöger (KU Eichstätt-Ingolstadt), Felix Voigtländer (KU Eichstätt-Ingolstadt)

Session S09_01

Monday, 25 September 2023 14:00 (Parksaal 1)

14:00 Hung-Hsu Chou (Ludwig-Maximilians-Universität München)
More is Less: Understanding Compressibility of Neural Networks via Neural Collapse and Implicit Bias

14:30 Christian Wald (Technische Universität Berlin,)
Generative Sliced MMD Flows with Riesz Kernels

15:00 Axel Flinth (Umeå University)
Restrict the model or augment the data? Optimization dynamics of deep neural networks and equivariance

15:30 Semih Cayci (RWTH Aachen University)
Sample Complexity and Overparameterization Bounds for Neural Reinforcement Learning

Session S09_02

Monday, 25 September 2023 16:30 (Parksaal 1)

16:30 Anna Veselovska (Technical University Munich & Munich Center for Machine Learning)
Quantization of Bandlimited Graph Signals

17:00 Alessandro Lupoli (TU Munich)
Quantization of Bandlimited Functions on the 2D−Torus

17:30 Dominik Stöger (KU Eichstätt-Ingolstadt)
Generalization and Convergence Guarantees for Overparameterized Asymmetric Matrix Sensing

Session S09_03

Tuesday, 26 September 2023 09:00 (Parksaal 1)

09:00 Paul Geuchen (Katholische Universität Eichstätt-Ingolstadt)
Universal approximation with complex-valued deep narrow neural networks

09:30 Laura Thesing (Ludwig-Maximilians-Universität München)
Sumformer: Universal Approximation for Efficient Transformers

10:00 Andrei Caragea (Katholische Universität Eichstätt-Ingolstadt)
Quantitative approximation rates for C^k functions with complex valued networks

10:30 Philipp Scholl (Ludwig-Maximilians-Universität München)
The Math Behind Physical Law Learning: Navigating the Uniqueness Problem

Session S09_04

Tuesday, 26 September 2023 14:00 (Parksaal 1)

14:00 Erik Lien Bolager (Technische Universität München)
Data-informed distributions for sampling neural network weights

14:30 Anna-Laura Sattelberger (KTH Royal Institute of Technology)
Algebraic Geometry of Equivariant Neural Networks

15:00 Johannes Maly (LMU Muenchen)
Algorithmic approaches to recovering simultaneously sparse and low-rank matrices

Session S09_05

Wednesday, 27 September 2023 16:45 (Parkcafé)

16:45 Sjoerd Dirksen (Utrecht University)
The separation capacity of random neural networks

17:45 Guido Montufar (UCLA and Max Planck Institute MiS)
Mildly Overparameterized ReLU Networks Have a Favorable Loss Landscape

More is Less: Understanding Compressibility of Neural Networks via Neural Collapse and Implicit Bias

Hung-Hsu Chou
(Ludwig-Maximilians-Universität München)

Despite their recent successes in various tasks, most modern machine learning algorithms lack theoretical guarantees, which are crucial to further development towards delicate tasks such as designing self-driving cars. One mysterious phenomenon is that, among infinitely many possible ways to fit data, the algorithms always find the "good" ones, even when the definition of "good" is not specified by the designers. In this talk I will cover the empirical and theoretical study of the connection between the good solutions in neural networks and the sparse solutions in compressed sensing with four questions in mind: What happens? When does it happen? Why does it happen? How can we improve it? The key concepts are neural tangent kernel, neural collapse, implicit bias/regularization, and weight normalization.

Generative Sliced MMD Flows with Riesz Kernels

Christian Wald[1], Johannes Hertrich, Fabian Altekrüger, Paul Hagemann
([1] Technische Universität Berlin,)

Generative models have gained increasing attention over the last years. A generative method which admits a nice mathematical interpretation is related to gradient flows in the space of probability measures. We considers a distance d on the space of probability measures $\mathcal{P}(\mathbb{R}^d)$ and look at Wasserstein gradient flows of the functional $\mathcal{F}(\mu) = d(\mu, \nu)$ for a target distribution ν [1]. We then approximate this gradient flow by neural networks and obtain a generative model. This construction suffers from high computational costs which we overcome by several techniques. As distance consider the Maximum Mean Discrepancy (MMD) which for a kernel $K : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ reads as

$$\mathsf{MMD}^2(\mu, \nu) = \int_{\mathbb{R}^d \times \mathbb{R}^d} K(x, y) \mathrm{d}(\mu - \nu)(x) \mathrm{d}(\mu - \nu)(y).$$

We use the Riesz kernel $K(x, y) = -\|x - y\|_2$ for which MMD defines a metric on the space of probability measures with finite first moments $\mathcal{P}_1(\mathbb{R}^d)$. We define $\mathcal{F}(\mu) = \mathsf{MMD}^2(\mu, \nu)$ where ν is the target distribution we want to sample from.
The Wasserstein gradient flow for empirical measures $\mu = \frac{1}{M}\sum_{i=1}^M \delta_{x_i}, \nu = \frac{1}{N}\sum_{i=1}^N \delta_{y_i}$ with respect to $\mathcal{F}$ can be written interpreted as a ODE as follows. Let $F_M((x_1, ..., x_M)) = \mathcal{F}\left(\frac{1}{M}\sum_{i=1}^M \delta_{x_i}\right)$ and

$$u' = -M\nabla F_M(u). \tag{1}$$

Then for a solution $u(t) = (x_1(t), \cdots, x_M(t))$ of (1) the path $\gamma(t) = \frac{1}{M}\sum_{i=1}^M \delta_{x_i(t)}$ is a Wasserstein gradient flow with respect to F_M [2]. Here the computation of ∇F_M is of complexity $O(MN + M^2)$. In the $1d$ case a simple sorting algorithm can be used to efficiently compute ∇F_M. We show that for the Riesz kernel $K(x, y) = -\|x - y\|_2$ in dimension d we can always reduce to the one dimensional case by slicing i.e.

$$K(x, y) = -c\mathbb{E}_{\xi \in S^{d-1}}\left[|\langle \xi, x\rangle - \langle \xi, y\rangle|\right]$$

where S^{d-1} is the $d-1$-sphere and c is a constant. This enables us to use the sorting algorithm in $1d$ even in higher dimensions and we can reduce the complexity for the computation of ∇F_M to $O((M + N)\log(M + N))$. In applications we only use finitely many slices P and we can show that the resulting error has complexity of only $O(\sqrt{d/P})$. To approximate (1) we use an explicit Euler scheme and approximate every gradient step of the Euler scheme by a neural network to obtain a generative model approximating a Wasserstein gradient flow. We demonstrate efficiency of our approach on the standard benchmark datasets for image generation MNIST, FashionMNIST and CIFAR10 [3]. Using conditional flows this approach can also be used for solving inverse problems in imaging.

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Restrict the model or augment the data? Optimization dynamics of deep neural networks and equivariance

Axel Flinth
(Umeå University)

A function is called equivariant to a group of transformations if transforming its inputs makes their outputs react accordingly. If we make a machine learning model equivariant, it will explicitly make use of a priori symmetries in the data. A popular way of inducing equivariance is to augment the data, i.e. transform it according to the symmetry group. The framework of Geometric Deep Learning provides an alternative approach: it allows the construction of neural networks which by design are equivariant. In this talk, we will present a theoretical comparison of the two approaches. Our main finding is that if a non-equivariant architecture is set up properly and trained on perfectly augmented data, the subset of equivariant architecture is an invariant, but not necessarily stable, set for the training dynamics. This has interesting consequences for equivariant deep learning. Based on joint work with Fredrik Ohlsson, Umeå University.

Sample Complexity and Overparameterization Bounds for Neural Reinforcement Learning

Semih Cayci [1], Niao He[2], Rayadurgam Srikant[3]
([1] RWTH Aachen University; [2] ETH Zürich; [3] UIUC)

Natural actor-critic (NAC) and its variants, equipped with the representation power of neural networks, have demonstrated impressive empirical success in solving Markov decision problems with large state spaces. In this talk, we present a finite-time analysis of NAC with neural network approximation, and identify the roles of regularization and optimization techniques (e.g., gradient clipping and averaging) to achieve sharp bounds on the sample complexity, iteration complexity and network width. In particular, we prove that (i) the uniform approximation power of the actor neural network is important for global optimality in policy optimization due to distributional shift, (ii) entropy regularization, averaging and gradient clipping ensure sufficient exploration to guarantee optimality under minimal assumptions on concentrability coefficients, and (iii) regularization leads to sharp sample complexity and network width bounds in the regularized MDPs, yielding a favorable bias-variance tradeoff in policy optimization.

Quantization of Bandlimited Graph Signals

Anna Veselovska
(Technical University Munich & Munich Center for Machine Learning)

Graph signals provide a natural representation for data in many applications, such as social networks, web information analysis, sensor networks and machine learning. Graph signal & data processing is currently an active field of mathematical research that aims to extend the well-developed tools for the analysis of conventional signals to signals on graphs while exploiting the underlying connectivity. A key challenge in this context is the problem of quantization, that is, finding efficient ways of representing the values of graph signal with only a finite number of bits.
In this talk, we address the problem of quantizing bandlimited graph signals. We introduce two classes of noise-shaping algorithms for graph signals that differ in their sampling methodologies. We demonstrate that these algorithms can be efficiently used to construct quantized representatives of bandlimited graph-based signals with bounded amplitude.
Inspired by the results obtained by Zhang et al. in 2022, for one of the algorithms, we provide theoretical guarantees on the relative error between the true signal and its quantized representative. As will be discussed, the incoherence of the underlying graph plays an important role in the process of quantization. Namely, bandlimited signals supported on graphs of lower incoherence allow for smaller relative error.
We support our findings by various numerical experiments showcasing the performance of the proposed quantization algorithms for bandlimited signals defined on graphs with different degrees of incoherence.

This is joint work with Felix Krahmer (Technical University of Munich), He Lyu (Michigan State University), Rayan Saab (University of California San Diego), and Rongrong Wang (Michigan State University).

Quantization of Bandlimited Functions on the 2D−Torus

Alessandro Lupoli, Felix Krahmer
(TU Munich)

The study of quantization techniques for bandlimited functions on manifolds has become increasingly important both for the results obtainable in some engineering applications (A/D converters, Digital Halftoning, Imaging), and for some Machine Learning and Deep Learning applications.
One of the best known in the literature is Sigma-Delta quantization, a method for mapping samples of a bandlimited function into a small set ($\{-1, 1\}$ in the case of this article), guaranteeing a good approximation of the reconstructed signal.
The main issue is highlighted when trying to apply these algorithms on closed manifolds, as cuts need to be introduced to apply them, and the schemes will typically not yield good reconstruction along the cuts. The first problem in this direction was addressed in [1], where the authors adapted $\Sigma\Delta$ schemes of order $m = 1, 2$ for functions defined on a circle. In that case, through an update of the samples, that is considering new samples of the type $\tilde{f_n} := f_n + \delta$, where δ is a constant estimated through an initial run of the algorithm, $\Sigma\Delta$ turns out to guarantee a good reconstruction of the function with absence of artifacts.
In the case of functions defined on a torus (the flat torus will be considered in the paper), fitting $\Sigma\Delta$ schemes becomes even more complicated. In this context the authors of [2] define weighted Sigma-Delta schemes to obtain stable algorithms, particularly for application to Digital Halftoning. However, when applied to functions $f \in {}^2\left(\mathbb{T}^2\right)$, they again encounter problems related to the presence of artifacts on the cuts introduced on the flat torus. This is due to the fact that the algorithms are initialized with predefined variables on some of the cuts of the torus, which, however, take on different instances on some others, although they should not change value due to periodicity.
In this paper a different sampling scheme is proposed, which allows to avoid the artifacts obtained with the weighted schemes. The basic idea is to sample the function along a spiral drawn along the torus, which leads to the use of rank 1-lattices instead of regular grids. The advantage of this approach consists in being able to obtain a representation formula similar to a weighted DFT, therefore easy to compute. Also, in this way, it is possible to use the well known theory developed for $1D$ functions and the results obtained in [1] to be able to eliminate the artifacts occurring at the end/beginning of the spiral. In the talk some numerical applications will be presented to show the effectiveness of the method for A/D converters and for digital-halftoned images. In the literature it is known that the use of $1D$ methods is suboptimal for representing images as sequences of dots, due to the presence of small artifacts in the direction orthogonal to the quantization direction. However these can be corrected using error diffusion techniques discussed in the engineering literature.

This is joint work with Felix Krahmer (TUM).

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Generalization and Convergence Guarantees for Overparameterized Asymmetric Matrix Sensing

Dominik Stöger
(KU Eichstätt-Ingolstadt)

There has been significant progress in understanding the convergence and generalization properties of gradient-based methods for training overparameterized learning models. However, many aspects including the role of small random initialization and how the various parameters of the model are coupled during gradient-based updates to facilitate good generalization remain largely mysterious. In this talk, we consider a general overparameterized low-rank matrix sensing problem where one wishes to reconstruct an asymmetric rectangular low-rank matrix from a few linear measurements. We show that in this setting, factorized gradient descent enjoys two implicit properties:

1. coupling of the trajectory of gradient descent where the factors are coupled in various ways throughout the gradient update trajectory
2. an algorithmic regularization property where the iterates show a propensity towards low-rank models despite the overparameterized nature of the factorized model.

These two implicit properties in turn allow us to show that the gradient descent trajectory from small random initialization moves towards solutions that are both globally optimal and generalize well.

Universal approximation with complex-valued deep narrow neural networks

Paul Geuchen, Thomas Jahn, Hannes Matt
(Katholische Universität Eichstätt-Ingolstadt)

We study the universality of complex-valued neural networks with bounded widths and arbitrary depths. Under mild assumptions, we give a full description of those activation functions $\varrho : \mathbb{C} \to \mathbb{C}$ that have the property that their associated networks are universal, i.e., are capable of approximating continuous functions to arbitrary accuracy on compact domains. Precisely, we show that deep narrow complex-valued networks are universal if and only if their activation function is neither holomorphic, nor antiholomorphic, nor $\mathbb{R}$-affine. This is a much larger class of functions than in the dual setting of arbitrary width and fixed depth. Unlike in the real case, the sufficient width differs significantly depending on the considered activation function. We show that a width of $2n + 2m + 5$ is always sufficient and that in general a width of $\max\{2n, 2m\}$ is necessary. We prove, however, that a width of $n + m + 4$ suffices for a rich subclass of the admissible activation functions. Here, n and m denote the input and output dimensions of the considered networks. This is joint work with Thomas Jahn and Hannes Matt.

Sumformer: Universal Approximation for Efficient Transformers

Laura Thesing
(Ludwig-Maximilians-Universität München)

Natural language processing made an impressive jump with the introduction of Transformers. ChatGPT is one of the most famous examples, changing the perception of the possibilities of AI even outside the research community. However, besides the impressive performance, the quadratic time and space complexity of Transformers with respect to sequence length pose significant limitations for handling long sequences. While efficient Transformer architectures like Linformer and Performer with linear complexity have emerged as promising solutions, their theoretical understanding remains limited. In this talk, we present the recent joint work on Sumformer, a novel and simple architecture capable of universally approximating equivariant sequence-to-sequence functions. We elaborate on how Sumformer are used to show the universality of Linformer and Performer and a new proof for Transformer.

Quantitative approximation rates for C^k functions with complex valued networks

Andrei Caragea[1], Paul Geuchen[1], Dae Gwan Lee[2], Johannes Maly[3], Goetz Pfander[1], Felix Voigtlaender[1]
([1] Katholische Universität Eichstätt-Ingolstadt; [2] Kumoh National Institute of Technology; [3] Ludwig-Maximilians-Universität München)

We discuss the approximation rates of complex valued feed forward neural networks for functions with C^k regularity (in the sense of real differentiation). We show that such functions can be approximated to precision ε by modReLU neural networks with no more than order $\varepsilon^{-2d/k}$ weights, where d is the input dimension. We also briefly discuss a more recent generalization to a wider class of activation functions.

The Math Behind Physical Law Learning: Navigating the Uniqueness Problem

Philipp Scholl[1], Aras Bacho[1], Holger Boche[2], Gitta Kutyniok[1]
([1] Ludwig-Maximilians-Universität München; [2] Technical University Munich)

Physical law learning is the ambiguous attempt at automating the derivation of governing equations with the use of machine learning techniques. Instead of approximating the solution, the goal of physical law learning is to learn the true law governing a phenomenon, to increase the interpretability and, therefore, trustworthiness of the model. Furthermore, interpretable formulas facilitate an understanding and scientific insights which would be impossible with blackbox models. The current literature focuses solely on the development of methods to achieve this goal, and a theoretical foundation is missing at present. This paper shall thus serve as a first step to build a comprehensive theoretical framework for learning physical laws, aiming to provide reliability to according algorithms. One key problem consists in the fact that the governing equations might not be uniquely determined by the given data. We will study this problem in the common situation that a physical law is described by an ordinary or partial differential equation. For various different classes of differential equations, we provide both necessary and sufficient conditions for a function to uniquely determine the differential equation which is governing the phenomenon. These results show how common the issue of non-uniqueness is, especially for higher order PDEs. Based on our theoretical results we then devise numerical algorithms to determine whether a function solves a differential equation uniquely. Finally, we provide extensive numerical experiments showing that our algorithms in combination with common approaches for learning physical laws indeed allow to determine if a unique governing differential equation is learnt, without assuming any knowledge about the function, thereby ensuring reliability.

Data-informed distributions for sampling neural network weights

Erik Lien Bolager[1], Felix Dietrich
([1] Technische Universität München)

Random feature methods construct internal neural network weights by randomly sampling them, typically from a Gaussian distribution. Thus, even in supervised learning problems, these methods do not utilize the information from the available data. Other attempts with more informed distributions, for example Bayesian neural networks, require a lot of computational power. In this talk, we present a method to construct the weights and biases of the hidden layers strictly from the space $\mathcal{X} \times \mathcal{X}$, where $\mathcal{X}$ is the domain of the underlying function, and then present a probability distribution over $\mathcal{X} \times \mathcal{X}$ that also uses the information of the function we approximate. By sampling weights and biases of the hidden layers in this way and then solving the linear system to obtain the parameters of the last layer, we can construct accurate neural networks in a gradient-free way. The construction is possible for shallow and deep feedforward neural networks. We will present theoretical results showing that even though we limit the space of weights and biases in the hidden layers, we do not limit the space of functions we can approximate, under mild assumptions on $\mathcal{X}$. We also bound the L_2 approximation error for Barron functions to $\mathcal{O}(m^{-1/2})$, where m is the width of the network. We provide an example of input spaces that break these mild assumptions and discuss why the networks fail to approximate certain functions. We end the talk by demonstrating the approximation quality of the proposed method on numerical examples.

Algebraic Geometry of Equivariant Neural Networks

Kathlén Kohn, Anna-Laura Sattelberger, Vahid Shahverdi
(KTH Royal Institute of Technology)

Function spaces of dense linear neural networks are determinantal varieties, which are well understood from the perspective of algebraic geometry. In contrast, function spaces of linear convolutional networks are semi-algebraic sets and were studied in detail in works of Kohn, Merkh, Montúfar, Shahverdi, and Trager. They provide a parameterization of the function space, a description of its singular locus, and determine where critical points of the loss typically are located. A solid knowledge of the geometry of the function space of a neural network hence is not only interesting per se, but is useful for understanding the training of the network. In ongoing work with Kathlén Kohn and Vahid Shahverdi, we investigate the function space of dense linear networks that are equivariant or invariant under certain group actions. This has direct impact on the construction of encoder-decoder models, as I am going to explain in my talk.

Algorithmic approaches to recovering simultaneously sparse and low-rank matrices

Johannes Maly
(LMU Muenchen)

In this talk, we consider the problem of recovering an unknown sparse and low-rank matrix from observations gathered in a linear measurement process. We discuss the challenges that come with leveraging several structures simultaneously and present two new algorithmic strategies to efficiently approach the problem. Both strategies come with local convergence guarantees.

The separation capacity of random neural networks

Sjoerd Dirksen
(Utrecht University)

Neural networks with random weights appear in a variety of machine learning applications, most prominently as the initialization of many deep learning algorithms and as a computationally cheap alternative to fully learned neural networks. The first goal of this talk is to enhance the theoretical understanding of random neural networks by addressing the following data separation problem: under what conditions can a random neural network make two classes (with positive distance) linearly separable? I will show that a sufficiently large two-layer ReLU-network with Gaussian weights and uniformly distributed biases can solve this problem with high probability. Building on the insights behind this result, I will next present a simple randomized algorithm to produce a small interpolating neural net for a given dataset with two classes. In both results, the size of the network is explicitly linked to geometric properties of the two classes and their mutual arrangement. This instance-specific viewpoint allows to overcome the curse of dimensionality. I will connect the presented results with related work on memorization and generalization.

This talk is based on joint works with Patrick Finke (Utrecht University, The Netherlands), Martin Genzel (Utrecht University, The Netherlands and Merantix Momentum), Laurent Jacques (UC Louvain, Belgium), and Alexander Stollenwerk (UC Louvain, Belgium and KPMG Germany).

Mildly Overparameterized ReLU Networks Have a Favorable Loss Landscape

Guido Montufar
(UCLA and Max Planck Institute MiS)

We study the loss landscape of two-layer mildly overparameterized ReLU neural networks on a generic finite input dataset for the squared error loss. Our approach involves bounding the dimension of the sets of local and global minima using the rank of the Jacobian of the parametrization map. Using results on random binary matrices, we show most activation patterns correspond to parameter regions with no bad differentiable local minima. Furthermore, for one-dimensional input data, we show most activation regions realizable by the network contain a high dimensional set of global minima and no bad local minima. We experimentally confirm these results by finding a phase transition from most regions having full rank to many regions having deficient rank depending on the amount of overparameterization. This is joint work with Kedar Karhadkar, Michael Murray, and Hanna Tseran.

S10 Numerical Mathematics and Scientific Computing

Section leaders: Dietmar Gallistl (Jena), Daniel Peterseim (Augsburg)

Session S10_01

Wednesday, 27 September 2023 09:00 (VP-Raum)

09:00 Johannes Storn (Universität Bielefeld)
Kacanov iterations for the p-Laplacian and modifications for minimal residual methods in $W^{-1,p}$

10:00 Ngoc Tien Tran (Friedrich-Schiller-Universität Jena)
Minimal residual methods for uniformly elliptic PDE of second order in nondivergence form

10:30 Dietmar Gallistl (FSU Jena)
A posteriori error control in the max morm for the Monge-Ampère equation

Session S10_02

Wednesday, 27 September 2023 16:45 (W9-Raum)

16:45 Wolfgang Arendt (Ulm University)
Space-time Error Estimates for Approximations of Linear Parabolic Problems with Generalized Time Boundary Conditions

17:15 Nadiia Derevianko (University of Göttingen)
Differential approximation of the Gaussian by short cosine sums with exponential error decay

17:45 Gero Junike (Carl von Ossietzky University)
How to handle the COS method for option pricing

18:15 Daniel Peterseim (Universität Augsburg)
Computational Polyconvexification of Isotropic Functions

Kacanov iterations for the p-Laplacian and modifications for minimal residual methods in $W^{-1,p}$

Anna Kh Balci, Lars Diening, Johannes Storn
(Universität Bielefeld)

The first part of this talk introduces an iterative scheme for the computation of the discrete minimizer to the p-Laplace problem. The iterative scheme is easy to implement since each iterate results only from the solve of a weighted, linear Poisson problem. It neither requires an additional line search nor involves unknown constants for the step length. The scheme converges globally and its rate of convergence is independent of the underlying mesh. In the second part of the talk we adjust this ansatz to compute the minimizer of a minimal residual method in $W^{-1,p}$. Such schemes remedy instabilities of finite element methods for problems like convection-dominated diffusion.

Minimal residual methods for uniformly elliptic PDE of second order in nondivergence form

Ngoc Tien Tran
(Friedrich-Schiller-Universität Jena)

The design of finite element methods for PDE of second order in nondivergence form is in general difficult due to the lack of a variational formulation. Minimal residual methods can be attractive in this context thanks to an easy access to a well-posed discrete problem. The focus of this talk is on those methods, whose residual arises from the Alexandrov-Bakelman-Pucci maximum principle. Convergence of FEM with respect to the maximum norm is established under suitable existence theory of strong solutions. The arguments are presented for linear PDE, but can be easily transferred to nonlinear examples such as the Pucci or (regularized) Monge-Ampère equation.

A posteriori error control in the max morm for the Monge-Ampère equation

Dietmar Gallistl, Ngoc Tien Tran
(FSU Jena)

This talk discusses a stability result for the Monge-Ampère operator in a (potentially regularized) Hamilton-Jacobi- Bellman format as a consequence of Alexandrov's classical maximum principle. The main application is guaranteed a posteriori error control in the L^∞ norm for the difference of the Monge-Ampère solution and the convex hull of a fairly arbitrary C^1-conforming finite element approximation.

Space-time Error Estimates for Approximations of Linear Parabolic Problems with Generalized Time Boundary Conditions

Wolfgang Arendt
(Ulm University)

A parabolic problem will be approximated by a scheme which discretizes time and space simultaneously. It is a nonconforming approximation by step functions. The error estimate is optimal in the same sense as in the celebrated Céa Lemma. It is interesting that the space of the limit functions of this kind of approximation can be characterized as a mixed Sobolev space introduced by Lions for the treatment of parabolic problems. Another special feature of our approach is the kind of boundary conditions with respect to time that we consider. They may have the form of an initial value condition, be periodic or, a mixture of both. The talk is based on joint work with I. Chalendar and R. Eymard (Paris).

Differential approximation of the Gaussian by short cosine sums with exponential error decay

Nadiia Derevianko, Gerlind Plonka
(University of Göttingen)

We propose a method to approximate the Gaussian function $\mathrm{e}^{t^2/2\sigma}$, $\sigma > 0$, on $\mathbb{R}$ by a short cosine sum. We prove that the optimal for our method frequency parameters $\lambda_1, \ldots, \lambda_N$ in the approximation problem $\min\limits_{\lambda_1,\ldots,\lambda_N,\gamma_1\ldots\gamma_N} \|\mathrm{e}^{-\cdot^2/2\sigma} - \sum\limits_{j=1}^{N} \gamma_j \,\mathrm{e}^{\lambda_j \cdot}\|_{L_2(\mathbb{R},\mathrm{e}^{-t^2/2\rho})}$, $\rho > 0$, are zeros of a scaled Hermite polynomial. This observation leads us to a numerically stable approximation method with low computational cost of $\mathcal{O}(N^3)$ operations. Furthermore, we derive a direct algorithm to solve this approximation problem based on a matrix pencil method for a special structured matrix. The entries of this matrix are determined by hypergeometric functions. For the weighted L_2-norm, we prove that the approximation error decays exponentially with respect to the length N of the sum.

How to handle the COS method for option pricing

Gero Junike
(Carl von Ossietzky University)

The Fourier cosine expansion (COS) method is used for pricing European options numerically very fast. For model calibration, it is of utmost importance to price European options efficienty. To apply the COS method one has to specify two parameters: a truncation range for the density of the log-returns and a number of terms N to approximate the truncated density by a cosine series. Using Markov's inequality, we derive a new formula to obtain the truncation range and prove that the range is large enough to ensure convergence of the COS method within a predefined error tolerance. The number of terms N depends on the smoothness of the density. In many models, the density of the log-returns are arbitrary smooth and we are able to find an explicit expression for N as well. Numerical experiments confirm the theoretical findings and show that the bounds for the truncation range and the number of terms are sharp enough to be useful in applications.
This talk is based on an unpublished working paper and on Junike, G., & Pankrashkin, K. (2022). Precise option pricing by the COS method—how to choose the truncation range. Applied Mathematics and Computation, 421, 126935.

Computational Polyconvexification of Isotropic Functions

Daniel Peterseim, Timo Neumeier, David Wiedemann, Malte Peter
(Universität Augsburg)

Based on the characterization of the polyconvex envelope of isotropic functions by their signed singular value representations, we propose a simple algorithm for the numerical approximation of the polyconvex envelope. Polyconvex envolopes are relevant for the relaxation of energy minimization problems in nonlinear elasticity. Instead of operating on the d^2-dimensional space of matrices (deformation gradients), the algorithm requires only the computation of the convex envelope of a function on a d-dimensional manifold, which is easily realized by standard algorithms. The significant speedup associated with the dimensional reduction from d^2 to d is demonstrated in a series of numerical experiments.

S11 Analysis & Differential Equations

Section leaders: Ralph Chill (TU Dresden), Christian Seifert (TU Hamburg)

Session S11_01

Monday, 25 September 2023 16:30 (Parksaal 3)

16:30 Daniel Grieser (Carl von Ossietzky University)
Construction of quasimodes for generalized semi-classical differential operators

17:00 Daniel Lenz (Friedrich Schiller Universität Jena)
Subshifts with leading sequences

17:30 Tomas Dohnal (Martin Luther University Halle-Wittenberg)
Bifurcation and asymptotics of nonlinear surface plasmons

18:00 Robert Lasarzik (Weierstrass Institute for Applied Analysis and Stochastics)
Solvability for viscoelastic materials via the energy-variational approach

18:30 Ralph Chill (TU Dresden)
Nonlinear nonlocal elliptic operators

Session S11_02

Wednesday, 27 September 2023 09:00 (Parksaal 3)

09:00 Wolfgang Arendt (Ulm University)
The Perron Solution for Elliptic Equations without the Maximum Principle

09:30 Boris Gulyak (Otto-von-Guericke Universität Magdeburg)
The graphical Willmore problem with weak Dirichlet boundary data

10:00 Amru Hussein (Universität Kassel)
Maximal L^p-regularity and H^∞-calculus for block operator matrices and applications

10:30 Christian Seifert (Technische Universität Hamburg)
Perturbation theory for non-autonomous second-order abstract Cauchy problems

Session S11_03

Wednesday, 27 September 2023 16:45 (Parksaal 2)

16:45 Thu Hien Nguyen (Universität Würzburg)
Some simple conditions for entire functions to have only real zeros

17:15 Spyridon Kakaroumpas (University of Würzburg)
Boundary preimages of linear combinations of iterates of Blaschke products

17:45 Olesia Zavarzina (V. N. Karazin Kharkiv National University)
Plasticity problem and related questions

18:15 Marcel Schmidt
Weak Hardy/Poincaré inequalities and criticality theory

Construction of quasimodes for generalized semi-classical differential operators

Daniel Grieser
(Carl von Ossietzky University)

We consider families P_h of differential operators on an interval that depend on a parameter $h \geq 0$ and degenerate as $h \to 0$. We consider the problem of constructing quasimodes, i.e. (families of) solutions u_h, $h > 0$, of $P_h u_h = O(h^\infty)$ as $h \to 0$. This can be used to find approximations of the spectrum of P_h for small positive h. A classical example is the semi-classical Schroedinger Operator $P_h = h^2\partial^2 + V$ where $\partial = d/dx$ and V is a smooth function. If V is positive then quasimodes can be found using the standard WKB method. At zeroes of V additional difficulties arise (solved by Olver long ago) due to different scaling behavior near and away from the zeroes. We construct full sets of quasimodes for very general families $P_h = P(x, \partial, h)$ of any order, including those where the coefficients depends analytically on x and h, under some mild genericity hypothesis. The generality of the setup leads to a high degree of combinatorial and analytic complexity, which can be handled using an efficient representation of the data by Newton polygons and of the result in terms of iterated blow-ups and a suitable class of oscillatory-polyhomogeneous functions. This is joint work with Dennis Sobotta.

Subshifts with leading sequences

Daniel Lenz
(Friedrich Schiller Universität Jena)

We consider uniformity of SL(2,R)-valued cocycles over subshifts. We show that uniformity holds for locally constant cocycles if the subshift satisfies what we call leading sequence condition. This condition is satisfied by simple Toeplitz subshifts and Sturmian subshifts. As a consequence we obtain that the associated Jacobi operators have Cantor spectrum of Lebesgue measure zero. This can be applied to study spectra of the Laplacians of Schreier graphs of spinal groups. (Joint work with Tatiana Nagnibeda, Rostislav Grigorchuk and Daniel Sell)

Bifurcation and asymptotics of nonlinear surface plasmons

Tomas Dohnal
(Martin Luther University Halle-Wittenberg)

Surface plasmons are electromagnetic waves localized at the interface of a metal and a dielectric, or more generally at the interface of dispersive (frequency dependent) materials. In the time harmonic case the linear Maxwell equations reduce to an operator pencil problem which is non-self-adjoint in the presence of metals. The frequency plays the role of a spectral parameter. We consider straight interfaces and analyze the linear spectral problem first. Next, the bifurcation of nonlinear surface plasmons from simple isolated eigenvalues and their asymptotic expansion are proved via a fixed point argument. In a PT-symmetric geometry the frequency remains real in the bifurcation. We consider both the TE and TM-polarizations. In the former case the problem become scalar and supports plasmons only in the case of two or more interfaces. In the latter case a system of two coupled ODEs can be derived. Numerical computations are provided to support the analysis.
This work includes collaborations with Malcolm Brown, Runan He, Michael Plum, Giulio Romani, and Ian Wood.

Solvability for viscoelastic materials via the energy-variational approach

Robert Lasarzik
(Weierstrass Institute for Applied Analysis and Stochastics)

An existence and weak-strong uniqueness result for energy-variational solutions is given for a general nonlinear evolutionary system of partial differential equations. The constructive existence proof is based on an incremental minimization process. During a short detour on the modeling of viscoelastic materials, certain models are identified that fit into the abstract framework. This gives some new existence results for models, where those seemed to be out of reach in other frameworks or are only possible by adding enough stress diffusion.

Nonlinear nonlocal elliptic operators

Ralph Chill
(TU Dresden)

We consider nonlocal elliptic operators on domains in $\mathbb{R}^N$, on Riemannian manifolds, on graphs or on random walk spaces with so-called Neumann, Robin or Dirichlet exterior conditions. The types of operators we consider are subgradients of convex energies, and in some special cases, they are „fractional powers“ of Laplace or p-Laplace type operators. A main ingredient in the definition of these operators is an operator j, a restriction operator, which appeared first in several articles by Arendt and ter Elst. We show that the nonlocal operators considered here generate order preserving, L^∞-contractive semigroups, that is, that the associated energies are (nonlinear) Dirichlet forms.
Joint work with Mahamadi Warma (George Mason University).

The Perron Solution for Elliptic Equations without the Maximum Principle

Wolfgang Arendt
(Ulm University)

In this talk we will consider the Dirichlet problem for an elliptic operator in divergence form. The only hypothesis is that $\{0\}$ is not a Dirichlet eigenvalue. Thus the maximum principle is not valid. If the underlying domain is Wiener regular, then the Dirichlet problem is well-posed in the classical sense; i.e., for each continuous function defined on the boundary there exists a unique harmonic function which is continuous up to the boundary and coincides with the given function at the boundary. If the domain is not Wiener regular we define a Perron solution of the Dirichlet problem. It can be described as the unique bounded continuous function which has as limit the given function at the boundary quasi everywhere. But we will give a variety of other descriptions of the Perron solution. The most interesting is a characterization via Sobolev spaces, and this one is even new for the Laplacian. The talk is based on joint work with Tom ter Elst (Auckland) and Manfred Sauter (Ulm).

The graphical Willmore problem with weak Dirichlet boundary data

Boris Gulyak
(Otto-von-Guericke Universität Magdeburg)

In the graphical setting, we want to present existence and regularity results for the Willmore equation with Dirichlet boundary conditions. In our case, the corresponding surface is embedded in $\mathbb{R}^3$ and the Willmore equation is described by an elliptic fourth-order operator with some right-hand side terms acting on function $u\colon \Omega \to \mathbb{R}$ on a $C^{1+\alpha}$ bounded domain. By using the linearization method in the framework of weighted second-order Sobolev spaces that bound the norm of the gradient, we can prove the existence of a smooth solution for small $C^{1+\alpha}$ boundary data. Here, we essentially use the recently derived divergence structure of the right-hand side of the graphical Willmore equation.

Maximal L^p-regularity and H^∞-calculus for block operator matrices and applications

Amru Hussein
(Universität Kassel)

Many coupled evolution equations can be described via 2×2-block operator matrices of the form $\mathcal{A} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ in a product space $X = X_1 \times X_2$ with possibly unbounded entries. Here, the case of diagonally dominant block operator matrices is considered, that is, the case where the full operator $\mathcal{A}$ can be seen as a relatively bounded perturbation of its diagonal part though with possibly large relative bound. For such operators the properties of sectoriality, $\mathcal{R}$-sectoriality and the boundedness of the H^∞-calculus are studied, and for these properties perturbation results for possibly large but structured perturbations are derived. Thereby, the time dependent parabolic problem associated with $\mathcal{A}$ can be analyzed in maximal L^p_t-regularity spaces, and this is illustrated by a number of applications such as different theories for liquid crystals, an artificial Stokes system, strongly damped wave and plate equations, and a Keller-Segel model.

The approach developed here is based in spirit on a combination of the theory by Kalton, Kunstmann and Weis (Perturbation and interpolation theorems for the H^∞-calculus with applications to differential operators. Math. Ann., 336(4):74-801, 2006) relating $\mathcal{R}$-sectoriality and the boundedness of the H^∞-calculus with concepts for diagonally dominant block operator matrices pioneered by Nagel (Towards a „matrix theory“ for unbounded operator matrices. Math. Z., 201(1):57-68, 1989) for C_0-semigroups.

The presentation is based on a joint work with Antonio Agresti, see https://arxiv.org/abs/2108.01962

Perturbation theory for non-autonomous second-order abstract Cauchy problems

Christian Seifert
(Technische Universität Hamburg)

In this talk, we focus on non-autonomous second order abstract Cauchy problems in Banach spaces, which model non-autonomous wave equations. First, we will study fundamental solutions and their existence by means of the corresponding non-autonomous first-order system. We then turn to a perturbation result where the perturbation consists of a suitably bounded family of operators. As an application, we study perturbations of wave equations.

The talk is based on joint work with Christian Budde (University of the Free State, Bloemfontein, South Africa).

Some simple conditions for entire functions to have only real zeros

Thu Hien Nguyen
(Universität Würzburg)

The Laguerre-Pólya class is a special class of entire functions, which appears to be the analytic closure of sets of univariate hyperbolic polynomials. We present some simple necessary and sufficient conditions for entire functions to belong to the Laguerre-Pólya class, or to have only real zeros, in terms of their Taylor coefficients. For an entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$, we define the second quotients of Taylor coefficients as $q_n(f) := \frac{a_{n-1}^2}{a_{n-2}a_n}$, $n \geq 2$ and find conditions on $q_n(f)$ for f to belong to the Laguerre-Pólya class. We also discuss the operators that preserve the Laguerre-Pólya class and its relation to the generating functions of totally positive sequences.

This is joint work with Anna Vishnyakova.

Boundary preimages of linear combinations of iterates of Blaschke products

Spyridon Kakaroumpas
(University of Würzburg)

Let f be a finite Blaschke product on the unit disk with $f(0) = 0$ that is not a rotation. Motivated by earlier results of M. Weiss on lacunary power series, J. J. Donaire and A. Nicolau proved that if $(a_n)_{n=1}^{\infty}$ is a sequence of complex numbers such that $\lim_{n\to\infty} a_n = 0$ and $\sum_{n=1}^{\infty} |a_n| = \infty$, then for all $w \in \mathbb{C}$, there exists $\xi \in \mathbb{D}$ such that $\sum_{n=1}^{\infty} a_n f^n(\xi) = w$, where f^n denotes the n-th iterate of f (i. e. its composition with itself n times).
In this talk we discuss an extension of this result, which shows that under the same conditions on the coefficients $(a_n)_{n=1}^{\infty}$, the set E of all $\xi \in \mathbb{D}$ such that $\sum_{n=1}^{\infty} a_n f^n(\xi) = w$ has Hausdorff dimension 1. We use an iterative construction of appropriate Cantor-type sets, which depends on a careful analysis of the machinery developed by Donaire-Nicolau.
We also explore the optimality of our result under imposing more restrictive conditions on the coefficients $(a_n)_{n=1}^{\infty}$. On the one hand, for any fixed gauge function φ that is "better" than any power function, we provide conditions which ensure that the set E has positive φ-Hausdorff content. On the other hand, we also provide conditions which ensure that E has zero φ-Hausdorff content, in the important special case that the Blaschke product f is of the form $f(z) = z^m$.
This is joint work with O. Soler i Gibert.

Plasticity problem and related questions

Olesia Zavarzina
(V. N. Karazin Kharkiv National University)

A metric space X is called Expand-Contract plastic (or simply plastic) if every non-expansive bijection $f\colon X \to X$ is an isometry. The only general result concerning metric spaces states that every totally bounded metric space is plastic. This result can be found in [5], however, the question about simple characterisation of plastic spaces is open. In [2] B. Cascales, V. Kadets, J. Orihuela and E. J. Wingler posed a question whether the unit ball of every Banach space is plastic. Although there are several partial positive results (see [1],[3],[4] and references therein), the answer is still unknown. In this talk we are going to discuss plasticity in metric spaces and to give some examples of Banach spaces with a plastic unit ball.
The speaker was partially supported by the Volkswagen Foundation grant within the frameworks of the international project „From Modeling and Analysis to Approximation“ and by Akhiezer Foundation grant, 2023.

1. C. Angosto, V. Kadets, O. Zavarzina Non-expansive bijections, uniformities and polyhedral faces, J. Math. Anal. Appl. 471 (1-2) (2019), 38-52.
2. B. Cascales, V. Kadets, J. Orihuela, E.J. Wingler Plasticity of the unit ball of a strictly convex Banach space, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas. 110(2) (2016), 723-727.
3. R. Haller, N. Leo, O. Zavarzina Two new examples of Banach spaces with a plastic unit ball, Acta et Commentationes Universitatis Tartuensis de Mathematica. 26 (1) (2022), 89-101.
4. V. Kadets, O. Zavarzina Plastic pairs of metric spaces, J. Math. Anal. Appl. https://doi.org/10.1016/j.jmaa.2023.127435
5. S. A. Naimpally, Z. Piotrowski, E. J. Wingler Plasticity in metric spaces, J. Math. Anal. Appl. 313 (2006), 38-48.

Weak Hardy/Poincaré inequalities and criticality theory

Marcel Schmidt

In this talk we discuss criticality theory for quadratic forms on $L^2(X,m)$ satsisfying the first Beurling Deny criterion with the help of weak Hardy/Poincaré inequalities. Examples for this setting are quadratic forms of Schrödinger operators and Dirichlet forms - a class which includes the quadratic forms of Laplacians on Riemannian manifolds, fractional Laplacians and discrete Laplacians.

Roughly speaking, criticality theory is the question of (non)existence of generalized ground states, which can be characterized through the validity of Hardy inequalities. In all examples in the literature there is a dichotomy between criticality (existence of generalized ground states) and subcriticality (validity of a Hardy inequality). In this talk we discuss why the dichotomy between criticality and subcriticality may fail in general and how this is related to an open question by Schep'07 on a characterization of the operator norm of positive operators on $L^p(X,m)$ through abstract Schur tests. Our motivation comes from a recent increase of interest in optimal Hardy inequalities for Schrödinger-type operators in various settings, which require the dichotomy mentioned above.

Weak Poincaré inequalities were introduced by Röckner and Wang '01 to quantify the speed of convergence to equilibrium of Markovian semigroups if their generators do not have a spectral gap. We explain why they and weak Hardy inequalities can be used to establish a criticality theory in great generality.

S12 Dynamical Systems

Section leaders: Marc Keßeböhmer (Bremen), Sabrina Kombrink (Birmingham), Tony Samuel (Birmingham)

Session S12_01

Tuesday, 26 September 2023 09:00 (Parksaal 3)

09:00 Jörn Steuding (Julius-Maximilians-Universität Würzburg)
Ergodic theorems for the Riemann zeta-function

09:30 Philipp Kunde (Jagiellonian University in Krakow)
(Anti-)classification results in Dynamical Systems and Ergodic Theory

10:00 Daniel Sell (Nicolaus Copernicus University in Toruń)
On entropy and invariant measures of B-free subshifts

10:30 Uta Renata Freiberg (TU Chemnitz)
The Einstein Relation on Metric Measure Spaces

Session S12_02

Tuesday, 26 September 2023 14:00 (Großer Saal)

14:00 Felix Pogorzelski (Leipzig University)
Colored Heisenberg groups and spectral approximation

14:30 Julia Slipantschuk (University of Warwick)
Distribution of resonances for Anosov maps on the torus

15:00 Manfred Denker
Remarks on iterated function systems

15:30 Markus Haase (Christian-Albrechts-Universität zu Kiel)
Conditional Functional Analysis and the Structure Theory of Dynamical Systems

Session S12_03

Wednesday, 27 September 2023 09:00 (Kleiner Saal)

09:00 Tanja Schindler
Limit Theorems for a class of unbounded observables with an application to "Sampling the Lindelöf hypothesis"

09:30 Maik Gröger
Continuity of Følner averages

10:00 Peter Koltai (University of Bayreuth)
Takens meets Koopman: Linear least squares prediction of nonlinear time series

10:30 Jonas Kirchhoff (Technische Universität Ilmenau)
A behavioural approach to port-Hamiltonian systems

Session S12_04

Thursday, 28 September 2023 13:00 (Parksaal 1)

13:00 Daniel Lenz (Friedrich Schiller Universität Jena)
Pure point spectrum and almost periodicity

13:30 Keivan Mallahi-Karai (Constructor University)
A Central limit theorem for random walks on horospherical products of Gromov hyperbolic spaces

14:00 Alexander Lohse (Universität Hamburg)
Large heteroclinic networks in low-dimensional state space

14:30 Tobias Oertel-Jäger (Friedrich Schiller University Jena)
Isomorphic extensions of irrational rotations that are not almost automorphic

Ergodic theorems for the Riemann zeta-function

Jörn Steuding
(Julius-Maximilians-Universität Würzburg)

The Riemann hypothesis about the location of the complex zeros of the Riemann zeta-function is still an open problem. The same applies to the Lindelöf hypothesis about the growth of the zeta function. In the talk we present an ergodic approach to these two unsolved problems.

(Anti-)classification results in Dynamical Systems and Ergodic Theory

Philipp Kunde
(Jagiellonian University in Krakow)

A fundamental theme in dynamics is the classification of systems up to appropriate equivalence relations. For instance, the equivalence relation of *topological conjugacy* preserves the qualitative behavior of topological dynamical systems. Smale's celebrated program proposes to classify topological or smooth dynamical systems up to topological conjugacy. In Ergodic Theory the *isomorphism problem* dates back to von Neumann's foundational paper and asks to classify measure-preserving transformations up to measure isomorphism.
These classification problems not only turn out to be hard but sometimes even to be impossible. In this talk, we give an overview of classification as well as anti-classification results and present some related projects.

On entropy and invariant measures of B-free subshifts

Daniel Sell
(Nicolaus Copernicus University in Toruń)

For $\mathcal{B} \subseteq \mathbb{N}$, the $\mathcal{B}$-free subshift X_η is the orbit closure of the characteristic function of $\mathcal{B}$-free numbers. Its unique minimal component is generated by a Toeplitz sequence η^*. I will present joint work with A. Dymek and J. Kułaga-Przymus, showing that if η^* is regular, then many results for measures and entropy of the hereditary closure of X_η have their analogues for X_η. An important tool is a result of G. Keller that for taut $\mathcal{B}$, the elements of X_η can be described as sequences between η^* and η. From this we obtain natural periodic approximations that we use frequently in our proofs.

The Einstein Relation on Metric Measure Spaces

Uta Renata Freiberg
(TU Chemnitz)

We review the Einstein relation, which connects the Hausdorff, local walk and spectral dimensions on a space, in the abstract setting of a metric measure space equipped with a suitable operator. This requires some twists compared to the usual definitions from fractal geometry. The main result establishes the invariance of the three involved notions of fractal dimension under bi-Lipschitz continuous isomorphisms between mm-spaces and explains, more generally, how the transport of the analytic and stochastic structure behind the Einstein relation works. While any homeomorphism suffices for this transport of structure, non-Lipschitz maps distort the Hausdorff and the local walk dimension in different ways. To illustrate this, we take a look at Hölder regular transformations and how they influence the local walk dimension and prove some partial results concerning the Einstein relation on graphs of fractional Brownian motions. We conclude by giving a short list of further questions that may help building a general theory of the Einstein relation. The talk is based on a joint work with Fabian Burghart, Uppsala University, Sweden.

Colored Heisenberg groups and spectral approximation

Ram Band[1], Siegfried Beckus[2], Tobias Hartnick[3], Felix Pogorzelski , Lior Tenenbaum[1]
([1] Technion - Israel Institute of Technology and Potsdam University; [2] Potsdam University; [3] Karlsruhe Institute of Technology (KIT); Leipzig University)

Consider a discrete Schrödinger operator with non-periodic, finite-valued potential on the Cayley graph of the Heisenberg group. Can one study its spectrum? This is a general and in many ways challenging program. One angle is to study approximations via finite-volume or periodic analogs. For potentials arising from fixed points of symbolic substitution systems (defined recently for lattices in homogeneous Lie groups by Siegfried Beckus, Tobias Hartnick and myself), we present some qualitiative results in this direction, including ongoing joint work with Ram Band, Siegfried Beckus, and Lior Tenenbaum.

Distribution of resonances for Anosov maps on the torus

Julia Slipantschuk
(University of Warwick)

I will present a complete description of Pollicott-Ruelle resonances for a class of rational Anosov diffeomorphisms on the two-torus. This allows us to show that every homotopy class of two-dimensional Anosov diffeomorphisms contains (non-linear) maps with the sequence of resonances decaying stretched-exponentially, exponentially or having only trivial resonances.

Remarks on iterated function systems

Manfred Denker

A new method is given to estimating Hausdorff dimension. This is based on a general concept for conformal measures and standard arguments in dimension theory.

Conditional Functional Analysis and the Structure Theory of Dynamical Systems

Markus Haase
(Christian-Albrechts-Universität zu Kiel)

The use of functional-analytic methods in ergodic theory, in particular Hilbert space operator theory, is as old as ergodic theory itself (mean ergodic theorem, systems with discrete spectrum vs. weakly mixing systems). However, the classical functional-analytic toolbox is not powerful enough to cover *extensions* of measure-preserving systems. For this, one needs „relative“ or „conditional“ versions of the classical functional-analytic objects and results. For example, the conditional version of a Hilbert space is a so-called *Kaplansky-Hilbert module.* Whereas this notion has long been known to specialists, its relevance for ergodic theory has been recognized only relatively recently, see [1].
In my talk, which is based on joint work with Nikolai Edeko (Zürich) and Henrik Kreidler (Wuppertal), I will try to explain the idea of a „conditional functional analysis“ and to give a small glimpse of its impact on ergodic theory.

[1] Edeko, N. and Haase, M. and Kreidler, H.: A Decomposition Theorem for Unitary Group Representations on Kaplansky-Hilbert Modules and the Furstenberg-Zimmer Structure Theorem, submitted.
https://arxiv.org/abs/2104.04865

Limit Theorems for a class of unbounded observables with an application to "Sampling the Lindelöf hypothesis"

Tanja Schindler

Many limit theorems in ergodic theory are proven using the spectral gap method. So one of the main ingredients for this method is to have a space on which the transfer operator has a spectral gap. However, most of the classical spaces, like for example the space of Hölder or quasi-Hölder function or BV functions don't allow unbounded functions. We will give such a space which allows observables with a pole at the fixed points of a piecewise expanding interval transformation and state a quantitative central limit theorem using Edgeworth expansions. As an application we give a sampling result for the Riemann-zeta function over a Boolean type transformation. This is joint work with Kasun Fernando.

Continuity of Følner averages

Maik Gröger

The notion of generic/mean points goes back to the seminal work of Krylov and Bogolyubov. The first to investigate the question of what happens when all points of a dynamical system are generic for some invariant measure seem to be Dowker and Lederer in 1964. As it turns out, combining this property with other topological regularity criteria yields measure-theoretic rigidity results of the dynamical system. For example, minimality of the system implies its unique ergodicity in this setting. Another natural topological criterion in place of minimality is to assume that the map, which assigns each point its invariant measure to which it is generic, is continuous. By several recent works by different authors, the following picture emerges for abelian group actions in this setting: each point is generic for some ergodic measure and even stronger, each orbit closure is uniquely ergodic. In my talk, I will show that this is no longer the case for general actions by topological amenable groups, providing concrete counter-examples involving the group of all orientation preserving homeomorphisms on the unit interval as well as the Lamplighter group.
This is joint work with G. Fuhrmann and T. Hauser.

Takens meets Koopman: Linear least squares prediction of nonlinear time series

Peter Koltai[1], Philipp Kunde[2]
([1] University of Bayreuth; [2] Jagiellonian University in Kraków)

The least squares linear filter, also called the Wiener filter, is a popular tool to predict the next element(s) of time series by linear combination of time-delayed observations. We consider observation sequences of deterministic dynamics, and ask: Which pairs of observation function and dynamics are predictable? If one allows for nonlinear mappings of time-delayed observations, then Takens' well-known theorem implies that a set of pairs, large in a specific topological sense, exists for which an exact prediction is possible. We show that a similar statement applies for the linear least squares filter in the infinite-delay limit, by considering the forecast problem in an invertible ergodic-theoretic setting and the Koopman operator on square-integrable functions.

A behavioural approach to port-Hamiltonian systems

Jonas Kirchhoff
(Technische Universität Ilmenau)

The port-Hamiltonian framework is used to model the interaction of physical systems with their environment through so called ports. We study the port-structure for port-Hamiltonian systems. The possible interactions between the state-space of the system, which is modeled as a smooth, finite-dimensional manifold M, and its neighbourhood are modeled by a smooth, finite dimensional vector bundle $\pi : E \to M$. When we consider the system together with its neighbourhood as a closed system, then we expect that this system is Hamiltonian, i.e. there is a Dirac structure $\mathcal{D} \subseteq \mathcal{T}E \oplus \mathcal{T}^*E$ so that the behaviour of the total system is given as

$$\mathfrak{B}(\mathcal{D}) := \left\{ e \in \mathcal{C}^\infty(I, E) \,\middle|\, I \subseteq \mathbb{R} \text{ an interval, } \exists H \in \mathcal{C}^\infty(E) : \left(\tfrac{\mathrm{d}}{\mathrm{d}t} e, \mathrm{d}_e H\right) \in \mathcal{D} \right\}.$$

Using parallel transport, we identify a local symmetry, which generalises fiberwise constant functions on the trivial vector bundle $\mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. We show that the symmetric behaviour

$$\mathfrak{B}_s(\mathcal{D}) := \left\{ e \in \mathfrak{B}(\mathcal{D}) \,\middle|\, \tfrac{\mathrm{d}}{\mathrm{d}t} e \text{ symmetric} \right\}$$

can be identified with the behaviour associated to an isotropic structure $\mathcal{I} \subseteq \mathcal{T}M \oplus \mathcal{T}^*M \oplus E \oplus E^*$,

$$\mathfrak{B}(\mathcal{I}) := \left\{ (y, u) \in \mathcal{C}^\infty(I, E \oplus E^*) \,\middle|\, I \subseteq \mathbb{R} \text{ an interval, } \exists H \in \mathcal{C}^\infty(M) : \left(\tfrac{\mathrm{d}}{\mathrm{d}t} \pi(y), \mathrm{d}_{\pi(y)} H, y, u\right) \right\}.$$

Properties of $\mathcal{D}$ under which $\mathcal{I}$ is a Dirac structure are studied. The latter Dirac structure defines the port-structure of port-Hamiltonian systems. This systematic procedure allows us to equip other systems with port-structures, e.g. metriplectic or Nambu systems, which is illustrated with examples.

Pure point spectrum and almost periodicity

Daniel Lenz
(Friedrich Schiller Universität Jena)

We report on recent results on dynamical systems with pure discrete spectrum. (Based on joint work with Timo Spindeler and Nicolae Strungaru.)

A Central limit theorem for random walks on horospherical products of Gromov hyperbolic spaces

Amin Bahramian[1], Behrang Forghani[2], Ilya Gekhtman[3], Keivan Mallahi-Karai
([1] Illinois State University; [2] College of Charleston; [3] Technion (Haifa); Constructor University)

Let G be a countable group acting by isometries on a metric space (M, d), and let μ denote a probability measure on G. The μ-random walk on M is the random process defined by

$$Z_n = X_n \dots X_1 o,$$

where $o \in M$ is a fixed base point, and X_i are independent μ-distributed random variables. Studying statistical properties of the displacement sequence $D_n := d(Z_n, o)$ has been a topic of current research. In this talk, which is based on a joint work with Amin Bahmanian, Behrang Forghani, and Ilya Gekhtman, I will discuss a central limit theorem for D_n in the case that M is the horospherical product of Gromov hyperbolic spaces.

Large heteroclinic networks in low-dimensional state space

Alexander Lohse[1], Sofia Castro[2]
([1] Universität Hamburg; [2] Universidade do Porto)

Heteroclinic connections lie in the intersection of the respective stable and unstable manifolds of invariant sets (nodes) in a dynamical system given through an ordinary differential equation. The nodes are often two steady states ξ_1 and ξ_2, that are thus connected in the sense that there is at least one solution converging to ξ_1 in backward time and to ξ_2 in forward time. Networks of heteroclinic connections can be represented as directed graphs, where vertices and edges in the graph correspond to nodes and connections in the network. Over the last decade the task of realizing a given graph as a heteroclinic network has drawn increasing attention. Several construction methods with different restrictions have been developed to produce suitable vector fields for this purpose.
In this talk we focus on the realization of so-called double-next-neighbour (DNN) graphs, where each vertex has two outgoing edges (connecting to the two subsequent vertices under some ordering). We present a construction method such that for the resulting heteroclinic network (i) all nodes lie on the same coordinate axis, (ii) the vector field is polynomial and most importantly (iii) the state space has dimension at most 6, regardless of the number of vertices in the graph. Some well-known networks of this type result from the Rock-Scissors-Paper and Rock-Scissors-Paper-Lizard-Spock games. Our result is in striking contrast to the existing methods where the dimension of the state space increases unboundedly when the number of nodes goes to infinity.

Isomorphic extensions of irrational rotations that are not almost automorphic

Tobias Oertel-Jäger, Lino Haupt
(Friedrich Schiller University Jena)

We use the well-known Anosov-Katok method to construct skew product torus diffeomorphisms which are mean equicontinuous and isomorphic extensions of the underlying irrational rotations, but have no injectivity points for the projection to the maximal equicontinuous factor. This provides an alternative route to examples first obtained by Downarowicz and Glasner in 2016.
Further, we show that lifts of the constructed torus diffeomorphisms produce no additional dynamical eigenvalues, but exhibit a singular continuous component in their dynamical spectrum. Thus, the measure-theoretic structure coincides with that of certain examples of Toeplitz flows constructed by Iwanik and Lacroix in 1994. However, similar examples with the same of measure-theoretic and topological strucure have - to the best of our knowledge - not been observed before.
Joint work with Lino Haupt (FSU Jena)
DMV-Meeting 2023, Ilmenau, Section 12 Dynamical Systems

MINISYMPOSIA

MS01 Data-Driven Methods for Dynamical Systems

Organized by Hannes Gernandt (TU Berlin), Friedrich Philipp (TU Ilmenau)

Session MS01_01

Wednesday, 27 September 2023 09:00 (W9-Raum)

09:00 Vladimir Kostic (Istituto Italiano di Tecnologia)
Koopman Operator Regression: Statistical Learning Perspective to Data-driven Dynamical Systems

09:30 Friedrich Philipp (Technische Universität Ilmenau)
Kernel Methods for data-based prediction of dynamics

10:00 Ion Victor Gosea (Max Planck Institute for Dynamics of Complex Technical Systems)
Non-intrusive balancing methods from input-output data: an overview and some new results

10:30 Jan Heiland (Max Planck Institute for Dynamics of Complex Technical Systems)
Low-complexity nonlinear systems approximations for efficient nonlinear feedback design

Koopman Operator Regression: Statistical Learning Perspective to Data-driven Dynamical Systems

Vladimir Kostic
(Istituto Italiano di Tecnologia)

Non-linear dynamical systems can be handily described by the associated Koopman operator, whose action evolves every observable of the system forward in time. While data-driven algorithms to reconstruct such operators are well known, their relationship with statistical learning is still largely unexplored. To bridge this gap, in this talk we propose a framework to learn the Koopman operator from finite data trajectories of the dynamical system using reproducing kernel Hilbert spaces. Introducing a notion of risk, our framework Koopman operator regression naturally yields different estimators for which we study generalisation properties. Not only we produce optimal nonasymptotic learning rates for most popular estimators Kernel Ridge Regression (KRR) and Principal Component Regression (aka extended DMD, full and projected) and a novel one Reduced Rank Regression (RRR), but also provide the generalisation guarantees for spectral decomposition of the Koopman operator. Finally, concentrating on time-reversal-invariant Markov chains, which includes important examples of stochastic dynamical systems such Langevin dynamics, our analysis shows how spurious eigenvalues, a phenomenon which has been empirically observed, may occur in learning even well-conditioned operators.

Kernel Methods for data-based prediction of dynamics

Friedrich Philipp[1], Manuel Schaller[2], Feliks Nüske[3], Sebastian Peitz , Karl Worthmann[2]
([1] Technische Universität Ilmenau; [2] Technische Universität Ilmenau (Ilmenau, Thüringen); [3] Max Planck Institute for Dynamics of Complex Technical Systems; Universität Paderborn)

We consider the data-driven approximation of the Koopman operator for stochastic differential equations on reproducing kernel Hilbert spaces (RKHS) by collecting data from either long-term ergodic simulations or independent uniformly distributed samples of the observed dynamics. We derive probabilistic bounds for the finite-data estimation error and a bound on the prediction error of observables in the RKHS using a finite Mercer series expansion. Further, assuming Koopman-invariance of the RKHS, we provide a bound on the full approximation error. The results are illustrated by means of experiments with the Ornstein-Uhlenbeck process.

Non-intrusive balancing methods from input-output data: an overview and some new results

Ion Victor Gosea[1], Björn Liljegren-Sailer[2]
([1] Max Planck Institute for Dynamics of Complex Technical Systems; [2] Trier University)

Non-intrusive reduced-order modeling techniques have become quite popular in recent years, especially because they do not require explicit access to the state-space formulation of the dynamical system to be reduced/approximated (from data). Instead, only measurements are used, either as snapshots of the state variable (in the time domain) or as input-output data. The latter can be samples of state-independent quantities such as the transfer function (in the frequency domain) or the impulse response (in the time domain). Singular Perturbation Approximation (SPA) is a model order reduction method for linear time-invariant systems that guarantees asymptotic stability and for which there exists an a priori error bound. In that respect, it is similar to Balanced Truncation (BT). We present a data-driven reinterpretation of SPA, referred to as QuadSPA, recently proposed in [Liljegren-Sailer/G. '23]. QuadSPA only requires input-output data, and thus, is realization-free. It is based, although only implicitly, on quadrature approximations of the infinite Gramians of the underlying system. It is, in that sense, closely related to the QuadBT method: a data-driven reinterpretation of BT that was proposed in [G./Gugercin/Beattie '22]. If time permits, we will go through both the frequency-domain and also the time-domain formulations of QuadBT and Quad SPA (depending on how the infinite integrals of the Gramians are formulated). The numerical performance is tested for several numerical benchmarks and appropriate comparisons with established intrusive MOR methods are made.

Low-complexity nonlinear systems approximations for efficient nonlinear feedback design

Jan Heiland
(Max Planck Institute for Dynamics of Complex Technical Systems)

Nonlinear feedback design via state-dependent Riccati equations is well established but unfeasible for large-scale systems because of computational costs. If the system can be embedded in the class of linear parameter-varying (LPV) systems with the parameter dependency being affine-linear, then the nonlinear feedback law has a series expansion with constant and precomputable coefficients. In this work, we propose a general method to approximating nonlinear systems such that the series expansion is possible and efficient even for high-dimensional systems. We lay out the stabilization of incompressible Navier-Stokes equations as application, discuss the numerical solution of the involved matrix-valued equations, and confirm the performance of the approach in a numerical example.

MS02 Combinatorial aspects of finite fields

Organized by Gohar Kyureghyan (Rostock), Alexander Pott (Magdeburg)

Session MS02_01

Monday, 25 September 2023 16:30 (Balkonzimmer)

16:30 Michael Kiermaier (Universität Bayreuth)
The degree of functions in the Johnson and q-Johson schemes

17:00 Violetta Weger (Technical University Munich)
CROSS: Signature scheme with restricted errors

17:30 Felicitas Hörmann (German Aerospace Center (DLR))
A Combinatorial Perspective on Sum-Rank-Metric Codes

18:00 Faruk Göloglu
Classification of cryptographic functions arising from projective polynomials

18:30 Anna-Maurin Graner (University of Rostock)
The factorization of $X^n - a$ and $f(X^n)$ over $\mathbb{F}_q$

Session MS02_02

Tuesday, 26 September 2023 09:00 (Balkonzimmer)

09:00 Alexandr Polujan (Otto-von-Guericke-Universität Magdeburg)
On the analysis and construction methods of bent functions

09:30 Tekgül Kalaycı (Sabancı University)
Generalized spread bent partitions and association schemes

10:00 Charlene Weiß (Paderborn University)
Packings and Steiner systems in polar spaces

10:30 Alfred Wassermann (Universität Bayreuth)
Higher incidence matrices and tactical decomposition matrices of designs

The degree of functions in the Johnson and q-Johson schemes

Michael Kiermaier[1], Jonathan Mannaert[2], Alfred Wassermann[1]
([1] Universität Bayreuth; [2] Vrije Universiteit Brussel)

For $q = 1$, let V be a set of finite size n, and for a prime power $q \geq 2$, let V be an $\mathbb{F}_q$-vector space of finite dimension n. We denote the set of all subsets (or subspaces) of V of size (or dimension) k by $\begin{bmatrix} V \\ k \end{bmatrix}$. By the ($q$-)Johnson scheme, the $\mathbb{R}$-vector space of all functions $\begin{bmatrix} V \\ k \end{bmatrix} \to \mathbb{R}$ is decomposed into the orthogonal direct sum $V_0 \perp \ldots \perp V_k$ of eigenspaces. We define the *degree* $\deg f$ of a function $f : \begin{bmatrix} V \\ k \end{bmatrix} \to \mathbb{R}$ as the smallest integer $d \geq 0$ such that $f \in V_0 \perp \ldots \perp V_d$. The degree $\deg \mathcal{F}$ of a set $\mathcal{F} \subseteq \begin{bmatrix} V \\ k \end{bmatrix}$ is defined as the degree of its characteristic function, which is commonly referred to as a Boolean function in this context.

The construction and classification of sets $\mathcal{F}$ of small degree is an intriguing and nontrivial combinatorial question. In the case $q = 1$ and $d = 1$, the classification is known: A set of degree 1 either consists of all k-subsets of V containing a fixed point $P \in V$, or of all k-subsets of V not containing a fixed point P. The case $q = 1$ and $d = 2$ is already open. In the case $q \geq 2$ and $k = 2$, sets $\mathcal{F}$ of degree 1 are the same as Cameron-Liebler sets, which have received quite a bit of attention in the last two decades and are yet far from a complete classification.

In this talk, fundamental properties of the above defined general notion of the degree will be worked out. The connection to the theory of (q-analogs of) designs will be discussed. Moreover, we present and investigate the result of a computer classification of sets of degree 2.

CROSS: Signature scheme with restricted errors

Violetta Weger
(Technical University Munich)

Due to the threat coming from capable quantum computers, the National Institute of Standards and Technology (NIST) is on the search for quantum-secure alternatives. This collaborative process has been of major interest within the cryptographic community over the last decade. This summer, the standardization call reopens solely for digital signature schemes. To find efficient and secure code-based signatures is thus one of the most pressing challenges in cryptography. In this talk, I will introduce the possible approaches to construct code-based signature schemes and discuss their advantages and limitations. Of particular interest will be the approach using zero-knowledge protocols. These have been deemed impractical for a long time, due to their large signature sizes. Recent approaches are able to decrease the signature sizes with the cost of increased running times and more complicated schemes. This raises the question if there is another way to make code-based zero-knowledge protocols practical? We go back to the very roots of the zero-knowledge protocol and identify the large sizes of Hamming isometries as an issue. One possibility to overcome this, is by changing the underlying problem. Instead of asking for an error vector of Hamming weight t, we will consider full weight error vectors where each entry lives in a fixed cyclic subgroup of the multiplicative group. This new restricted decoding problem is still NP-complete and we will see how the algebraic structure of the considered ambient space for the error vectors helps to obtain signatures of competitive sizes.

A Combinatorial Perspective on Sum-Rank-Metric Codes

Felicitas Hörmann
(German Aerospace Center (DLR))

Code-based cryptography is one of the promising approaches for quantum-resistant cryptographic schemes that remain secure in the advent of powerful quantum computers. Most of the recent proposals rely on the McEliece cryptosystem which is based on a secretly chosen error-correcting code $\mathcal{C}$ and a known efficient decoding algorithm for $\mathcal{C}$. The public key is a disguised representation of $\mathcal{C}$ that does not allow to derive any efficient decoder. Encryption then encodes the message by means of the public representation and adds a randomly chosen error of a pre-determined weight to the codeword such that the secret decoder can recover the message.

McEliece-like systems based on algebraic Hamming-metric codes suffer from extremely large keys and are thus neither competitive with respect to lattice- or isogeny-based schemes nor desirable for applications with restrictions in terms of bandwidth or storage. One approach to mitigate the key-size issue is to switch to alternative decoding metrics in which the underlying generic syndrome-decoding problem has a higher attack complexity than its Hamming-metric counterpart. This is for example the case for the sum-rank metric, which generalizes both the Hamming metric and the rank metric. In fact, the sum-rank metric covers a whole spectrum of metrics corresponding to how vectors of a fixed length are subdivided into blocks.

We present recent results [1] that investigate the suitability of linearized Reed-Solomon (LRS) codes for McEliece-like cryptosystems in the sum-rank metric. As LRS codes are the natural generalization of Reed-Solomon (RS) and Gabidulin codes, we show if and how the square-code attack on RS codes in the Hamming metric and Overbeck's attack on Gabidulin codes in the rank metric carry over to LRS codes. Note that LRS codes have more secret parameters than RS and Gabidulin codes, as their construction introduces one additional secret evaluation parameter per code block. Interestingly, these secret evaluation parameters make it much harder to recover the secret parameters from disguised LRS codes than from disguised representations of RS and Gabidulin codes, respectively. In fact, the generalizations of the square-code and Overbeck's attack only run in polynomial time when the exact evaluation parameters are known.

References

[1] F. Hörmann, H. Bartz, and A.-L. Horlemann, „Distinguishing and Recovering Generalized Linearized Reed-Solomon Codes,“ in *Code-Based Cryptography 2022*, pp. 1-20, Springer Nature Switzerland, 2023.

Classification of cryptographic functions arising from projective polynomials

Faruk Göloglu

Perfect nonlinear (PN) functions are essential to cryptography. They are used as building blocks of frequently used ciphers. Mathematical investigation of such functions are connected to several areas of algebra, number theory, geometry and combinatorics.
Monomial functions, which are evaluations of monomials in the polynomial ring $\mathbb{F}_q[X]$, as well as quadratic functions are the first classes to consider. Recently **biprojective** functions are introduced which generalizes the monomial quadratic functions in a structural way. Underlying structure (e.g., automorphism groups) of quadratic monomials over $\mathbf{F}_{p^m}$ are intimately connected with $GL(1, p^m)$. Biprojective functions are naturally defined so that its underlying structure is $GL(2, p^{\frac{m}{2}})$ for even m. This generalization was shown to produce many examples of cryptographic functions as well as interesting mathematical objects.
In this talk, we address solutions of classification problems regarding cryptographic functions and related permutation polynomials and rational functions. These classification results solve many recent open problems on the subject.

The factorization of $X^n - a$ and $f(X^n)$ over $\mathbb{F}_q$

Anna-Maurin Graner
(University of Rostock)

The polynomial $X^n - 1$ and its factorization over $\mathbb{F}_q$ have been studied for a long time. Many results on this, and the closely related problem of the factorization of the cyclotomic polynomials, exist. We study the factorization of the polynomial $X^n - a$ over $\mathbb{F}_q$ for any $a \in \mathbb{F}_q^*$. This factorization is needed for the construction of a-constacyclic codes and has been studied for the case that there exist at most three distinct prime factors of n. Furthermore, it is known that if there exists an element $b \in \mathbb{F}_q$ such that $b^n = a$, the factorization of $X^n - a = X^n - b^n = b^n \cdot ((\frac{X}{b})^n - 1)$ can easily be derived from the factorization of $X^n - 1$. Our proof has a recursive structure which allows us to use short arguments in each step and to consider any positive integer n and any $a \in \mathbb{F}_q^*$. Additionally, we use our results to factor the composition $f(X^n)$, where f is an irreducible polynomial over $\mathbb{F}_q$. The factorization of $f(X^n)$ is known for the case $\gcd(n, \mathrm{ord}(f) \cdot \deg(f)) = 1$. Our results allow us to give the factorization of $f(X^n)$ for any positive integer n.

Keywords: Factorization, irreducible polynomials, composition.

This talk is based on the preprint arXiv:2306.11183.

On the analysis and construction methods of bent functions

Alexandr Polujan
(Otto-von-Guericke-Universität Magdeburg)

Let $\mathbb{F}_2 = \{0, 1\}$ be the finite field with two elements and let n be even. A Boolean function $f\colon \mathbb{F}_2^n \to \mathbb{F}_2$ is called *bent* if it is as far away as possible from all affine functions on $\mathbb{F}_2^n$, i.e., for all $b \in \mathbb{F}_2$ and all non-zero $a \in \mathbb{F}_2^n$ the equation $f(x + a) + f(x) = b$ has 2^{n-1} solutions $x \in \mathbb{F}_2^n$. Since their introduction by Rothaus in the mid-1960s, bent functions play an important role in combinatorics and finite fields due to their broad connections with Hadamard difference sets, certain optimal codes and (partial) spreads. One of the main problems in the theory of bent functions is the design of new generic construction methods of these functions. However, despite significant progress in this direction during the last five decades, most of bent functions in $n = 8$ variables are still not understood: only a tiny portion of size $\approx 2^{76}$ stems from two fundamental classes $\mathcal{PS}$ (the partial spread class) and $\mathcal{MM}$ (the Maiorana-McFarland class), whereas the total number of bent functions on $\mathbb{F}_2^8$ is approximately 2^{106}.

In this talk, I present the results regarding the analysis and construction methods of bent functions f on $\mathbb{F}_2^n$ using the algebraic structure called an $\mathcal{M}$-*subspace*, i.e., a vector subspace $V \subset \mathbb{F}_2^n$ s.t. for all $a, b, \in V$ the equation $f(x + a + b) + f(x + a) + f(x + b) + f(x) = 0$ holds for all $x \in \mathbb{F}_2^n$. By imposing restrictions on permutations π of $\mathbb{F}_2^{n/2}$, we specify the conditions, such that Maiorana-McFarland bent functions $f(x, y) = x \cdot \pi(y) + h(y)$ admit a unique $\mathcal{M}$-subspace of dimension $n/2$. On the other hand, we show that permutations π with linear structures give rise to Maiorana-McFarland bent functions that do not have this property. In this way, we contribute to the classification of Maiorana-McFarland bent functions, since the number of $\mathcal{M}$-subspaces is invariant under equivalence. Additionally, we give several generic methods of specifying permutations π so that $f \in \mathcal{MM}$ admits a unique $\mathcal{M}$-subspace. Most notably, using the knowledge about $\mathcal{M}$-subspaces, we show that using the concatenation of four suitably chosen Maiorana-McFarland bent functions, one can in a generic manner construct bent functions on $\mathbb{F}_2^n$ outside the $\mathcal{MM}$ class (up to equivalence) for any even $n \geq 8$. Remarkably, with our construction methods, it is possible to obtain inequivalent bent functions on $\mathbb{F}_2^8$ not stemming from two primary classes $\mathcal{PS}$ and $\mathcal{MM}$. In this way, we contribute to a better understanding of the origin of bent functions in eight variables.

This talk is based on the recent work [1].

References

[1] Enes Pasalic, Alexandr Polujan, Sadmir Kudin, and Fengrong Zhang. „Design and analysis of bent functions using $\mathcal{M}$-subspaces.“ arXiv preprint arXiv:2304.13432 (2023).

Generalized spread bent partitions and association schemes

Tekgül Kalaycı [1], Nurdagül Anbar[1], Wilfried Meidl[2]
([1] Sabancı University; [2] Alpen-Adria-Universität Klagenfurt)

A partition Ω of a $2m$-dimensional vector space $\mathbb{V}_{2m}^{(p)}$ over the prime field $\mathbb{F}_p$ into subsets $A_1, \ldots, A_K$, is called a bent partition if every function f from $\mathbb{V}_{2m}^{(p)}$ to $\mathbb{F}_p$ with the following property is a bent function: Every $c \in \mathbb{F}_p$ has precisely K/p of the sets A_i in its preimage set. The classical examples of bent partitions are obtained from spreads or partial spreads. From (pre)semifields whose duals satisfy a linearity property over a subfield of $\mathbb{F}_{p^m}$, we construct a large class of bent partitions of $\mathbb{F}_{p^m} \times \mathbb{F}_{p^m}$, which we call generalized semifield spreads. We show that generalized semifield spreads do not arise from any spread and one obtains not only p-ary and vectorial bent functions, but also bent functions into finite abelian groups from generalized semifield spreads. By employing the Bannai-Muzychuk criterion, we show that generalized semifield spreads give rise to amorphic association schemes on $\mathbb{F}_{p^m} \times \mathbb{F}_{p^m}$. This is joint work with Nurdagül Anbar and Wilfried Meidl.

Packings and Steiner systems in polar spaces

Charlene Weiß
(Paderborn University)

A finite classical polar space of rank n consists of the totally isotropic subspaces of a finite vector space equipped with a nondegenerate form such that n is the maximal dimension of such a subspace. A t-Steiner system in a finite classical polar space of rank n is a collection Y of totally isotropic n-spaces such that each totally isotropic t-space is contained in exactly one member of Y. Nontrivial examples are known only for $t = 1$ and $t = n - 1$. We give an almost complete classification of such t-Steiner systems, showing that such objects can only exist in some corner cases. This classification result arises from a more general result on packings in polar spaces, which we obtain by studying the association scheme arising from polar spaces and applying the powerful linear programming method from Delsarte.
This is a joint work with Kai-Uwe Schmidt.

Higher incidence matrices and tactical decomposition matrices of designs

Alfred Wassermann, Michael Kiermaier
(Universität Bayreuth)

In 1985, Janko and Tran Van Trung published an algorithm for constructing symmetric 2-designs with prescribed automorphisms. This algorithm is based on equations by Dembowski (1958) for tactical decompositions of point-block incidence matrices. In the sequel, the algorithm has been generalized and improved in many articles.
In parallel, higher incidence matrices for t-designs with arbitrary strength $t \geq 2$ have been introduced by Wilson in 1982. They have proven useful for obtaining several restrictions on the existence of designs. For example, a short proof of the generalized Fisher's inequality makes use of these incidence matrices.
In this talk we present a unified approach to higher tactical decomposition matrices and incidence matrices. It works for both combinatorial and subspace designs alike. As a result, we obtain a generalization of Fisher's inequality for tactical decompositions of combinatorial and subspace designs of arbitrary strength. Moreover, our approach is explored for the construction of combinatorial and subspace designs for $t > 2$.

Reference:

M. Kiermaier and A. Wassermann: *Higher incidence matrices and tactical decomposition matrices*, Glasnik Matematički, to appear.

MS03 Where optimisation and mathematical systems theory meet

Organized by Simone Göttlich (Mannheim), Karl Worthmann (TU Ilmenau)

Session MS03_01

Monday, 25 September 2023 14:00 (Parksaal 2)

14:00 Thomas Berger (Universität Paderborn)
Funnel MPC - a brief introduction

14:30 Dario Dennstädt (Paderborn University)
Learning-based Robust Funnel MPC

15:00 Karl Worthmann (Technische Universität Ilmenau (Ilmenau, Thüringen))
Stability analysis of eDMD-based model predictive control

15:30 Feliks Nüske (Max Planck Institute for Dynamics of Complex Technical Systems)
Efficient Approximation of the Koopman Operator with Kernels and Random Fourier Features

Session MS03_02

Monday, 25 September 2023 16:30 (Parksaal 2)

16:30 Andrii Mironchenko (University of Passau)
Quadratic Lyapunov functions for linear infinite-dimensional control systems

17:00 Timo Reis (TU Ilmenau)
Boundary controlled Oseen equations in port-Hamiltonian formulation

17:30 Manuel Schaller (Technische Universität Ilmenau)
Stability in elliptic optimal control

18:00 Katharina Klioba (Hamburg University of Technology)
Taming uncertain systems: Approximation of Random Evolution Equations

Session MS03_03

Tuesday, 26 September 2023 09:00 (Parksaal 2)

09:00 Patrick Mehlitz (BTU Cottbus-Senftenberg)
A nonsmooth augmented Lagrangian method and its application to Poisson denoising and sparse control

09:30 Nadja Vater (Julius-Maximilians-Universität Würzburg)
Randomized iterative methods for the solution of nonlinear least squares problems

10:00 Simon Weissmann (University of Mannheim)
Gradient flow structure and convergence analysis of the ensemble Kalman inversion

10:30 Mathias Staudigl (Universität Mannheim)
Hessian Barrier Algorithms for noncovex conic optimization

Funnel MPC - a brief introduction

Thomas Berger
(Universität Paderborn)

MPC is a well-established control technique which relies on the iterative solution of optimal control problems (OCPs). Recently, funnel-like ideas were introduced to overcome some limitations in MPC. The latter means that „artificial“ assumptions are imposed to find an initially feasible solution and to ensure recursive feasibility of MPC (i.e., solvability of the OCP at a particular time instant automatically implies solvability of the OCP at the successor time instant). It was shown that these assumptions are superfluous when „funnel-like“ stage costs are introduced so that the costs grow unbounded when the tracking error approaches the funnel boundary. More precisely, in contrast to simply adding the constraints on the tracking error to the OCP with standard quadratic stage costs, funnel MPC is initially and recursively feasible, without imposing state constraints or terminal conditions and independent of the length of the prediction horizon. This is shown for a large class of nonlinear systems with relative degree one. Available extensions to higher relative degree require so called feasibility constraints, but recent results (which are also presented) aim at avoiding those.
Another recent development is the combination of funnel MPC with an additional funnel control feedback loop, which leads to a control scheme that achieves the tracking objective even in case of severe model-plant mismatches. This resolves another limitation of classical MPC: It requires a sufficiently accurate model to predict the system behavior and compute the optimal control in each step. A second extension of this approach is to improve the model by learning it's parameters from data, while it is still safeguarded by the funnel controller component.

Learning-based Robust Funnel MPC

Dario Dennstädt
(Paderborn University)

Recently, robust funnel Model Predictive Control (MPC), a two component control scheme, has been developed, which achieves output-reference tracking within prescribed performance boundaries for a class of unknown nonlinear multi-input multi-output systems. Consisting of the model-based funnel MPC, which uses a particular stage cost mimicking the high-gain idea of funnel control, and using the adaptive model-free funnel control as an outer feedback-loop, it is robust against structural model-plant mismatches, disturbances, and uncertainties. Extending this control scheme by a learning component allows to continually adapt the underlying model to the system data and thereby improve the prediction capability required in MPC and the overall controller performance. The combined three component controller maintains predefined guarantees on the evolution of the tracking error while the learning component is able to perform safe online learning – even without an initial model or offline training.

Stability analysis of eDMD-based model predictive control

Karl Worthmann[1], Manuel Schaller[2], Lea Bold[3], Lars Grüne
([1] Technische Universität Ilmenau (Ilmenau, Thüringen); [2] Technische Universiät Ilmenau; [3] TU Ilmenau; Universität Bayreuth)

Model Predictive Control (MPC) is nowadays a well-established technique for set-point stabilisation and more-general control task. In this talk, we present a data-based technique using the extended Dynamic Mode Decomposition (eDMD) to generate a surrogate model for the optimisation step in MPC. Further, we rigorously analyse the stability behaviour of the resulting MPC closed loop invoked recently established finite-data error bounds on the approximation error in the so-called Koopman framework, which encompasses both, the projection and the estimation error.

Efficient Approximation of the Koopman Operator with Kernels and Random Fourier Features

Feliks Nüske

(Max Planck Institute for Dynamics of Complex Technical Systems)

Many interesting properties of complex dynamical systems can be inferred from the spectrum of the Koopman operator or its generator. In this context, the Variational Approach to Markov Processes (VAMP) provides a rigorous way of discerning the quality of different approximate models. Kernel methods have been shown to provide accurate and robust estimates for Koopman eigenvalues, but are sensitive to hyper-parameter selection, and require the solution of large-scale generalized eigenvalue problems, which can easily become computationally demanding for large data sizes. In this contribution, we employ a stochastic approximation of the kernel based on random Fourier features (RFFs) to derive a small-scale dual eigenvalue problem which can easily be solved. We provide an interpretation of this procedure in terms of a finite randomly generated basis set. By combining the RFF approach and model selection by means of the VAMP score, we show that kernel parameters can be efficiently tuned. The approach is validated using several benchmarking simulations of biological macromolecules.

Quadratic Lyapunov functions for linear infinite-dimensional control systems

Andrii Mironchenko

(University of Passau)

Quadratic Lyapunov functions are a classical tool to analyze the exponential stability of linear systems. At the same time, for infinite-dimensional systems, quadratic Lyapunov functions constructed by solving the operator Lyapunov equation are not necessarily coercive (which motivated the development of the non-coercive Lyapunov theory).

In this talk, we shed light on the interplay of the quadraticity and the coercivity of Lyapunov functions. We show that an exponentially stable linear infinite-dimensional system in a Hilbert space possesses a coercive quadratic Lyapunov function if and only if the semigroup is similar to a contraction semigroup. If the semigroup is, in addition, analytic, we can explicitly construct the corresponding quadratic Lyapunov function.

Next, we consider the infinite-dimensional systems with inputs (boundary control systems). We show that the input-to-state stability (ISS) of a linear system does not imply the existence of a coercive quadratic ISS Lyapunov function, even if the input operator is bounded. If, however, the semigroup is similar to a contraction semigroup on a Hilbert space, then a quadratic ISS Lyapunov function always exists for any input operator that is bounded, or more generally, p-admissible with $p < 2$. Finally, we construct a family of non-coercive ISS Lyapunov functions for analytic ISS systems under weaker assumptions on B.

This is joint work with Felix Schwenninger (TU Twente, the Netherlands). The ArXiV manuscript can be found here:
https://arxiv.org/abs/2303.15093

Boundary controlled Oseen equations in port-Hamiltonian formulation

Timo Reis[1], Manuel Schaller[2]

([1] TU Ilmenau; [2] Technische Universiät Ilmenau)

We consider Oseen equations, i.e., Navier-Stokes equations which are linearized around a steady state. First we discuss classical boundary conditions for this type, such as velocity prescription, no-slip or "do-nothing" conditions. Thereafter, we present a port-Hamiltonian formulation of boundary-controlled Oseen equations. Our formulation will be on the basis of the framework on so-called "system nodes" by O. Staffans. As a benchmark example, we discuss a pipeline with in- and outflow.

Stability in elliptic optimal control

Manuel Schaller[1], Simone Göttlich[2], Karl Worthmann[1]
([1] Technische Universität Ilmenau; [2] Universität Mannheim)

The stability and dissipativity of dynamic optimal control problems with respect to time horizon and initial state is well studied. It serves as a central property in the analysis model predictive control schemes and manifests itself e.g. in the turnpike property. While this stability is well studied for dynamic optimal control problems, we present an approach to analyse the stability of elliptic optimal control problems. Here, we show that under suitable dissipativity conditions on the involved operators, perturbations of the optimal state, the control and the corresponding adjoint decay exponentially in space. Such a property finds application in the separability of optimal value functions or in adaptive finite elements. We will substantiate all theoretical results with numerical examples.

Taming uncertain systems: Approximation of Random Evolution Equations

Katharina Klioba
(Hamburg University of Technology)

A common challenge in modelling control systems consists in finding appropriate system parameters, such as diffusion coefficients. One possibility to overcome this challenge is to model them by a random variable, resulting in a random evolution equation. Since an analytical solution typically is out of reach, numerical discretisation is needed. Solving evolution equations with random coefficients numerically requires a discretisation in space, in time, and of random coefficients. Methods to treat these three problems separately are well-known, including rates of convergence.
In this talk, conditions are presented under which these rates of convergence are conserved as a joint convergence rate for the fully discretised solution. Uncertainty quantification is performed by means of a polynomial chaos expansion (PCE). To illustrate this interplay of different discretisation schemes, results are discussed for the heat equation with random diffusion coefficients.
This is joint work with Christian Seifert from Hamburg University of Technology.

A nonsmooth augmented Lagrangian method and its application to Poisson denoising and sparse control

Christian Kanzow[1], Fabius Krämer[2], Patrick Mehlitz[3], Gerd Wachsmuth[3], Frank Werner[1]
([1] Julius-Maximilians-Universität Würzburg; [2] University of Bonn; [3] BTU Cottbus-Senftenberg)

Nonsmooth optimization problems in Banach spaces with finitely many inequality constraints, an equality constraint within a Hilbert space framework, and an additional abstract constraint are considered. First, an augmented Lagrangian method for the solution of such problems is suggested and a derivative-free convergence theory is provided. Second, two associated applications from image denoising and sparse optimal control are discussed theoretically, and numerical results are presented as well.

Randomized iterative methods for the solution of nonlinear least squares problems

Nadja Vater, Alfio Borzì
(Julius-Maximilians-Universität Würzburg)

A random preconditioned gradient scheme for the solution of nonlinear least squares problems is presented. The random preconditioner is constructed by employing a random sketching technique developed in the framework of randomized numerical linear algebra [1]. It is shown that the preconditioned gradient update approximates the Gauss-Newton update in every step. Additionally, the applicability of randomized sketching schemes for second order methods is investigated.
In this talk, theoretical and computational aspects of the resulting schemes are discussed. Further, results of numerical experiments for problems arising in the context of the training of artificial neural networks are presented, which demonstrate the effectiveness of methods based on randomized sketching compared to the corresponding schemes without preconditioning.

[1] N. Vater and A. Borzì. *Preconditioned Gradient Descent for Data Approximation with Shallow Neural Networks. Machine Learning, Optimization, and Data Science: 8th International Workshop, LOD, 2022, Certosa di Pontignano, Italy, September 19–22, 2022, Revised Selected Papers, Part II*, 2023, (pp. 357–372)

Gradient flow structure and convergence analysis of the ensemble Kalman inversion

Simon Weissmann
(University of Mannheim)

The ensemble Kalman filter (EnKF) is a widely used methodology for data assimilation problems and has been recently generalized to inverse problems, known as ensemble Kalman inversion (EKI). We view the method as a derivative-free optimization method for a nonlinear least-squares misfit functional and discuss various variants of the scheme such as covariance inflation and regularization. This opens up the perspective to use the method for a wide range of applications, e.g. imaging, groundwater flow problems, biological problems as well as in the context of the training of neural networks. Based on the continuous time formulation we quantify the gradient flow structure of the scheme and present accuracy results of the EKI estimate.

Hessian Barrier Algorithms for noncovex conic optimization

Mathias Staudigl [1], Pavel Dvurechensky[2]
([1] Universität Mannheim; [2] Weierstrass Institute for Applied Analysis and Stochastics)

A key problem in mathematical imaging, signal processing and computational statistics is the minimization of non-convex objective functions that may be non-differentiable at the relative boundary of the feasible set. This paper proposes a new family of first- and second-order interior-point methods for non-convex optimization problems with linear and conic constraints, combining logarithmically homogeneous barriers with quadratic and cubic regularization respectively. Our approach is based on a potential-reduction mechanism and, under the Lipschitz continuity of the corresponding derivative with respect to the local barrier-induced norm, attains a suitably defined class of approximate first- or second-order KKT points with worst-case iteration complexity $O(\epsilon^{-2})$ (first-order) and $O(\epsilon^{-3/2})$ (second-order), respectively. Based on these findings, we develop new path-following schemes attaining the same complexity, modulo adjusting constants. These complexity bounds are known to be optimal in the unconstrained case, and our work shows that they are upper bounds in the case with complicated constraints as well. To the best of our knowledge, this work is the first which achieves these worst-case complexity bounds under such weak conditions for general conic constrained non-convex optimization problems.
This is joint work with Pavel Dvurechensky (WIAS Berlin).

MS04 Extremal and Probabilistic Combinatorics

Organized by Alberto Espuny Díaz (TU Ilmenau), Olaf Parczyk (FU Berlin)

Session MS04_01

Wednesday, 27 September 2023 16:45 (Parksaal 3)

16:45 Simona Boyadzhiyska (University of Birmingham)
Ramsey goodness of loose paths

17:15 Domenico Mergoni Cecchelli (London School of Economics)
The Ramsey number of square of paths

17:45 Pranshu Gupta (University of Passau)
A general approach to transversal versions of Dirac-type theorems

18:15 Eng Keat Hng (Czech Academy of Sciences)
Approximating fractionally isomorphic graphons

Session MS04_02

Thursday, 28 September 2023 13:00 (Kleiner Saal)

13:00 Arturo Merino (Universität des Saarlandes)
Hamilton Cycles in Symmetric Graphs

13:30 Alexander Allin (Technische Universität Ilmenau)
Analogues of Posa's Hamilitonicity theorem for randomly perturbed graphs

14:00 Bertille Granet (Universität Heidelberg)
Random perfect matchings in regular graphs

14:30 Yannick Mogge (Hamburg University of Technology)
Walker-Breaker Games on Random Boards

Ramsey goodness of loose paths

Simona Boyadzhiyska, Allan Lo
(University of Birmingham)

The *Ramsey number* of a pair of graphs (G, H), denoted by $R(G, H)$, is the smallest integer n such that, for every red/blue-coloring of the edges of the complete graph K_n, there exists a red copy of G or a blue copy of H. In the 1980s, Burr showed that, if G is large and connected, then $R(G, H)$ is bounded below by $(v(G)-1)(\chi(H)-1)+\sigma(H)$, where $\chi(H)$ is the chromatic number of H and $\sigma(H)$ stands for the minimum size of a color class over all proper $\chi(H)$-colorings of H. We say that G is *H-good* if $R(G, H)$ is equal to this general lower bound. This notion was first studied systematically by Burr and Erdős and has received considerable attention from researchers since its introduction. Among other results, it was shown by Burr that, for any graph H, every sufficiently long path is H-good.
These concepts generalize in the natural way to k-graphs, and in this talk we will explore the notion of Ramsey goodness when G is an ℓ-path for some $1 \leq \ell \leq k-1$. We will show that, while long loose paths are not always H-good, they are very close to being H-good for every k-graph H. As we will see, this is in stark contrast to the behavior of ℓ-paths for larger ℓ.

The Ramsey number of square of paths

Domenico Mergoni Cecchelli[1], Peter Allen, Jozef Skokan, Barnaby Roberts[2]
([1] London School of Economics; [2] Department of education)

The problem of finding explicit values of the Ramsey number $R(H, H)$ has been studied extensively, but the explicit value of $R(H, H)$ is known only for a few graphs H. Among the most relevant results in the area, Gerencsér and Gyárfás proved in 1967 that $R(P_{2n}, P_{2n}) = 3n - 1$, where P_{2n} is the path on $2n$ vertices.
We denote by P_{3n}^2 the square of the path on $3n$ vertices, which is the graph over $3n$ vertices obtained from P_{3n} by adding edges between vertices of distance 2. We prove that $R(P_{3n}^2, P_{3n}^2) = 9n - 3$ for n large enough.

A general approach to transversal versions of Dirac-type theorems

Pranshu Gupta[1], Fabian Hamann, Alp Müyesser, Olaf Parczyk, Amedeo Sgueglia
([1] University of Passau)

Given a collection of hypergraphs $\mathbf{H} = (H_1, \ldots, H_m)$ with the same vertex set, an m-edge graph $F \subset \cup_{i\in[m]} H_i$ is a transversal if there is a bijection $\phi : E(F) \to [m]$ such that $e \in E(H_{\phi(e)})$ for each $e \in E(F)$. How large does the minimum degree of each H_i need to be so that $\mathbf{H}$ necessarily contains a copy of F that is a transversal? Each H_i in the collection could be the same hypergraph, hence the minimum degree of each H_i needs to be large enough to ensure that $F \subseteq H_i$. Since its general introduction by Joos and Kim [Bull. Lond. Math. Soc., 2020, 52(3): 498–504], a growing body of work has shown that in many cases this lower bound is tight. In this paper, we give a unified approach to this problem by providing a widely applicable sufficient condition for this lower bound to be asymptotically tight. This is general enough to recover many previous results in the area and obtain novel transversal variants of several classical Dirac-type results for (powers of) Hamilton cycles. For example, we derive that any collection of rn graphs on an n-vertex set, each with minimum degree at least $(r/(r + 1) + o(1))n$, contains a transversal copy of the r-th power of a Hamilton cycle. This can be viewed as a rainbow version of the Pósa-Seymour conjecture.

Approximating fractionally isomorphic graphons

Eng Keat Hng
(Czech Academy of Sciences)

Fractional isomorphism of finite graphs is an important and well-studied concept at the intersection of graph theory and combinatorial optimization. It has many different characterizations that involve a range of very different and seemingly unrelated properties of graphs. Recently, Grebík and Rocha developed a theory of fractional isomorphism for graphons, i.e. limits of dense graph sequences, where they showed that every characterization of fractional isomorphism in graphs has a well-defined graphon counterpart and that these are indeed all equivalent characterizations of fractional isomorphism for graphons. They asked whether fractionally isomorphic graphons can be characterized as the limits of graph sequences which are entrywise fractionally isomorphic (in the graph sense). We answer their question in the affirmative. This is based on joint work with Jan Hladký.

Hamilton Cycles in Symmetric Graphs

Arturo Merino
(Universität des Saarlandes)

A classic conjecture due to Lovász states that (up to finitely many exceptions) every vertex-transitive graph has a Hamilton cycle. While we are still far from the general conjecture, some general techniques are available for building Hamilton cycles when the graph is highly symmetric.
In this talk, we will discuss two such techniques: *cycle gluing*, which involves adequately gluing a 2-factor into a Hamilton cycle, and *cycle lifting*, which lifts a cycle from a quotiented version of the graph into the original one. In particular, we will show how these techniques allow for Hamilton cycle constructions in all Kneser graphs (except the Petersen graph) [M., Mütze, Namrata 2023+] and for (near-)optimal *symmetric* Hamilton cycles in hypercubes, permutahedra, and more [Gregor, M., Mütze 2023+].

Analogues of Posa's Hamilitonicity theorem for randomly perturbed graphs

Alexander Allin, Alberto Espuny Díaz
(Technische Universität Ilmenau)

We consider Hamilton cycles in randomly perturbed graphs, that is, graphs obtained as the union of a deterministic graph H and a random graph. For this random pertubation we consider both the binomial random graph $G(n,p)$ and the geometric random graph $G(n,r)$. While most research into randomly perturbed graphs assumes a minimum degree condition on H, here we consider conditions on its degree sequence. Under the assumption of a degree sequence of H which is comparable with the classical condition of Posa (dependent on a parameter α analogous to the minimum degree condition in typical results in the area), we prove that there exists some constant $C = C(\alpha)$ such that taking $p = C/n$ suffices to a.a.s. obtain a Hamilton cycle in $H \cup G(n,p)$. Under the same conditions on H we further prove that there is a constant $K = K(\alpha)$ such that $r = \sqrt{K/n}$ ensures a.a.s. that $H \cup G(n,r)$ is Hamiltonian. Our results are best possible both in terms of the degree sequence condition and the asymptotic value of p and r, and extend the known results about Hamiltonicity in randomly perturbed graphs.
This is joint work with Alberto Espuny Díaz.

Random perfect matchings in regular graphs

Bertille Granet
(Universität Heidelberg)

We prove that in all regular robust expanders G, every edge is asymptotically equally likely contained in a uniformly chosen perfect matching M. We also show that given any fixed matching or spanning regular graph N in G, the random variable $|M \cap E(N)|$ is approximately Poisson distributed. This in particular confirms a conjecture and a question due to Spiro and Surya, and complements results due to Kahn and Kim who proved that in a regular graph every vertex is asymptotically equally likely contained in a uniformly chosen matching. Our proofs rely on the switching method and the fact that simple random walks mix rapidly in robust expanders.
This is joint work with Felix Joos.

Walker-Breaker Games on Random Boards

Dennis Clemens[1], Pranshu Gupta[2], Yannick Mogge[1]
([1] Hamburg University of Technology; [2] University of Passau)

The Maker-Breaker connectivity game and Hamilton cycle game belong to the best studied games in positional games theory, including results on biased games, games on random graphs, and fast winning strategies. Recently, the Connector-Breaker game variant, in which Connector has to claim edges such that her graph stays connected throughout the game, as well as the Walker-Breaker game variant, in which Walker has to claim her edges according to a walk, have received growing attention.
For instance, London and Pluhár [2] studied the threshold bias for the Connector-Breaker connectivity game on a complete graph K_n, and showed that there is a big difference between the cases when Maker's bias equals 1 or 2. Moreover, a recent result [1] shows that the threshold probability p for the $(2:2)$ Connector-Breaker connectivity game on a random graph $G \sim G_{n,p}$ is of order $n^{-2/3+o(1)}$. We extent this result further to Walker-Breaker games and prove that this probability is also enough for Walker to create a Hamilton cycle.

References

[1] Dennis Clemens, Laurin Kirsch, and Yannick Mogge, *Connector-Breaker games on random boards*, The Electronic Journal of Combinatorics, Volume 28, Issue 3, 2021, P3.10.

[2] András London and András Pluhár, *Spanning tree game as Prim would have played*, Acta Cybernetica, Volume 23 No 3, 2018, pp. 921-927.

MS05 Towards a digital infrastructure for mathematical research

Organized by Tabea Bacher (MPI MiS Leipzig), Thomas Koprucki (WIAS Berlin), Moritz Schubotz (FIZ Karlsruhe), Karsten Tabelow (WIAS Berlin), Olaf Teschke (FIZ Karslruhe)

Session MS05_01

Monday, 25 September 2023 14:00 (VP-Raum)

14:00 Olaf Teschke (FIZ Karlsruhe - Leibniz-Institut für Informationsinfrastruktur)
Exploring the Mathematical Research Data Initiative's Knowledge Graph

14:30 Kai Kortus; Sofia Raible
Math4VIP - Digital dimensions of accessibility in STEM

15:00 Daniel Mietchen (FIZ Karlsruhe - Leibniz-Institut für Informationsinfrastruktur)
Mathematics and Wikidata

15:30 Anita Eppelin (Technische Informationsbibliothek (TIB))
Supporting researchers in the publication process with open, algorithm-based tools

Session MS05_02

Monday, 25 September 2023 16:30 (VP-Raum)

16:30 Antony Della Vecchia (TU Berlin)
A FAIR File Format for Mathematical Software

17:00 Olaf Teschke (FIZ Karlsruhe - Leibniz-Institut für Informationsinfrastruktur)
API solutions for interlinking research data: the case study of DLMF, OEIS, and zbMATH Open

17:30 Marco Reidelbach (Zuse Institute Berlin)
MaRDMO – Document and Retrieve Interdisciplinary Workflows using the MaRDI Portal.

18:00 Pavan L Veluvali (Max Planck Institute for Dynamics of Complex Technical Systems)
A concrete and abstract CSE workflow framework for FAIR numerical experiments

Exploring the Mathematical Research Data Initiative's Knowledge Graph

Olaf Teschke, Moritz Schubotz
(FIZ Karlsruhe - Leibniz-Institut für Informationsinfrastruktur)

This presentation offers an in-depth review of the current state of data available in the Knowledge Graph of the Mathematical Research Data Initiative (MaRDI). It illustrates various techniques to access this expansive resource. Following a detailed presentation of statistics related to the quantity of nodes available within the Knowledge Graph, we will provide a practical demonstration of how this data can be accessed via the MaRDI portal (https://portal.mardi4nfdi.de). Our aim is to illuminate the potential and utility of this initiative, emphasizing the accessibility and applicability of mathematical research data in the contemporary digital environment.

Math4VIP - Digital dimensions of accessibility in STEM

Kai Kortus, Sofia Raible

The study of mathematics and other STEM subjects is a great challenge for students with visual impairments due to the visual contents and representations employed throughout lectures, seminars

and textbooks. Visually impaired students are thus largely underrepresented in STEM subjects.
The goal of the Math4VIP project, a joint venture between the University of Marburg and ACCESS@KIT, is to create a central platform that contains information about access to mathematics and how to provide it to visually impaired students. In cooperation with MaRDI, new standards will be developed, and guidelines will be written and publicized through appropriate outreach.
During the talk, we will introduce and explain the merits of using MaRDI's central research database for the distribution of these materials and as a junction between several online resources such as tactiles.eu. We will present the work of Math4VIP and invite an open discussion and cooperation concerning ideas and opportunities for the project.

Mathematics and Wikidata

Daniel Mietchen
(FIZ Karlsruhe - Leibniz-Institut für Informationsinfrastruktur)

Wikidata is an open and collaborative database that anyone can edit, which thousands do on a regular basis. Launched a decade ago, FAIR (Findable, Accessible, Interoperable and Reusable) right from the start and closely integrated with its sister sites in the Wikipedia ecosystem, it has since become the edit button of the semantic web and is increasingly being integrated with scholarly databases and workflows spanning across all fields of research. This presentation will consider Wikidata through the lense of mathematics, covering content, infrastructure and community aspects and how each of these are curated and interlinked both within and beyond Wikidata. In terms of content, coverage of mathematical concepts will be explored, including objects of mathematical research, software and other methods used in mathematical research, along with mathematical aspects of research in other fields as well as mathematical literature and linguistic knowledge about mathematical terminology across natural languages. In terms of infrastructure, support mechanisms for describing, displaying and analyzing mathematical objects in Wikidata contexts will be discussed. In terms of communities, we will cover producers, curators and users of mathematical knowledge and data, along with community structures engaged in any aspect of the life cycle of mathematical entities, both in scholarly and Wikidata contexts.

Supporting researchers in the publication process with open, algorithm-based tools

Anita Eppelin[1], Elias Entrup[2]
([1] Technische Informationsbibliothek (TIB); [2] Research Group Visual Analytics, Technische Informationsbibliothek (TIB))

There is a whole range of web-based services out there designed to assist researchers in the publication process - in their role as authors, editors, reviewers - at its various stages. Few of those, however, are inherently aligned with the needs and interests of the research community: the underlying procedures and data sources are not transparent and the source code not publicly available, their sustainability perspective is unclear, they are often provided by commercial actors. In our talk, we present two approaches with which we want to do things differently: B!SON, an open journal recommender that is already available, and our ideas for a reviewer recommender, based on the preliminary work and procedures from the B!SON development.
B!SON is a tool for identifying suitable Open Access journals for a manuscript, developed and provided by TIB (Technische Informationsbibliothek) and SLUB (Saxonian State and University Library Dresden). It uses community-based, open data sources, discloses its recommendation algorithm based on semantic and bibliometric similarity detection, handles user data sensitively and has been designed with sustainability in mind. The simple user interface, with input fields for a manuscript's title, abstract and references, provides users with a list of Open Access journals in which similar research has been published. For each journal listed, a similarity score is provided, along with the underlying articles where similarity was found, to make the recommendation verifiable. With the development of B!SON, we primarily aim at supporting researchers in their decision-making process and promote open access publishing options. At the same time, however, we want to support libraries as well as other institutions providing publishing advice, but also journals and publishers, especially smaller ones.
With our new project we want to build on the semantic and bibliometric similarity detection of B!SON while applying the same principles of openness in the development, but investigate a different use case: the need for an open tool for reviewer search, especially in the emerging landscape of Open Access journals run by the scientific community. Providing a tool to better manage the peer review process that can be integrated into existing publication infrastructures will support these infrastructures and the publications appearing there, and eventually enhance their reputation and attractiveness. Its use carries the potential to promote a faster, appropriate and less biased review process. In addition to reducing the workload for the selection of reviewers by editors, this can increase the confidence of authors in community-based open access publications and ultimately the quality of these publications. Furthermore, such a tool, which draws on large data corpora of already published literature and other scientific output, makes it easier to find reviewers for innovative and unique as well as inter- or transdisciplinary topics for the respective sub-disciplines than is possible with manual searches, and it can also provide initial indications of possible conflicts of interest.

A FAIR File Format for Mathematical Software

Antony Della Vecchia, Michael Joswig, Benjamin Lorenz
(TU Berlin)

Due to the complexity of mathematical objects, storing data so it is FAIR has its challenges. We describe a generic `JSON` based file format which is suitable for computations in computer algebra, highlighting some design decisions and demonstrating some of the benefits with an emphasis on interoperability. This is implemented in the computer algebra system `OSCAR`, but we also show how it can be used in a different context.

API solutions for interlinking research data: the case study of DLMF, OEIS, and zbMATH Open

Matteo Petrera, Olaf Teschke
(FIZ Karlsruhe - Leibniz-Institut für Informationsinfrastruktur)

zbMATH Open is a recognized information system that connects a broad variety of resources relevant for mathematics research, facilitated by the current opportunities in the framework of Open Access and Open Data. Mathematical research data are an essential additional information layer in such a network and provide a necessary tool for the advancement of mathematics . For the interlinking of research data, zbMATH Open provides Application Programming Interface (API) solutions. Among other APIs recently implemented at zbMATH Open, the so-called Links API is aimed to document interconnections between our database and external platforms which display mathematical literature indexed at zbMATH Open.
The Digital Library of Mathematical Functions (DLMF) has the first resource for interlinking and their data have been integrated in zbMATH Open in 2021. Recently, the Online Encyclopedia of Integer Sequences (OEIS), a renowned digital database of number sequences which is especially relevant for combinatorics and number theory, has been added.
The purpose of this talk to discuss the scientific potential of interlinking data in mathematical research, with particular emphasis on the latest developments brought by the Links API. In particular, we will show how starting from a document that is cited by DLMF or OEIS is now immediate to be redirected to the specific web pages.

MaRDMO – Document and Retrieve Interdisciplinary Workflows using the MaRDI Portal.

Marco Reidelbach , Eloi Ferrer, Marcus Weber
(Zuse Institute Berlin)

The Mathematical Research Data Initiative (MaRDI) has set itself the goal of making information about mathematical objects, e.g. algorithms or models, available in a structured and easy-to-find manner in the form of a knowledge graph. The linking of all these objects with concrete research questions, input and output data, software and hardware is done in specific workflows. To achieve reproducibility of these, often interdisciplinary, workflows, detailed documentation is required. For this purpose, a standardized workflow documentation template was developed in the MaRDI project, which can be completed by answering a simple questionnaire in the widely used Research Data Management Organiser (RDMO). Workflows recorded in this way can be stored locally or published directly on the MaRDI portal. In addition, central information of the documentations is integrated into the MaRDI Knowledge Graph and connected to other data sources, e.g. wikidata. Next to the pure documentation of workflows, MaRDMO offers the possibility to retrieve existing workflows from the MaRDI Knowledge Graph in order to provide researchers with suggestions for future projects and to document workflows based on these suggestions. Thus, MaRDMO creates a community-driven knowledge loop that could help to overcome the replication crisis.

A concrete and abstract CSE workflow framework for FAIR numerical experiments

Pavan L Veluvali, Jan Heiland, Peter Benner
(Max Planck Institute for Dynamics of Complex Technical Systems)

Numerical algorithms and computational tools are essential for managing and analyzing complex data processing tasks. With the availability of meta-data and parameter-driven simulations, the demand for automated workflows to reproduce computational experiments across platforms has grown. In general, a computational workflow is defined as a step-by-step description for accomplishing a scientific objective, expressed in terms of tasks and their data dependencies. Characterized through their input-output relation, workflows are designed such that the associated meta-data can be used interchangeably and redundantly. In the present work, we develop a computational framework, namely, MaRDIFlow, that focuses on the automation of abstracting meta-data embedded in an ontology of mathematical objects while negating the underlying execution and environment dependencies into multi-layered descriptions. Notably, by allowing the complete range between abstract descriptions and concrete numerical realizations (or even plain input-output data) of the tasks to serve equivalently and possibly redundantly in the definition of the workflows, we provide the lowest possible barrier for findable, accessible, interoperable and reusable (i.e. FAIR) workflow definitions. We showcase minimum working examples and how they are systematically incorporated into our workflow tool and data provenance framework.

MS06 Mathematics and Arts

Organized by Milena Damrau (Bielefeld), Martin Skrodzki (TU Delft)

Session MS06_01

Monday, 25 September 2023 16:30 (Großer Saal)

16:30 Opening
Opening

16:40 Susanne Spies (Uni Siegen)
"Maker of patterns." - Mathematics as a result of artistic creation.

17:30 Ana Oosting
An artist's view on Folds, Tesselations, Curvature, and other Science

18:00 Marlene Knoche
Hilbert’s Holidays - A mathematical point-and-click adventure game

Session MS06_02

Wednesday, 27 September 2023 16:45 (Großer Saal)

16:45 Tim Kunt (Zuse Institute Berlin)
Designing a Runway Choreography with Integer Programming

17:15 Kevin Walker
The mathematics of looking less mathematical

17:45 Alex Kontorovich (Rutgers University)
Polyplane: Exploring the natural laws of shape

18:15 Gabriel Dorfsman-Hopkins (St. Lawrence University)
Twenty-Seven

Session MS06_03

Thursday, 28 September 2023 13:00 (Parkcafé)

13:00 Diaaeldin Taha (Heidelberg University)
Mathematics and Visualization in Action: A Selection of Student Projects from the Heidelberg Experimental Geometry Lab

13:30 Pauline Hengst[1]; Abel de Bruijn; Rens Dur; Julia van der Kris ([1] Delft University of Technology)
Transforming hand-drawn level curves to augmented reality for classroom use

14:00 Shereen ElBedewy (JKU)
STEAM A for Arts or Architecture!

14:30 Christopher R H Hanusa (Queens College, City University of New York)
Mathematical Art on the AxiDraw Pen Plotter

Session MS06_04

Thursday, 28 September 2023 15:30 (Parkcafé)

15:30 Lucia Rossi (Montanuniversität Leoben)
Inter affine tiles

16:00 Joshua Holden (Rose-Hulman Institute of Technology)
Twist, Turn, and Shout: The Symmetries of Braided Cords

16:30 Audrey Nasar (The Fashion Institute of Technology)
"Symmetry" Card Game

17:00 Aaron Fenyes
Geodesic music

Opening

Opening

"Maker of patterns." - Mathematics as a result of artistic creation.

Susanne Spies
(Uni Siegen)

"A Mathematician, like a painter or a poet, is a maker of patterns. [. . .] A painter makes patterns with shapes and colors, a poet with words. [. . .] A mathematician, on the other hand, has no material to work with but ideas." (G.H. Hardy)
Statements like this by the famous British mathematician G.H. Hardy, which places mathematics as an art in the ranks of the fine arts, are as widespread among mathematicians as they are at least astonishing to non-mathematicians: How can a proving science that operates with numbers, signs and formulae and that is committed to a logical-deductive structure of its results be regarded as an art form? - How is creative artistlike work conceivable here? This apparent contradiction is usually met by mathematicians by claiming a multitude of analogies between mathematics and art forms of all kinds. These will be traced in proposed talk. Thereby a special focus will be placed on the material of the creative process within mathematics: How can one imagine the patterns mentioned? How is a creative approach to these patterns possible? How do the mathematicians works of art become accessible?

An artist's view on Folds, Tesselations, Curvature, and other Science

Ana Oosting
(nan)

In my talk, I will report on my journey from science to art and how my art practice has evolved over time (with specific focus on the role of science and the role of material agency in my practice). Following this, I will speak about folds and tessellations and take the remaining time to highlighting three recent projects and the role maths play in them (one on the Gaussian curvature, one on the use of Cryo-EM and the discovery of protein shapes, and one on tipping points in ecosystems).

Hilbert's Holidays - A mathematical point-and-click adventure game

Marlene Knoche

To aim at mathematically interested members of the general public, my approach in mathematics outreach has been to develop an illustrated point-and-click adventure game about the lives and history of famous mathematicians. Colourful illustrations and animations pair with quirky music, humour and love for little details in the game "Hilbert's Holidays". The player sets off on a vacation in the famous Hilbert Hotel - a thought experiment by mathematician David Hilbert - to meet different mathematicians and learn about their life and research, solve some riddles and learn about mathematics in a uniquely fun way. David Hilbert himself as well as Kurt Gödel, Évariste Galois, Emmy Noether and more will share their stories in conversations and with the help of historic documents like a radio speech and pictures.
The game is currently being developed as part of my fellowship at the MIP.labor, which is a research project for innovative science journalism in mathematics, computer science and physics. The MIP.labor is located at Freie Universität Berlin and funded by Klaus Tschira Stiftung.

Designing a Runway Choreography with Integer Programming

Tim Kunt
(Zuse Institute Berlin)

For fashion shows, stage design, choreography, and seating arrangement inevitably impose what Kondo [1] calls the politics of seating. We implemented integer programming as part of a tool to choreograph the winter 2022/2023 runway show at the Berlin University of Arts, Institute of Experimental Fashion and Textile Design. The choreography aims to create an equal experience for all viewers, maximizing visibility of all looks through their positions on stage and the running order. A hybrid design approach combined mathematical optimization for combinatorial designs and the traditional iterative process of rehearsal and curation.
As the mathematical core of the approach potentially translates to a broader range of applications, we further describe a general framework for choreographies in performing arts.

The mathematics of looking less mathematical

Kevin Walker

One can start with a mathematical idea and turn it into art. One can also start with an artistic goal and create math to realize that goal. This talk will be about the latter process.
I'll give several examples (each with many illustrations) of creating and refining algorithms to realize and enhance artistic ideas which start out and pen and paper doodling. This will involve math techniques such as Voronoi tessellations, spanning trees, random walks, and the discrete Poisson equation.
Much of my work is concerned with the interplay of regularity and irregularity. With hand-crafted art regularity is sometimes difficult to achieve and irregularity is easy. With algorithmic art the reverse is true: regularity comes easily and irregularity requires more effort. I'll discuss approaches to balancing regularity and irregularity at different spatial scales.

Polyplane: Exploring the natural laws of shape

Glen Whitney[1], Alex Kontorovich[2]
([1] Studio Infinity; [2] Rutgers University)

Few fundamental theorems of mathematics are as accessible to direct physical observation as Euler's Polyhedron Formula. To capitalize on this opportunity, the authors have collected numerous examples

[1] D. Kondo. About face: Performing race in fashion and theater. Routledge, 2014.

of polyhedra from contributing mathematical artists, popular culture, and consumer products. In the traveling exhibition entitled *Polyplane*, they display each one of these shapes precisely located in three dimensions at coordinates determined by its numbers of faces, edges, and vertices. This exhibition therefore allows visitors not only to appreciate the variety and beauty of polyhedra and art based on polyhedral forms, but to see Euler's formula made manifest in the spontaneous planar configuration of the items when „plotted“ in space in this way. The presenters will discuss the collection and construction of the exhibition as well as the didactic and communication issues involved in using the exhibition to help a broad audience understand and appreciate a major milestone in mathematical discovery.

Twenty-Seven

Gabriel Dorfsman-Hopkins [1], Daniel Rostamloo[2]
([1] St. Lawrence University; [2] University of Washington)

Twenty-Seven is an interactive sculpture depicting a cubic surface, a mathematical object arising in algebraic geometry. The interior of the surface is left empty except for several glowing acrylic cables which attach to the frame of the sculpture, illustrating the remarkable and mysterious fact that a cubic surface contains exactly 27 lines. We aim to give participants an interactive opportunity to engage directly with this beautiful theory, hoping to transport mathematics outside the realm of pure abstract thought and into the physical and emotional world. In this talk, we will discuss our designs (and redesigns), and the choices and goals that led us to this multimedia art piece incorporating 3D printed parts, with electronics and LEDs controlled by an Arduino microcontroller.

Mathematics and Visualization in Action: A Selection of Student Projects from the Heidelberg Experimental Geometry Lab

Diaaeldin Taha
(Heidelberg University)

The Heidelberg Experimental Geometry Lab (HEGL) is a community of researchers at Heidelberg University connected by their interest in exploring various facets of mathematics, including experimental methods of inquiry, visualization techniques, and interdisciplinary applications. In this talk, we showcase the output of a selection of student projects from HEGL which involved creating mathematical illustrations and artistic objects, such as interactive software, 3D printed sculptures, virtual reality simulations, laser engraved artworks, and community-created outreach installations. We discuss how these projects highlight the connection between mathematics and the arts and how they foster creativity, curiosity, and a sense of community among the students. We also reflect on the challenges and benefits of organizing such projects. We conclude with a message: Visualization projects are a valuable and innovative way to engage students in mathematics and promote interdisciplinary learning and research.

Transforming hand-drawn level curves to augmented reality for classroom use

Pauline Hengst[1], Abel de Bruijn, Rens Dur, Julia van der Kris, Jonas van Marrewijk, Beryl van Gelderen, Dennis den Ouden-van der Horst
([1] Delft University of Technology)

We present a mobile application designed to enhance students' understanding of directional derivatives and level curves in first years calculus. Developed in collaboration with PRIME (Programme of Innovation in Mathematics Education) at TU Delft, the application offers visual tools and gamified learning to provide an engaging educational experience. The application aims for ease of use, for teachers and its ability to convey key insights on the relationship between gradients and the shape of a model generated from hand-drawn level curves. Through this presentation, attendees will gain a comprehensive understanding of the application's features and the benefits it offers to students in comprehending directional derivatives.

STEAM A for Arts or Architecture!

Shereen ElBedewy , Zsolt Lavisca
(JKU)

Abstract
Title: STEAM A for Arts or Architecture! Authors: Shereen El Bedewy, Prof. Zsolt Lavisca In this presentation, we will present our work including transdisciplinary STEAM (Science, Technology, Engineering, Arts, Mathematics) practices connecting real-life examples such as architecture to foster mathematics education using mathematical modelling. Architecture includes an implicit relationship with art (Omale & Ogunmakinde, 2018). Architecture includes aesthetic features that create the forms ensuring buildings' symmetry and proportional balance. Moreover, some architecture includes artistic geometrical patterns, statues or colours that sometimes have a cultural relationship (Hamilton, 2019). All of these features are elements of arts and strengthen the architecture arts connections. Therefore, we utilise these connections in designing transdisciplinary STEAM educational practices that encourage learners to model architecture using CAD software such as GeoGebra. Furthermore, we include technologies such as Augmented reality and 3D printing to visualise these architectural models in digital and physical forms. Moreover, we will discuss the theoretical frameworks and methodology used in designing these STEAM practices We will highlight the implementation of these STEAM practices cross-culturally that resulted in diverse outcomes. Furthermore, we will discuss the qualitative analysis of these outcomes showing the emerged themes focusing on how participants utilised arts features in the architectural models while implementing these STEAM practices. “There is a tendency to judge the aesthetics of architecture from its external appearance, much like a painting or sculpture. We observe pleasing proportions, suitable materials, interesting colours, fit with the cultural or regional context, and the craft of appropriate detailing”(Hamilton, 2019, p.1). Figure1:
Figure 2: Architectural modelling examples by participating teachers from Libya
References: Omale, R. P., & Ogunmakinde, O. (2018). Comparative Analysis between Art and Architecture. Online Journal of Art and Design, 6(2), 15-31. Hamilton, D. K. (2019). Architecture: Object and Experience. HERD: Health Environments Research & Design Journal, 12(1), 87–90. https://doi.org/10.1177/1937586719832211.

Mathematical Art on the AxiDraw Pen Plotter

Christopher R H Hanusa
(Queens College, City University of New York)

In this talk I discuss the project-based course „Mathematical Design“ that I developed at Queens College to give non-math students a glimpse into the beauty of mathematics. Students learn the mathematics of transformations of functions, polar equations, and parametric equations, as well as the elements of art and design. They use this knowledge to create artwork using the online graphing software Desmos and realize their artwork physically using AxiDraw Pen Plotters in the Queens College Makerspace. The final project asks students to branch out to realize their mathematical art on other machines in the Makerspace, including the 3D printer, the laser cutter, and the embroidery machine. Students get practice in design thinking and experience with productive failure. The culmination of the semester is an art exhibition that celebrates everyone's progress as a mathematician, an artist, and as a maker. I will share information about the course's implementation, best practices for student success, lessons learned, and plenty of pictures of student work.

Inter affine tiles

Lucia Rossi
(Montanuniversität Leoben)

A number system is, intuitively, a way of representing numbers using a base and a set of digits. In spaces like the complex plane, number systems are associated with fractal sets that give rise to tilings. This project is devoted to an example in which the base is an algebraic number $\alpha = -1 + 3i$ and the digits are $\mathcal{D} = \{0, 1, 2, 3, 4\}$. We define the set $\mathcal{F}$ of 2 fractional parts of this number system, which lives in a space that has a certain p-adic component, so it is a fractal in a non Euclidean space. The rational nature of is the reason for introducing this p-adic space (where $p = 1+i$ is a Gaussian prime). This idea is based on the work of Steiner and Thuswaldner. To tackle the issue of depicting an object in a p-adic space, we consider slices of $\mathcal{F}$ that form a tiling of the complex plane. Each tile is made up of a shrunken union of other tiles, a property that we call inter-affinity. The art piece consists of tiles made up of laser cut wood and hand painted, and form a total of 3 puzzles with different numbers of pieces. The tiles can be placed in a vertical structure, and the height where the pieces go corresponds to their p-adic height, which is also indicated by the colors. This gives an approximation of $\mathcal{F}$. The vertical structure is 3D printed, the tiles from the 9 piece puzzle can be placed in it and can be moved around a central axis. The project was done together with Edmund Harriss and Reilly Dickens-Hoffman from the University of Arkansas.

Twist, Turn, and Shout: The Symmetries of Braided Cords

Joshua Holden
(Rose-Hulman Institute of Technology)

Cords made from braided fibers are found in many cultures. The group of symmetries of an infinite cord is a three-dimensional line group, which can be thought of as one of the 17 wallpaper groups wrapped around a cylinder. Alternatively, it can be seen as one of the 7 frieze groups combined with a generalized translation, i.e. a helical axis, a strict translation, a zigzag translation, or a glide plane reflection. Not all wallpaper groups can be wrapped around a cylinder, and some can be wrapped in multiple ways. Similarly, not every frieze group can be combined with every generalized translation. In all, there are 13 infinite families of three-dimensional line groups. Previous work by the author has found two of these families in the Kongo Gumi style of Japanese braiding and three more in the Naiki style. We will attempt to find examples from all 13 families, drawing primarily from the rich tradition of Japanese braiding (kumihimo) but also bringing in examples from other cultures such as European, Middle Eastern, African, Native American, and Polynesian as well as other Asian styles.

“Symmetry” Card Game

Audrey Nasar
(The Fashion Institute of Technology)

This presentation will showcase “Symmetry,” a card game that teaches players about rosette symmetry groups in a fun and interactive way. The card game was inspired by a geometry course for undergraduate art and design students at FIT. In “Symmetry,” each card features a character interacting with an object. The objects, which are also shown in the corners of each card, can be classified according to the symmetry group they belong to. In other words, they can be classified by their reflection lines and rotations. The instructions describe three different games: SYMMETRY MEMORY, SYMMETRY DROP, and SYMMETRY RUMMY. These games are designed to cater to different players’ skills and interest. It is suggested to start with SYMMETRY MEMORY and then to move on to SYMMETRY DROP and then SYMMETRY RUMMY once a comfort with identifying the correct symmetry group for an object is achieved. Furthermore, since the cards are similar to standard playing cards, many other card games can be played with little adjustment. The presenter has experimented playing "Symmetry" with players ranging from ages 6 to 73 and has found the games to be successful in making the concept of symmetry groups accessible and enjoyable to a wider audience.

Geodesic music

Aaron Fenyes

One way to describe the shape of a hyperbolic surface is to triangulate it by stretching taut strings from cusp to cusp, clamping the infinite ends of the strings under a circle around each cusp, and then measuring the finite distances between the circular clamps. This sounds a lot like building a string instrument, and Robert Penner—the geometer who first explored this description—recently suggested taking the analogy seriously. Turning lengths into tones gives a scale that describes each triangulated hyperbolic surface, and switching triangulations modulates between scales. I'll show you a virtual prototype of Penner's instrument, designed to be played in a web browser on a tablet or a touchscreen table. I'll also describe the geometry behind the instrument, and talk about some obstacles—both mathematical and interface-related—to implementing it in the nicest imaginable way.

MS07 Mathematics and society – embedding models in discussions of societal challenges

Organized by Anina Mischau (FU Berlin), Joshua Wiebe (FU Berlin), Sarah Wolf (FU Berlin)

Session MS07_01

Tuesday, 26 September 2023 09:00 (W9-Raum)

09:00 Sarah Wolf (Freie Universität Berlin)
Welcome and Introduction

09:30 Joshua Wiebe; Sarah Wolf (Freie Universität Berlin)
Two examples of Decision Theatres

10:00 Céline Dillmann (LEAF Inspiring Change)
Engaging Stakeholders through Companion Modelling and Serious Games in Natural Resource Management

10:30 Sinah Guertler (Freie Universität Berlin)
Mathematical modelling in the context of societal challenges: An innovative approach for higher education

Welcome and Introduction

Sarah Wolf
(Freie Universität Berlin)

Two examples of Decision Theatres

Joshua Wiebe, Sarah Wolf
(Freie Universität Berlin)

To set the background for the minisymposium on mathematics and society - embedding models in discussions of societal challenges - this talk introduces the Decision Theatre as a transdisciplinary method. A Decision Theatre is a workshop with scientists and stakeholders (such as experts from the policy, administration, or business worlds, and citizens) who discuss a societal challenge supported by visualisations of data as well as mathematical models and their simulations. In small groups, participants can compose scenarios; the corresponding model results are then explored and discussed with regard to their plausibility and social desirability.
As two examples, we present a Decision Theatre on sustainable mobility and one on COVID19. Further, we discuss our approach of enriching this format for pupils from grade ten and older with supporting workshops that give deeper insight into the mathematics and also computer science aspects of the underlying models. Comparing the two cases, we want to explore the potential of the Decision Theatre in illustrating the use of mathematical modeling for societal phenomena.

Engaging Stakeholders through Companion Modelling and Serious Games in Natural Resource Management

Céline Dillmann
(LEAF Inspiring Change)

In today's complex world, addressing environmental and societal challenges requires innovative, inclusive, and participatory approaches. This presentation explores the transformative potential of the Companion Modeling approach (`www.commod.org`) in designing participatory models for sustainable development. The first part of the talk delves into the principles and methodology of the Companion Modeling approach, developed over 20 years ago, to build serious games using the agent-based modelling framework with a diverse range of stakeholders. The second part draws on case studies from Gabon and Côte d'Ivoire to present the use of serious games to foster stakeholder dialogue in environmental issues. These real-world examples showcase how this approach facilitates dialogue, cooperation, and informed decision-making among stakeholders to co-create more sustainable solutions for pressing environmental challenges.

Mathematical modelling in the context of societal challenges: An innovative approach for higher education

Sinah Guertler
(Freie Universität Berlin)

New approaches that improve the quality of university mathematics education are needed. With this goal in mind, we developed a seminar at Freie Universität Berlin that implements an innovative curriculum in terms of both content and methodology/didactics. The seminar aims to develop students' understanding of the structure and functionality of mathematical models and their relevance in the context of complex societal challenges, such as sustainable mobility or epidemics. The starting and reference point of the seminar is a Decision Theatre, which is a new interactive format for examining societal challenges with applied mathematics. Based on the Decision Theatre, the students developed their own model and were involved in further developing the Decision Theatre. This type of format not only expands the image of mathematics but also enables a co-production of knowledge from which both the students, as well as the university teaching and research benefited. In this session I will present the content and methodology of this seminar as well as first teaching experiences.

MS08 Soziale Dimensionen der Mathematik

Organized by Nicola Oswald (Würzburg), Gudrun Thäter (Karlsruhe)

Session MS08_01

Monday, 25 September 2023 14:00 (Balkonzimmer)

14:00 Carina Geldhauser (Technical University Munich & Munich Center for Machine Learning)
Soziale Herkunft – Ein relevanter Diversity-Faktor in der Mathematik?

14:30 Anja Fetzer (University of Tuebingen)
Mathematikbezogene Lehrveranstaltungen zum Thema Gender und Diversity – Angebote an deutschen Universitäten und Vorstellung eines Praxisbeispiels

15:00 Buesra Sert (Deutsches Zentrum für Luft- und Raumfahrt e.V. (DLR))
Die Deutsche Mathematiker*innen-Vereinigung?

15:30 Kai Kortus; Sofia Raible
Math4VIP - Working towards accessibility in STEM

Session MS08_02

Tuesday, 26 September 2023 14:00 (Kleiner Saal)

14:00 Anna Ransiek (Freie Universtiät Berlin)
Picture a Mathematician. Erklärungen für bestehende Geschlechterdisparitäten in einem mathematischen Exzellenzcluster

14:40 Deborah Kant (University of Hamburg)
DMRCP - a network for the research on the diversity of mathematical research cultures and practices

15:00 Podiumsdiskussion
„Soziale Dimensionen in der Mathematik“

Soziale Herkunft – Ein relevanter Diversity-Faktor in der Mathematik?

Carina Geldhauser
(Technical University Munich & Munich Center for Machine Learning)

Standardmäßig werden 6 Dimensionen von Vielfalt benannt, nämlich Alter, Ethnische Herkunft/Nationalität, Geschlecht, geschlechtliche Identität, sexuelle Orientierung, Körperliche und geistige Fähigkeiten, sowie Religion und Weltanschauung.
Die Charta der Vielfalt entschied sich 2022, die Soziale Herkunft neue Vielfaltsdimension der Charta zu verankern.
Dieser Beitrag liefert einen Überblick über den Stand der Forschung in Bezug auf die soziale Vielfalt der Studierendenschaft, deren Vulnerabilitäten und potentielle Risiken für Diskriminierung und stellt Thesen zur Debatte, in wieweit die Vielfaltsdimension „soziale Herkunft“ im Diversity-Monitoring eine Rolle spielen sollte.

Mathematikbezogene Lehrveranstaltungen zum Thema Gender und Diversity – Angebote an deutschen Universitäten und Vorstellung eines Praxisbeispiels

Anja Fetzer
(University of Tuebingen)

Angesichts bestehender Geschlechterunterschiede in der Mathematik ist die Ausbildung von Genderkompetenz bei Lehrkräften von besonderer Bedeutung. Im Vortrag werden Einblicke in das interdisziplinäre Seminar „Genderperspektive auf mathematisch-naturwissenschaftliche Fächer in der Schule. Problemfelder und Chancen“ als Praxisbeispiel gegeben. Zudem werden die Ergebnisse einer Umfrage vorgestellt, die im Februar 2023 erstellt wurde, um das Angebot und den Bedarf an mathematikbezogenen Lehrveranstaltungen zum Thema Gender und Diversity abzufragen.

Die Deutsche Mathematiker*innen-Vereinigung?

Buesra Sert
(Deutsches Zentrum für Luft- und Raumfahrt e.V. (DLR))

Change is in the nature of languages. Naturally, the German language has also been evolving. Today, gender-inclusive forms are a part of the language, and are used commonly. As a result, the German Research Foundation (DFG) enforces gender-inclusive language in research in its guidelines. However, the DMV still refers to mathematicians using the traditional masculine form in its name. While, there have been many debates on this issue throughout the history of DMV, change is yet to happen. On the other hand, changes require time, effort and determination. A petition started six months ago brings the issue on the table again and (re-)initiates discussions. This talk will be a follow up discussion on the petition.

Math4VIP - Working towards accessibility in STEM

Kai Kortus, Sofia Raible

Studying mathematics and other STEM subjects is a great challenge for students with visual impairments due to the visual contents and representations employed throughout lectures, seminars and textbooks. Visually impaired students are thus largely underrepresented in STEM subjects.
The goal of the Math4VIP project, a joint venture between the University of Marburg and ACCESS@KIT, is to create a central platform that provides information about access to mathematics and how to provide it to visually impaired students. In cooperation with the Mathematical Research Data Initiative (MaRDI), new standards will be developed, and guidelines will be written and publicized through appropriate outreach.
During the talk, participants will have a chance to explore and experience some of the methods and technologies available to make mathematics accessible to visually impaired students and gain an understanding of the challenges posed to these students. We will present the work of Math4VIP and invite an open discussion and cooperation concerning ideas and opportunities for the project.

Picture a Mathematician. Erklärungen für bestehende Geschlechterdisparitäten in einem mathematischen Exzellenzcluster

Anna Ransiek[1], Anina Mischau[2]
([1] Freie Universtiät Berlin; [2] Freie Universität Berlin)

Sekundärstatistische Auswertungen von Hochschuldaten des Statistischen Bundesamtes sowie die jährlichen statistischen Erhebungen der Gemeinsamen Wissenschaftskonferenz (GWK) zur Entwicklung des Frauenanteils an Hochschulen und außeruniversitären Forschungseinrichtungen zeigen für die Mathematik in Deutschland seit Jahren einen (langsamen, aber) stetigen Anstieg des Frauenanteils für alle wissenschaftlichen Qualifikations- und Statusstufen. Dennoch ist das vorhandene Rekrutierungspotenzial von der vorherigen bis zur nächsten wissenschaftlichen Qualifikations- bzw. Statusebene bei weitem nicht ausgeschöpft. Das Bild der sogenannten *Leaky Pipeline* besitzt somit weiterhin Gültigkeit. Infolgedessen sind Frauen in wissenschaftlichen oder akademischen Führungspositionen (einschließlich Professuren) weiterhin unterrepräsentiert (GWK, 2022). Ziel des vorgestellten soziologischen Forschungsprojektes ist es, die Reproduktionsmechanismen für möglicherweise bestehender Geschlechterdisparitäten in einem mathematischen Exzellenzcluster aufzuzeigen und wissenschaftlich basierte Hinweise für die Überwindung dieser Disparitäten in der Mathematik im Allgemeinen und im Cluster im Speziellen zu generieren. Der Cluster bietet die einmalige Gelegenheit, sowohl den Zugang zu einer exzellenten Forschungsumgebung auf den unterschiedlichen Karrierestufen (bspw. durch Aus- und Zuwahl und Betreuungsentscheidungen seitens von Gatekeeper*innen), als auch Karriereentscheidungen von PhD's und Postdocs an den Statusübergänge in den Blick zu nehmen. Gleichzeitig untersuchen wir einen Kontext der sich Gleichstellung als explizites Ziel gesetzt und bereits verschiedene Maßnahmen zur Umsetzung etabliert hat. Basierend auf Analysen leitfadengestützter Interviews mit 44 Wissenschaftler*innen in Leitungspositionen im Cluster sowie 21 PhD's und Postdocs, die ebenfalls im Clusterzusammenhang tätig sind oder waren, werden weiterhin bestehende (mögliche) Barrieren für sogenannte Nachwuchswissenschaftlerinnen in der Mathematik, auch in Verbindung mit weiterhin vorherrschenden Geschlechterstereotypen in den Blick genommen.

Quelle

GWK (2022). Chancengleichheit in Wissenschaft und Forschung. 26 Fortschreibung des Datenmaterials (2020/2021) zu Frauen in Hochschulen und außer-hochschulischen Forschungseinrichtungen. Heft 82, Bonn.

DMRCP - a network for the research on the diversity of mathematical research cultures and practices

Deborah Kant
(University of Hamburg)

DMRCP relies on an active and energetic research community whose aim is to gain understanding of mathematical research as a human activity and the mutual dependency between cultural practices and mathematical research. The traditional image of mathematics is that of a cerebral discipline performed by individuals in pensive isolation without social interaction. This misconception is reflected in traditional philosophy of mathematics and some of the attitudes of sociology of science. In the past twenty years, it has been challenged by philosophers, mathematicians, and sociologists. One of the major research challenges in the field is the lack of reliable and representative descriptions of mathematical research practices based on empirical data. DMRCP aims to coordinate the global research activities studying mathematical research cultures and practices, with an emphasis on using methods of the humanities and empirical social sciences, creating institutional cohesion for the research community that is dispersed across several disciplinary contexts.

„Soziale Dimensionen in der Mathematik“

Podiumsdiskussion

Mit u.a. Lara Gildehaus (Paderborn), Nicola Oswald (Würzburg), Gudrun Thäter (Karlsruhe).

MS09 Generalizations of complex analysis

Organized by Swanhild Bernstein (Freiberg), Dmitrii Legatiuk (Erfurt)

Session MS09_01

Tuesday, 26 September 2023 09:00 (VP-Raum)

09:00 Irene Sabadini (Politecnico di Milano)
Slice topology as a new approach to hyperholomorphic functions

09:30 Rolf Soeren Krausshar (Universität Erfurt)
The notion of proximate orders for monogenic functions in $\mathbb{R}^{n+1}$ and applications to infinite order differential operators

10:00 Sebastian Bock (Bauhaus-Universität Weimar)
Special classes of monogenic functions in IH

10:30 Helmuth Malonek (University of Aveiro (PT 501 461 108))
Polyanalytic Functions, their Generalization and Application in Hypercomplex Analysis

Session MS09_02

Wednesday, 27 September 2023 09:00 (Balkonzimmer)

09:00 Uwe Kähler (Universidade de Aveiro, VN 501 461 108)
Fock spaces over ternary Grassmann algebras and its applications to stochastic processes in hypersymmetry

09:30 Roman Lávička (Faculty of Mathematics and Physics, Charles University)
Generalized Cauchy-Riemann equations

10:00 Swanhild Bernstein (TU Bergakademie Freiberg)
An Overview of q-Analysis Theories

10:30 Martha Zimmermann (TU Bergakademie Freiberg)
Introduction to a Clifford-Jackson calculus

Slice topology as a new approach to hyperholomorphic functions

Irene Sabadini
(Politecnico di Milano)

As it is well known, in quaternionic slice analysis the Representation Formula plays a crucial role. Its validity requires that functions are defined on axially symmetric slice domains. It is a natural question to ask if it is possible to extend a slice hyperholomorphic function from an open set (in the Euclidean topology) to its axially symmetric completion, but this question has, in general, a negative answer. In this talk we discuss the role of the topology and we show that the so-called slice topology is the natural tool to deal with slice hyperhomolomorphic functions. We discuss a Representation Formula, the extension property, domains of slice hyperholomorphy and we show how this setting can suggest generalizations to the case of real alternative algebras, to functions with values in a Euclidean space (with even dimension) and, in all these settings, to the several variables case. The talk is based on joint works with X. Dou and G. Ren.

The notion of proximate orders for monogenic functions in $\mathbb{R}^{n+1}$ and applications to infinite order differential operators

Rolf Soeren Krausshar[1], Fabrizio Colombo[2], Stefano Pinton[2], Irene Sabadini[2]
([1] Universität Erfurt; [2] Politecnico di Milano)

In this talk we consider homomorphisms that act on Clifford algebra valued functions that are null-solutions to the generalized Cauchy-Riemann system. To study them we introduce the notion of the

proximate order for the set of entire monogenic functions and prove some fundamental properties generalizing relations that are known in complex analysis.
As main application we are able to characterize infinite order differential operators that act continuously on spaces of monogenic entire functions. These operators in turn provide crucial new tools in the study of the Schrödinger equation giving new insight in recent mathematical physical problems. This is joint work with F. Colombo, S. Pinton and I. Sabadini from the Department of Mathematics from the Politecnico di Milano, Italy.

Special classes of monogenic functions in IH

Sebastian Bock
(Bauhaus-Universität Weimar)

In complex analysis, orthogonal and special classes of holomorphic functions play an important role in solving both theoretical and applied problems of interpolation and approximation theory, such as in the definition of canonical series expansions (Taylor, Fourier & Laurent) and integral transforms, or in constructing solution representations for boundary value problems in mathematical physics. An important advantage of (complex) function theory compared to other approaches is that these function systems have special properties and relationships that are very beneficial for modeling, numerical computation, and problem analysis. In the talk, higher dimensional generalizations of the most important classes of holomorphic functions are presented in the framework of a hypercomplex function theory and essential properties of the different function classes, such as orthogonality, Appell property, three-term recurrence relation, singularities, as well as their relations to each other are discussed. Finally, it will be shown that, as in complex analysis, the different classes of monogenic functions can be classified in a uniform and hitherto complete scheme.

Polyanalytic Functions, their Generalization and Application in Hypercomplex Analysis

Helmuth Malonek
(University of Aveiro (PT 501 461 108))

In complex analysis, polyanalytic functions are defined as polynomials with respect to the variable $\bar{z} = x–iy$ with coefficients which are analytic functions of the variable $z = x + iy$. They may also be defined as solutions of the Cauchy-Riemann equation of order n, i.e. $\frac{\partial\ n}{\partial \bar{z}\ n} f = 0$. These ideas go back to Goursat and Almansi at the end of the 19th century (cf.[1], [2], [6]).
Since the generalized hypercomplex Cauchy-Riemann operator and its conjugate factorize the higher dimensional Laplace operator, one can expect that through this property the consideration of generalized polyanalytic functions in higher dimensions is useful not only for its own sake, but also for studying polyharmonic functions in higher-dimensions by function theoretic methods (cf.[1]). In order to solve in this way questions of bi-harmonic functions in the theory of elasticity in $\mathbb{R}$ 3 we mention [2]. Very recently, one can notice a renewed interest in the use of Fueter variables (cf.[1]) together with results on integral transforms and suitable function spaces (cf.[1], [4], [5]), combined with polyanalytic functions in the hypercomplex setting (cf.[3]).
These interesting recent contributions mainly concerned with quaternionic valued slice functions suggest the search for an intrinsic hypercomplex definition of polyanalytic functions, i.e. with respect to their genuine polynomial expansions, depending from the corresponding differential operators and resulting generalized holomorphic or anti-holomorphic variables. As in [1], we will make extensive use of the Fueter variables in our contribution to answering this question.

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Fock spaces over ternary Grassmann algebras and its applications to stochastic processes in hypersymmetry

Uwe Kähler
(Universidade de Aveiro, VN 501 461 108)

Classic supersymmetry is based $\mathbb{Z}_2$-graded algebras, like Clifford and Grassmann algebras which still allows us to consider a Fock space of monogenic function and build most of the necessary ingredients for a theory of entire functions. But more general settings like quarks need a more general type of supersymmetry based on a $\mathbb{Z}_2$-grading (also called hypersymmetry). In this talk we present the groundwork for an Itô/Malliavin stochastic calculus and Hida's white noise analysis in the context of a supersymmetry with $\mathbb{Z}_3$-graded algebras. To this end, we establish a ternary Fock space and the corresponding strong algebra of stochastic distributions and present its application in the study of stochastic processes in this context.

Generalized Cauchy-Riemann equations

Roman Lávička
(Faculty of Mathematics and Physics, Charles University)

Clifford analysis is nowadays a well-developed function theory of the Dirac equation in Euclidean spaces for spin 1/2 fields. For any spin, as a proper analogue of the massless field equations in Euclidean spaces, we have suggested the so-called generalized Cauchy-Riemann equations (GCR). In this talk, we explain recent results about solutions of GCR. The talk is based on joint work with F. Brackx, H. De Schepper, V. Souček and W. Wang

An Overview of q-Analysis Theories

Swanhild Bernstein
(TU Bergakademie Freiberg)

The development of q-analysis started in the 1740s when Euler initiated the theory of partitions, also called additive analytic number theory. It has numerous applications in mathematics and physics. For example, special functions, q-functions, orthogonal polynomials, and umbral calculus. All of these theories are one-dimensional analysis and uses a q-difference or q-differential operator:

$$D^q f(x) = \frac{f(qx) - f(x)}{qx - x}.$$

Another important application is a quantum theory based on quantum groups. These considerations are multidimensional and uses q-commutative variables, i.e. $x_i x_j = q x_j x_i, i < j$. Quantum groups arose in the work of L.D. Faddeev and the Leningrad school on the inverse scattering method to solve integrable models. The algebra $U_q(\mathfrak{sl}_2)$ appeared first in 1981 in a paper by P.P. Kulish and N. Yu. Reshetikin on the study of integrable XYZ models with the highest spin. Around 1985 V.G. Drinfeld and M. Jimbo discovered a class of Hopf algebras which can be considered as one-parameter deformations of universal enveloping algebras of semisimple complex Lie algebras. There are links with Lie theory, Lie algebras and their representations, special functions, knot theory, operator algebras, and noncommutative geometry. All settings can give rise to higher dimensional theories with surprising differences. We will discuss some of the different approaches in the context of complex function theory.

Introduction to a Clifford-Jackson calculus

Martha Zimmermann
(TU Bergakademie Freiberg)

We consider an extension of Jackson calculus into higher dimensions and specifically into Clifford analysis for the case of commuting variables. In this case, Dirac is the operator of the first q-partial derivatives (or q-differences) ${}_q\mathcal{D} = \sum_{i=1}^n e_i \, {}_q\partial_i$, where ${}_q\partial_i$ denotes the q-partial derivative with respect to x_i. This Dirac operator factorizes the q-deformed Laplace operator. Similar to the case of classical Clifford analysis, we then consider the q-deformed Euler and Gamma operators and their relations to each other. Nullsolutions of this q-Dirac equation are called q-monogenic. Using the Fischer decomposition, we can decompose the space of homogeneous polynomials into spaces of q-monogenic polynomials. Using the q-deformed Cauchy-Kovalevskaya extension theorem, we can construct q-monogenic functions. Further, we will consider q-harmonic polynomials in this setting. Overall, we show the analogies and the differences between classical Clifford and Jackson-Clifford analysis. In particular, q-monogenic functions need not to be monogenic and vice versa.

MS10 Mathematical Analysis of Complex Quantum Systems

Organized by Simone Rademacher (LMU), Heinz Siedentop (LMU)

Session MS10_01

Monday, 25 September 2023 14:00 (Parkcafé)

14:00 Volker Bach (TU Braunschweig)
Convergent Renormalization Group Flow of Spectral Problems in Quantum Field Theory

14:30 Cristina Caraci (University of Zürich)
Fluctuations of N-Particle Quantum Dynamics around the Gross-Pitaevskii Equation

15:00 Lea Boßmann (LMU München)
Focusing NLS and Bogoliubov correction for dilute Bose gases in the instability regime

15:30 Sabine Jansen (LMU Muenchen)
Markov processes, orthogonal polynomials, and a probabilistic representation of the SU(1,1) current algebra

Session MS10_02

Tuesday, 26 September 2023 09:00 (Kleiner Saal)

09:00 Horia Cornean (Aalborg University)
Bulk–edge correspondence for unbounded Dirac-Landau operators

09:30 Benjamin Hinrichs (Universität Paderborn)
A Ballistic Lieb-Robinson Bound for Many-Body Fermions

10:00 Martin Ravn Christiansen (Ludwig-Maximilians-Universität München)
Spectral Estimates for Fermionic n-Body Operators - Old and New

10:30 Charlotte Dietze (LMU München)
Semiclassical estimates for Schrödinger operators with Neumann boundary conditions on Hölder domains

Session MS10_03

Wednesday, 27 September 2023 09:00 (Parkcafé)

09:00 David Mitrouskas (Institute of Science and Technology Austria (ISTA))
The strongly coupled polaron

09:30 Valentin Kußmaul
Pointwise bounds on eigenstates in non-relativistic quantum field theory

10:00 Simone Rademacher (LMU München)
Exponential bounds of the condensation for dilute Bose gases

10:30 Marius Lemm (Universität Tübingen)
Information propagation in long-range quantum many-body systems

Convergent Renormalization Group Flow of Spectral Problems in Quantum Field Theory

Volker Bach, Sorour Karimi Dehbroki
(TU Braunschweig)

The renormalization group based on the isospectral Feshbach-Schur map introduced about 25 years ago is an important tool for the spectral analysis of Hamiltonians of quantum field theory, especially, for models of nonrelativistic quantum electrodynamics (Pauli-Fierz Hamiltonians). The renormalization transformation $\mathcal{R}_\rho$ is defined on a closed subset $\mathcal{D}$ of an infinite-dimensional Banach space $\mathcal{W}$ of sequences of coupling functions which parametrize the Hamiltonians. One of its key properties is its codimension-one contractivity, i.e., the contractivity of $\mathcal{R}_\rho$ on $\mathcal{D}$ up to a single coupling function. The new result presented in the talk is that, if $\mathcal{W}$ is replaced by a Banach (sub-)space $\mathcal{W}'$ of coupling functions of higher regularity, then there is a closed subset $\mathcal{D}' \subseteq \mathcal{W}'$ such that R is a (genuine) contraction on $\mathcal{D}'$. Its unique fixed point is the effective Hamiltonian presenting the essential spectral properties of the original model at small momenta, i.e., in the infrared limit. Most of the physically relevant models possess the required higher regularity, and their Hamiltonians belong to $\mathcal{D}'$, for small magnitudes of the coupling constant, indeed. This is joint work with Sorour Karimi.

Fluctuations of N-Particle Quantum Dynamics around the Gross-Pitaevskii Equation

Cristina Caraci[1], Jakob Oldenburg, Benjamin Schlein
([1] University of Zürich)

We consider the quantum dynamics of N interacting bosons in the Gross-Pitaevskii regime. We obtain a norm-approximation for the many-body evolution of initial states exhibiting Bose-Einstein condensation in terms of a unitary Fock space evolution with a quadratic generator for the fluctuations. In addition, using this result, we provide the proof of a central limit theorem for the fluctuations of bounded one-particle observables. This is a joint work with Jakob Oldenburg and Benjamin Schlein.

Focusing NLS and Bogoliubov correction for dilute Bose gases in the instability regime

Lea Boßmann, Charlotte Dietze, Phan Thành Nam
(LMU München)

We consider the dynamics of a 2d Bose gas with attractive interactions. The interactions may be chosen in the instability regime, where the corresponding focusing nonlinear Schrödinger equation (NLS) has a blow-up. We show that the evolution of the condensate is effectively described by this NLS for all times before a possible blow-up. Moreover, we prove the validity of the Bogoliubov approximation for the dynamics of the fluctuations, resulting in a norm approximation of the many-body dynamics for all times up to the NLS blow-up.

Markov processes, orthogonal polynomials, and a probabilistic representation of the SU(1,1) current algebra

Sabine Jansen
(LMU Muenchen)

The algebraic approach to duality links generators of Markov processes to representations of Lie algebras and dualities to changes of representations. Often orthogonal polynomials (Hermite, Charlier, Meixner) appear. I will present recent extensions of the algebraic approach from particles on lattices to particles in R^d, focusing on the SU(1,1) algebra and Meixner polynomials. Joint work with S.Floreani, F. Redig and S. Wagner.

Bulk–edge correspondence for unbounded Dirac-Landau operators

Horia Cornean
(Aalborg University)

We consider two-dimensional unbounded magnetic Dirac operators, either defined on the whole plane or with infinite mass boundary conditions on a half-plane. Our main results use techniques from elliptic PDEs and integral operators, while their topological consequences are presented as corollaries of some more general identities involving magnetic derivatives of local traces of fast decaying functions of the bulk and edge operators. One of these corollaries leads to the so-called Středa formula: if the bulk operator has an isolated compact spectral island, then the integrated density of states of the corresponding bulk spectral projection varies linearly with the magnetic field as long as the gaps between the spectral island and the rest of the spectrum are not closed, and the slope of this variation is given by the Chern character of the projection. The same bulk Chern character is related to the number of edge states that appear in the gaps of the bulk operator. This is joint work with Massimo Moscolari and Kasper Sørensen.

A Ballistic Lieb-Robinson Bound for Many-Body Fermions

Benjamin Hinrichs
(Universität Paderborn)

Lieb-Robinson bounds have found wide use in the treatment of discrete quantum systems, but their generalization to continuous systems has proven somewhat difficult. In this talk, we consider a system of fermions in $\mathbb{R}^d$ with two-body interactions and present the proof for a ballistic Lieb-Robinson bound using propagation speed estimates for (one-body) Schrödinger operators. We also discuss applications of these bounds, such as locality of observables and exponential clustering.
This talk is based on joint work with Marius Lemm and Oliver Siebert.

Spectral Estimates for Fermionic n-Body Operators - Old and New

Martin Ravn Christiansen
(Ludwig-Maximilians-Universität München)

We consider bounds on the eigenvalues of the n-body operators associated to fermionic N-particle states. We review the classic norm estimates of Yang, and present a recent estimate on the Hilbert-Schmidt norm of 2-body operators.
The Hilbert-Schmidt estimate is interesting since it is of the same order as the optimal norm estimate, $O(N)$. This implies that although a fermionic 2-body operator can have eigenvalues of size comparable to N, it can not have "too many" large eigenvalues.

Semiclassical estimates for Schrödinger operators with Neumann boundary conditions on Hölder domains

Charlotte Dietze
(LMU München)

We prove a universal bound for the number of negative eigenvalues of Schrödinger operators with Neumann boundary conditions on bounded Hölder domains, under suitable assumptions on the Hölder exponent and the external potential. Our bound yields the same semiclassical behaviour as the Weyl asymptotics for smooth domains. We also discuss different cases where Weyl's law holds and fails.

The strongly coupled polaron

David Mitrouskas
(Institute of Science and Technology Austria (ISTA))

In the polaron model, an electron interacts with a polarizable crystal, described by a nonrelativistic continuous quantum field. If the interaction between the electron and the field is strong, one can effectively describe the polaron using a semiclassical model, where the field is treated as a classical variable. In this talk, we present new results on the rigorous derivation of the semiclassical approximation and the corresponding quantum fluctuations, starting from the microscopic Hamiltonian. Specifically, we will discuss three results: (1) The behavior of the ground state energy as a function of the conserved total momentum. Here, we show that the interaction with the field effectively leads to a free electron with largely increased mass. (2) The existence of excited bound states for sufficiently strong coupling. (3) An asymptotic series for the eigenvalues in inverse powers of the coupling constant. If time permits, we will also provide a brief review of the relation between the time-dependent Landau-Pekar equations, the Hamiltonian equations of the semiclassical energy, and the full quantum dynamics. The talk is based on collaborations with M. Brooks, J. Lampart, N. Leopold, K. Mysliwy, S. Rademacher, B. Schlein, and R. Seiringer.

Pointwise bounds on eigenstates in non-relativistic quantum field theory

Valentin Kußmaul, Marcel Griesemer

We establish subsolution estimates for vector-valued Sobolev functions obeying a very mild subharmonicity condition. Our results generalize and improve a well known subsolution estimate in the scalar-valued case, and, most importantly, they apply to models from non-relativistic quantum field theory: for eigenstates of the Nelson and Pauli-Fierz models we show that an L^2-exponential bound in terms of a Lipschitz function implies the corresponding pointwise exponential bound.

Exponential bounds of the condensation for dilute Bose gases

Simone Rademacher, Phan Thành Nam
(LMU München)

Bose–Einstein condensation (BEC) is a special phenomenon of the thermal equilibrium of Bose gases at low temperatures where a macroscopic fraction of particles occupy a common one-body quantum state. We justify BEC for a class of dilute Bose gases where the number of particles outside of the condensation is controlled in a strong sense. More precisely, we consider N bosons on the unit torus $= [0,1]^3$ in the Gross-Pitaevski regime where the interaction potential scales as $N^2V(N(x-y))$. We prove that the thermal equilibrium at low temperatures exhibits BEC in a strong sense, namely the probability of having n particles outside of the condensation decays exponentially in n. This is joint work with Phan Thành Nam.

Information propagation in long-range quantum many-body systems

Marius Lemm[1], Carla Rubiliani, Israel Michael Sigal, Jingxuan Zhang
([1] Universität Tübingen)

We present rigorous bounds on information propagation in bosonic quantum many-body lattice systems with long-range interactions. These bounds have applications to estimate the minimal time needed for quantum messaging, for the propagation of quantum correlations, and for quantum state control. The proofs are based on a maximal velocity bound and the light-cone approximation of the dynamics including Lieb-Robinson-type bounds. Our results yield a linear light cone for suitable initial states even for long-range interactions decaying as $|x-y|^{-\alpha}$ with $\alpha \in (d+2, 2d+1)$ which poses unforeseen limitations on the applicability of fast-transfer and entanglement-generation protocols developed for breaking linear light cones in long-range and/or bosonic systems. Joint work with C. Rubiliani, I.M.Sigal, and J. Zhang.

MS11 Decoding the Disciplines - an introduction to the process

Organized by Thomas Skill (HS Bochum)

Session MS11_01

Monday, 25 September 2023 16:30 (W9-Raum)
Please notice that German will be spoken in this minisymposium and only continuous participation in this session makes sense.

16:30 Ingrid Scharlau (Paderborn University)
Decoding my discipline - an introduction

18:00 Harald Hofberger (OTH Amberg-Weiden)
Bottleneck "Limes" in der Mathematik-Lehre für Informatiker und Ingenieure

Decoding my discipline - an introduction

Ingrid Scharlau
(Paderborn University)

Decoding the Disciplines is an approach developed in history teaching from where is spread to other fields and has become increasingly important for fostering student learning. The focus is on recurrent learning difficulties of students which are called bottlenecks. These bottleneck difficulties are ascribed to that the teaching staff solves these problems by implicit knowledge. Decoding the Disciplines is a process that enhances student learning by narrowing the gap between expert thinking and student efforts in learning that thinking. The process begins by identifying the characteristic discipline-related bottlenecks and attempts to decode important, but usually hidden, thinking patterns of subject matter experts. Decoding the Disciplines is at the same time a process of interdisciplinary and collegial collaboration. This comes into play especially in decoding discipline-specific patterns of thought and action: teachers are interviewed by teachers from another discipline who, by asking appropriate questions, help them to describe discipline-specific thought and action in such a way that it can be communicated more effectively to students. The introductory talk will provide an overview of the approach and open it up for discussion. The first 30 minutes will provide an overview of the approach and open it up for discussion. This will be followed by a workshop phase in which participants will engage in guided discussion and collegial exchange about decoding in their discipline. It would be perfect if the participants could bring with them an obstacle that they are aware of in their teaching experience. However, unclear experiences of failure are also more than enough, and finally, the interest to deal with one's own implicit practices (or practices that have become implicit) is the most important prerequisite. Please note that the symposium is interrelated and all parts must be attended. It is not possible to join later.

Bottleneck "Limes" in der Mathematik-Lehre für Informatiker und Ingenieure

Harald Hofberger
(OTH Amberg-Weiden)

(Deutsch) Die Einführung bzw. (Schulstoff-)Wiederholung der Grenzwertrechnung stellt für Studenten von Anwendungsdisziplinen eine hohe Hürde dar: Zum Einen weil das Thema per se schwierig ist, insbesondere wenn man es auf Konzept-Ebene behandelt; zum Anderen weil man als Lehrender und Experte "implizite Expertise" einbringt, die einem selbst nicht (mehr) bewusst ist. Der Autor hat die Methode "Decoding the Disciplines" als geeignet erfahren, das implizite Wissen aufzudecken und der Lehrplanung je nach Anwendungsdisziplin (wieder) zugänglich zu machen.
(English) The introduction or repetition of limit value calculation represents a high hurdle for students of application disciplines: On the one hand, because the topic is difficult per se, especially if it is dealt with at the concept level; on the other hand, because as a teacher and expert you bring in "implicit expertise" that you yourself are not (or no longer) aware of. The author has found the "Decoding the Disciplines" method to be suitable for uncovering implicit knowledge and making it (again) accessible for curriculum planning, depending on the application discipline.

MS12 Random graphs and statistical network analysis

Organized by Hajo Holzmann (Marburg), Anja Sturm (Göttingen)

Session MS12_01

Monday, 25 September 2023 14:00 (Kleiner Saal)

14:00 Peter Mörters (Universität zu Köln)
The contact process on evolving scale-free networks

14:40 Benedikt Jahnel (Technische Universität Braunschweig)
Subcritical percolation phases for generalized weight-dependent random connection models

15:20 Hanna Döring (Universität Osnabrück)
Random graphs and Stein's method

Session MS12_02

Monday, 25 September 2023 16:30 (Kleiner Saal)

16:30 Göran Kauermann (LMU München)
Recent Achievements and new Challenges in Statistical Network Data Analysis

17:10 Carsten Jentsch (TU Dortmund University)
Goodness-of-fit testing based on graph functionals for homogeneous Erdös-Rényi graphs

17:50 Alexander Kreiss (Leipzig University)
Testing For Global Covariate Effects in Dynamic Interaction Event Networks

The contact process on evolving scale-free networks

Peter Mörters
(Universität zu Köln)

We study the contact process on scale-free inhomogeneous random graphs evolving according to a stationary dynamics, where the neighbourhood of each vertex is updated with a rate depending on its strength. We identify the full phase diagram of metastability exponents in dependence on the tail exponent of the degree distribution and the rate of updating. This talk is based on joint work with Emmanuel Jacob (Lyon) and Amitai Linker (Santiago de Chile).

Subcritical percolation phases for generalized weight-dependent random connection models

Benedikt Jahnel
(Technische Universität Braunschweig)

We present results on the existence of a subcritical percolation phase for a wide range of continuum percolation models where each vertex is embedded into Euclidean space and carries an independent weight. In contrast to many established models, the presence of an edge is not only allowed to depend on the distance and weights of its end vertices but can also depend on the surrounding vertex set. Our result can be applied in particular to models combining heavy-tailed degree distributions and long-range effects, which are typically well connected. Moreover, we establish bounds on the tail-distribution of the number of points and the diameter of the subcritical component of a typical point. This is joint work with Lukas Lüchtrath.

Random graphs and Stein's method

Hanna Döring
(Universität Osnabrück)

I will start with a central limit theorem for the indegree in a preferential attachment graph. The proof relies on Stein's method. A generalization of this method to the setting of Poisson point processes allows applications to dynamic Boolean models which are studied in the context of telecommunication networks. The first result is joint work with Carina Betken and Marcel Ortgiese, the latter one with Stephan Bussmann.

Recent Achievements and new Challenges in Statistical Network Data Analysis

Göran Kauermann
(LMU München)

The talks give an overview of new approaches and open challenges in statistical network data analysis. This includes classical network models like the exponential random graph model (ERGM) but also touches on ideas based on so-called graphons. We demonstrate how network data analyses can be used in practice and demonstrate the flexibility and generality of common network approaches. Still, coping with the inherent endogeneity in the network leads to open questions, some of which will be elaborated on in more depth in the presentations.

Goodness-of-fit testing based on graph functionals for homogeneous Erdös-Rényi graphs

Barbara Brune[1], Jonathan Flossdorf[2], Carsten Jentsch[2]
([1] TU Wien, Vienna; [2] TU Dortmund University)

The Erdös-Rényi graph is a popular choice to model network data as it is parsimoniously parametrized, straightforward to interprete and easy to estimate. However, it has limited suitability in practice, since it often fails to capture crucial characteristics of real-world networks. To check the adequacy of this model, we propose a novel class of goodness-of-fit tests for homogeneous Erdös-Rényi models against heterogeneous alternatives that allow for non-constant edge probabilities. We allow for asymptotically dense and sparse networks. The tests are based on graph functionals that cover a broad class of network statistics for which we derive limiting distributions in a unified manner. The resulting class of asymptotic tests includes several existing tests as special cases. Further, we propose a parametric bootstrap and prove its consistency, which allows for performance improvements particularly for small network sizes and avoids the often tedious variance estimation for asymptotic tests. Moreover, we analyse the sensitivity of different goodness-of-fit test statistics that rely on popular choices of subgraphs. We evaluate the proposed class of tests and illustrate our theoretical findings by extensive simulations.

Testing For Global Covariate Effects in Dynamic Interaction Event Networks

Alexander Kreiss
(Leipzig University)

In statistical network analysis it is common to observe so called interaction data. Such data is characterized by actors forming the vertices and interacting along edges of the network, where edges are randomly formed and dissolved over the observation horizon. In addition covariates are observed and the goal is to model the impact of the covariates on the interactions. We distinguish two types of covariates: global, system-wide covariates (i.e. covariates taking the same value for all individuals, such as seasonality) and local, dyadic covariates modeling interactions between two individuals in the network. Existing continuous time network models are extended to allow for comparing a completely parametric model and a model that is parametric only in the local covariates but has a global non-parametric time component. This allows, for instance, to test whether global time dynamics can be explained by simple global covariates like weather, seasonality etc. The procedure is applied to a bike-sharing network by using weather and weekdays as global covariates and distances between the bike stations as local covariates.

Springer

Tuesday, 26 September 2023 14:00 (Parkcafé)

Publishing with Springer Spektrum
Iris Ruhmann
(Springer Spektrum)

Ihre Buchpublikation bei Springer Spektrum
Springer Spektrum ist die Springer-Marke für deutschsprachige Buchpublikationen im Bereich Mathematik und Naturwissenschaften. In einem kurzen Vortrag stellen wir die Vielfalt an Möglichkeiten vor, um bei uns als Autor:in aktiv zu werden:

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- Digitale Möglichkeiten (Flashcards, Einbindung von Videos, Autotranslations…)

Sie sind natürlich herzlich eingeladen, eigene Fragen zu stellen. Unabhängig davon stehen wir (im Anschluss an den Vortrag oder unabhängig davon an unserem Büchertisch) auch für Einzelgespräche zur Verfügung.

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Of course, you're welcome to ask questions. After the presentation and during the conference, we will be available for one-on-one discussions at our booth.

Lehrertag (Teachers' Day) - 27. September 2023

Angebote für interessierte Lehrerinnen und Lehrer (kostenfrei).

14:00 Prof. Dr. Norbert Henze (Karlsruher Institut für Technologie)
Sinnstiftende Stochastik in der Schule: Bernoulli-„Kette“, Binomialkoeffizienten und die Binomialverteilung - Kleiner Saal

15:00 Kaffeepause

15:30 Prof. Dr. Thomas Hotz (Technische Universität Ilmenau)
Mathematik für die Götter - Kleiner Saal

17:00 Thomas Richard (Maplesoft Europe GmbH)
Maple 2023 – Neue Wege, um Ihren Schülerinnen und Schülern beim Lernen zu helfen! - Kleiner Saal

19:00 Public Lecture: Alex Kontorovich (Rutgers University)
Mirror Mirror on the Wall: the story of reflection groups and fractal sphere-packings - Großer Saal

Sinnstiftende Stochastik in der Schule: Bernoulli-"Kette", Binomialkoeffizienten und die Binomialverteilung

Norbert Henze
(Karlsruher Institut für Technologie)

Im Vortrag geht es um alte Bekannte der Stochastik, wobei aber die grundlegenden Konzepte zum Zwecke nachhaltiger Erkenntnis in den Vordergrund gestellt werden. Für die Schule zentrale Themen – unabhängige Bernoulli-Versuche, Binomialkoeffizienten und Binomialverteilung – dienen als Beispiele, um die Wertigkeit von Konzepten und Rezepten zu überdenken, mit dem Ziel, stupides Rechnen durch kurzes Nachdenken zu ersetzen. Binäre Tupel spielen dabei eine Schlüsselrolle, um stochastischen Grundkonzepten im Unterricht wieder einen breiteren Raum zu geben.

Mathematik für die Götter

Thomas Hotz
(Technische Universität Ilmenau)

Nordindien, 800 v. Chr. Kriegerische Stämme aus Zentralasien erobern das Land. Um sich den Beistand ihrer Götter zu sichern, konstruieren sie Altäre, doch diese müssen präzise geometrische Formen besitzen, beispielsweise exakte Quadrate darstellen; sonst kann das Opfer misslingen – mit katastrophalen Folgen! Doch wie konstruiert man im Feld ein Quadrat? Um das herauszufinden, probieren wir es aus – mit Stöcken und Schnüren im Stadtpark! Dort folgen wir den Anweisungen des Baudhāyana-Śulbasūtra und konstruieren gemeinsam ein Quadrat. Dabei klären wir auch, wieso das gelingt – und über welche beieindruckenden mathematischen Kenntnisse die Inder zu dieser Zeit sonst noch verfügten. (Bei schlechtem Wetter findet der Workshop innen statt.)

Maple 2023 – Neue Wege, um Ihren Schülerinnen und Schülern beim Lernen zu helfen!

Thomas Richard
(Maplesoft Europe GmbH)

Wir stellen neue Features in Maple 2023 vor, die Studierenden und Lehrenden helfen, mehr Sicherheit im Umgang mit mathematischen Aufgaben und Lösungen zu erlangen. Konkrete Beispiele kommen aus den Themen Differentialgleichungen, Ableitung impliziter Funktionen, quadratische Ergänzung sowie dem schriftlichen Dividieren von Polynomen oder ganzen Zahlen. Daneben zeigen wir, wie man interaktive Aufgaben (inkl. Randomisierung) erstellt und in Maple oder wahlweise in der Online-Umgebung Maple Learn bearbeitet. Technische Fragen und Feature-Wünsche nehmen wir gern entgegen.

Social Program: Memorable Ilmenau

The social program aims to highlight what Ilmenau stands for: Goethe, mining and the university, glass-making, and winter sports. Learn more about these traditions and get involved!

All events start at the given time with a guided walk at the fountain in front of Hotel Tanne (Lindenstrasse 38, a 5 min walk from the conference venue).

Please **register online** through the conference registration platform; participation is limited and will be determined on a first come, first serve basis. Fees have to be paid in cash (EUR) at the event (if any).

Guided tour through the town centre — *Tuesday, 26 September 2023, 09:00-10:00*

number of participants: min. 5, max. 20
free of charge

Trace Ilmenau's history while walking through its town centre, and learn about its origins, its connection to Goethe, and about its university.

Visiting a glassblower — *Tuesday, 26 September 2023, 14:00-16:00*

number of participants: max. 20
fee: 2 EUR p. p.

Ilmenau has a long tradition of glass-making, in particular featuring many small workshops run by local craftsmen as a side-business. Visit one such glasblower and see for yourself how christmas tree decorations, which originate from the Thuringian Forest, are made - or even try it yourself!

Luge — *Wednesday, 27 September 2023, 16:45*

number of participants: min. 10, max. 20
fee: 5 EUR p. p.

Ilmenau has been and still is home to many succesfull athletes which brought quite a few olympic medals to town, in particular in luge and bob sleigh. For training in summer, it features one of only two summer toboggan runs in Germany. Get on the luge and experience it yourself! (Protective equipment will be provided.)

Presenting Authors