Workshop on Optimization

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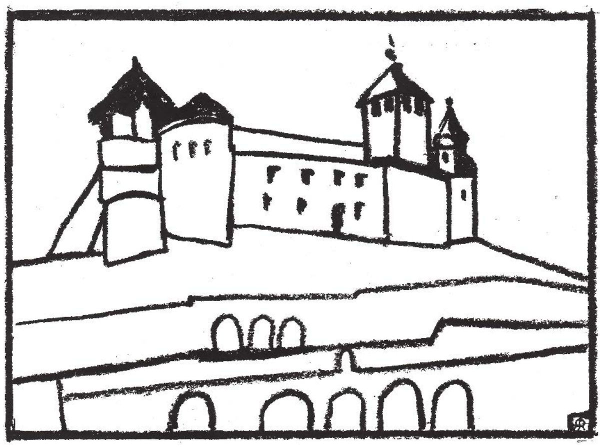
Abstracts



Organizer: Gabriele Eichfelder, Tobias Gerlach, Regina Hildenbrandt
Technische Universität Ilmenau

Content

18 unglaubliche Monate im Computer-Go

Ingo Althöfer

Friedrich-Schiller-Universität Jena, Institut für Mathematik

Ende Januar 2016 erschien in NATURE ein Artikel zum Computer-Go: Das bis dahin völlig unbekannnte Programm AlphaGo einer fast unbekannnten Londoner Firma DeepMind hatte in einem geheimen Wettkampf im Oktober 2015 den besten europäischen Spieler Fan Hui mit 5-0 abgefertigt. Im März 2016 war dann mit dem Südkoreaner Lee Sedol die Nummer 4 der Weltrangliste dran: 4-1 für DeepMind. Ende Dezember 2016 spielte eine verbesserte Version von AlphaGo im Internet inkognito 60 Schnellpartien gegen die besten Profispieler der Welt und siegte mit 60-0. Im Vortrag soll diese Entwicklung und die Vorgeschichte diskutiert werden.
Für alle Freunde der A-Capella-Musik ein kurzer Aufwärmer: AlphaGo - Our Silicon Overlord https://www.youtube.com/watch?v=dh_mfGo183Y
Die ersten 31 Sekunden reichen aus.

A Linear Vector Programming Approach to Quasi-concave Global Minimization

Daniel Ciripoi (Co-authors: Andreas Löhne and Benjamin Weißing)

Friedrich-Schiller-Universität Jena, Institut für Mathematik

Constrained global quasi-concave minimization problems are known to be NP-hard in general. There do not exist local criteria to characterize a point as a global minimum. In the case of bounded polyhedral constraints it is well known that a quasi-concave function attains its minimum in a vertex of the feasible region. This property shall be exploited to the purpose of applying a version of Benson's algorithm [1] for the solution of the minimization problem.
Ehrgott and Shao introduced in [2] a variant of Benson's algorithm to solve certain linear multiplicative problems under polyhedral constraints. To the end of applying their basic idea, a transformation of the scalar global problem into a linear multiple criteria problem is shown. With respect to certain assumptions, a generalized algorithm can be formulated to solve a wider range of problems. A monotonicity-property of the objective-function, induced by a convex pointed solid cone, is needed therefore.
Assume we have an objective function whose monotonicity-cone is non-solid. For instance take the cone $\{0\}$, which signifies the case of no polyhedral monotonicity at all. Recent polyhedral projection results, see [3], can be employed to the purpose of solving this broader class of problems. This will enable said algorithm to solve any quasi-concave mimization problem subject to a bounded polyhedral feasible region regardless of the corresponding monotonicity-cone's solidity. This improvement requires an additional image space dimension.
The talk involves the description of the generalized version of Ehrgott and Shao's algorithm and details about the extension to arbitrary polyhedral cones.

References

[1] A. Hamel and A. Löhne and B. Rudloff, *Benson type algorithms for linear vector optimization and applications*, Journal of Global Optimization, 59 (2014), pp. 811-863.

[2] M. Ehrgott and L. Shao, *Primal and dual multi-objective linear programming algorithms for linear multiplicative programmes*, Optimization, 65 (2015), pp. 415-431.

[3] A. Löhne and B. Weissing, *Equivalence between polyhedral projection, multiple objective linear programming and vector linear programming*, Math. Meth. Oper. Res. 84 (2016), pp. 411-426.

A Cutting Plane Approach for the Two-dimensional Strip Packing Problem

Isabel Friedow

Technische Universität Dresden, Institut für Numerische Mathematik

Given a strip of fixed width and unrestricted height, the two-dimensional strip packing problem consists in packing a set of rectangles into the strip with the aim of minimizing the strip height used. We consider a relaxation of the strip packing problem, in which each rectangle is represented by a single one-dimensional item-type where an item-type is characterized by its size that equals the width of the corresponding rectangle and its order demand which equals the height of the related rectangle. The approach starts with a relaxation based on the horizontal bar relaxation, the Gilmore-Gomory model of the binary cutting stock problem with the requirement of meeting order demands exactly. As known from literature, in order to represent a solution of the strip packing problem, a solution of the horizontal bar relaxation has to satisfy further conditions. First, there must exist such an ordering of one-dimensional cutting patterns that all items of one type are located in consecutive patterns. Secondly, the positions of all items belonging to the same type have to be equal in each cutting pattern. Because of that, by successively adding constraints, or rather cutting planes, to our relaxation, we persue the two objectives: creating of consecutive cutting patterns within the LP solution of the relaxation and ensuring constant locations of item-types.

Relationships between Constrained and Unconstrained Multi-Objective Optimization and Applications

Christian Günther

Martin-Luther-Universität Halle-Wittenberg, Institut für Mathematik

In this talk we investigate relationships between constrained and unconstrained multi-objective optimization problems.
Let $f = (f_1, \ldots, f_m) : \mathbb{R}^n \to \mathbb{R}^m$ be a vector-valued function and $X \subseteq \mathbb{R}^n$ be a convex and closed set in $\mathbb{R}^n$ with nonempty interior. We will study relationships between the set of Pareto efficient solutions of the constrained multi-objective optimization problem

$$\begin{cases} f(x) = (f_1(x), \ldots, f_m(x)) \to \min \quad \text{w.r.t } \mathbb{R}^m_+ \\ x \in X \end{cases} \tag{$\mathcal{P}_X$}$$

and the set of Pareto efficient solutions of the unconstrained multi-objective optimization problem involving an additional objective function given by a gauge function $\mu : \mathbb{R}^n \to \mathbb{R}$:

$$\begin{cases} f^\mu(x) = (f_1(x), \ldots, f_m(x), \mu(x - \tilde{x})) \to \min \quad \text{w.r.t } \mathbb{R}^{m+1}_+ \\ x \in \mathbb{R}^n, \end{cases} \tag{$\mathcal{P}^\mu_{\mathbb{R}^n}$}$$

where $\tilde{x}$ is an arbitrarily given point in the interior of X, and the associated unit ball of the gauge μ is given by $B_\mu := -\tilde{x} + X$.
In our considerations we mainly focus on generalized convex multi-objective optimization problems, i.e., the objective function f is a componentwise generalized convex (e.g., quasi-convex or semi-strictly quasi-convex) function. We derive a characterization of the set of Pareto efficient solutions of the multi-objective optimization problem $(\mathcal{P}_X)$ using characterizations of the sets of Pareto efficient solutions of two unconstrained multi-objective optimization problems, namely the free problems $(\mathcal{P}_{\mathbb{R}^n})$ and $(\mathcal{P}^\mu_{\mathbb{R}^n})$.
We demonstrate the usefulness of the results by applying it on constrained multi-objective location problems. Using our new results we show that special classes of constrained multi-objective location problems (e.g., point-objective location problems, Weber location problems and center location problems) can be completely solved with the help of algorithms for the unconstrained case.
At the end of the talk, we present some information about the current development of the MATLAB-based software tool "Facility Location Optimizer" (see www.project-flo.de).

Notwendige Optimalitätsbedingungen für gewisse Klassen nichtkonvexer Standortoptimierungsprobleme

Marcus Hillmann

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Die Lokalisierung eines neuen Standortes unter Beachtung von bereits existierenden Standorten mit sowohl anziehender als auch abstoßender Wirkung ist eine Problemstellung von großer praktischer Bedeutung, beispielsweise in der Wirtschaftsplanung, im Industriedesign oder in städtebaulichen Projekten. Leider ist dieses Problem aufgrund der Verwendung negativer Gewichte im Allgemeinen nichtkonvex, so dass es nicht sinnvoll mit den etablierten Algorithmen der Standortoptimierung gelöst werden kann.
Daher präsentieren wir einen neuen Ansatz, mit welchem wir unter Nutzung der verallgemeinerten Subdifferentiale von Ioffe und Kruger/Mordukhovich notwendige Optimalitätsbedingungen für verschiedene Klassen solcher Probleme herleiten können. Obgleich eine Vielzahl theoretischer Resultate zu diesen Subdifferentialen existiert, war es bisher dennoch kaum möglich, sie für solche praktischen Zwecke zu nutzen.
Nach einer kurzen Wiederholung der notwendigen Definitionen und Eigenschaften der genutzten Subdifferentiale werden wir diese für einige Abstandsfunktionen exakt berechnen. Unter Nutzung spezieller Strukturen leiten wir dann notwendige Optimalitätsbedingungen für gewisse Klassen dieser Standortoptimierungsprobleme her und diskutieren diese. Den Abschluss des Vortrags bildet ein Ausblick auf offene Fragestellungen und mögliche Anknüpfungspunkte.

A Variant of the Periodic Event Scheduling Problem and its Industrial Application

Tobias Hofmann

Technische Universität Chemnitz, Fakultät für Mathematik

The use of industrial robots in the automotive industry noticeably changed the view of production plants and led to a tremendous increase in productivity. Nonetheless, rising technological complexity, the parallelization of production processes, as well as the crucial need for respecting safety issues pose new challenges for man and machine. Furthermore, the progress shall proceed - production cannot be too fast, too safe or too cheap.
The talk will be about optimizing the schedule of industrial robots to ensure desired cycle times while respecting collision constraints. We consider periodic timetables as they typically appear in the context of train scheduling problems as well as why and how the mathematical models used in this field are well applicable to our scenario.
We adapt a variant of the periodic event scheduling problem introduced by Serafini and Ukovich, yielding integer program formulations, whereby we focus on the cycle periodicity formulation.

Phase-only Discrete Receive Beamforming

Johannes Israel

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In wireless communications, beamforming is a common technique to adjust the radiation pattern of an antenna array. This can be realized by variable phase shifters and amplifiers that are connected to the individual antenna elements. We want to determine antenna weights, i.e., phase shifts and amplifications at the antenna elements, such that the signal-to-interference-plus-noise ratio (SINR) is maximized. The Capon beamformer is known to be a solution in case of continuous weights.

In this talk we focus on the SINR-maximization problem with discrete antenna weights, i.e., the phase shifters and amplifiers have finite resolution only. We introduce a branch-and-bound algorithm for an exact solution and present how upper bounds can be tightened by means of fractional programming techniques. In case of fixed amplitudes we consider phase-only receive beamforming, which is more difficult in the continuous case, but allows for further improvements of bounds in the discrete approach. Finally, simulation results are provided.

Modified Augmented Lagrangian Methods in Finite and Infinite Dimensions

Christian Kanzow (Co-authors: Daniel Steck and Daniel Wachsmuth)

Julius-Maximilians-Universität Würzburg, Institut für Mathematik

The classical augmented Lagrangian method belongs to the standard approaches for the solution of constrained optimization problems.

Recent modifications by Andreani, Birgin, Martinez and Co-workers for finite-dimensional optimization problems turn out to have stronger properties than the classical method. In this talk, we begin with a review of these classical and modified augmented Lagrangian methods. We then present an extension of the modified method to optimization problems in Banach spaces. Some numerical results illustrate the reliability of the proposed technique.

A Bundle Trust-Region Method for Marginal Functions Using Continuous Outer Subdifferentials

Martin Knossalla

Friedrich-Alexander-Universität Erlangen-Nürnberg, Department Mathematik

Typically, exact information of the whole subdifferential is not available for intrinsically nonsmooth objective functions such as for marginal functions. Therefore, the semismoothness of the objective function cannot be proven or is even violated. In particular, in these case standard nonsmooth methods cannot be used. Basing on the so-called continuous outer subdifferentials (COS), this talk presents a strategy for nonsmooth optimization problems with local Lipschitz continuous objectve function. At first, we describe a descent method for arbitrary local Lipschitz continuous functions, which is realized by projections on the COS of the function. Possibly, it can happen that the computation of the whole COS is too heavy. For this reason we will approximate COSs especially for marginal functions. Further, we develop on this basis a bundle-trust region method and prove its global convergence.

Generalized Set Order Relations and Their Numerical Treatment

Elisabeth Köbis (Co-authors: Daishi Kuroiwa and Christiane Tammer)

Martin-Luther-Universität Halle-Wittenberg, Institut für Mathematik

We introduce very general definitions of set order relations and propose their unified characterization by means of a prominent scalarizing functional from vector optimization.

Furthermore, we propose new numerical methods for obtaining minimal elements of a family of finitely many sets. Most set optimization problems, even if given in a continuous framework, need to be handled in a discrete manner concerning computations. Therefore, given a finite discrete family of sets, in this talk we propose several numerical methods that first sort out non-minimal elements and then determine all minimal elements of the family of sets.

These methods can be interpreted as extensions of the well-known Jahn-Graef-Younes method to set optimization. When the involved sets are compared by means of a generalized set order relation, we use the characterization of set order relations by the aforementioned scalarizing functional. Numerical tests justify that our approaches are useful and the numerical effort is drastically reduced.

This talk is based on a joint work with Christiane Tammer and Daishi Kuroiwa entitled *Generalized Set Order Relations and Their Numerical Treatment*, which will appear in the journal *Applied Analysis and Optimization* (Yokohama Publishers, 2017).

An Introduction to the Skiving Stock Problem

John Martinovic

Technische Universität Dresden, Institut für Numerische Mathematik

We consider the one-dimensional skiving stock problem (SSP) which is strongly related to the dual bin packing problem (DBPP) in literature. In the classical formulation, $m \in \mathbb{N}$ different (small) item lengths $l_1, \ldots, l_m$ with corresponding availabilities $b_1, \ldots, b_m$ shall be used to combine a maximum number of larger objects each having a minimum length of L. Hence, the SSP can be considered as a natural counterpart of the extensively studied one-dimensional cutting stock problem, where larger items have to be cut into smaller ones such that a given demand is satisfied. Despite sharing a common practical background and a similar mathematical structure (e.g. as regards their input data), both problems are not dual formulations in the sense of linear optimization. Consequently, the SSP represents an independent challenge in the field of cutting and packing. Besides its original application in industrial production and recycling, the skiving stock problem plays an important role whenever an efficient and sustainable use of given resources is desired, e.g. in the allocation of wireless users within a given frequency range.
This talk presents an introduction to the skiving stock problem and introduces some of the most common modeling formulations. Moreover, basic properties regarding the quality of the continuous relaxation shall be discussed.

On the Concepts of W-, M-, and S-stationarity for MPCCs in Banach Spaces

Patrick Mehlitz (Co-author: Gerd Wachsmuth)

Technische Universität Bergakademie Freiberg, Institut für Numerische Mathematik und Optimierung

Many real-world problems can be modelled as a mathematical program with complementarity constraints (MPCC for short). It is well-known that these programs suffer from an inherent lack of regularity and, thus, in the past, huge effort was put in constructing suitable constraint qualifications and optimality conditions as well as numerical methods which can be used to handle MPCCs. However, most of the literature deals with the situation where the complementarity is induced in the space $\mathbb{R}^n$. On the other hand, several applications from optimal control as optimal control problems with mixed control-state complementarity constraints or the obstacle problem comprise complementarity constraints in more complex Banach spaces like $L^2(\Omega; \mathbb{R}^n)$ or $H_0^1(\Omega)$ where $\Omega \subseteq R^d$ is a bounded domain.
In this talk, we introduce and justify reasonable notions of weak (W-), Mordukhovich (M-), and strong (S-) stationarity for a general complementarity constrained program in Banach spaces. From the finite-dimensional case, we expect any S-stationary point to be M-stationary, and any M-stationary point to be W-stationary. Conditions are presented which ensure that these relations hold in the abstract situation as well.

New Discarding Tests for a Branch-and-Bound-Algorithm for Multiobjective Problems

Julia Niebling

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A Branch-and-Bound-algorithm to find a covering of all globally optimal solutions of a multiobjective optimization problem with twice continuously differentiable objective functions and convex constraints is introduced. The procedure consists of selection, discarding and termination rules. The so-called discarding tests examine in different ways whether some parts of the domain can contain solutions. In case of the non-existence of any solutions in a subbox of the domain this subbox will be discarded. The new discarding tests are based on convex underestimators from the αBB-method, ideal points and Bensons outer approximation algorithm. The new discarding tests and some numerical results will be presented.

Time and Dynamic Consistency of Risk Averse Stochastic Programs

Alois Pichler (Co-author: Alexander Shapiro)

Technische Universität Chemnitz, Fakultät für Mathematik

In various settings time consistency in dynamic programming has been addressed by many authors going all the way back to original developments by Richard Bellman. The basic idea of the involved dynamic principle is that a policy designed at the first stage, before observing realizations of the random data, should not be changed at the later stages of the decision process. This is a rather vague principle since this leaves a choice of optimality criteria at every stage of the process conditional on an observed realization of the random data. In this paper we discuss this from the point of view of modern theory of risk averse stochastic programming. In particular we discuss time consistent decision making by addressing risk measures which are recursive, nested, dynamically or time consistent. It turns out that the paradigm of time consistency is in conflict with various desirable, classical properties of general risk measures.

A Chance Constraint Model for Multi-Failure Resilience in Communication Networks

Sebastian Richter

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For ensuring network survivability in case of single component failures many routing protocols provide a primary and a back up routing path for each origin destination pair. We address the problem of selecting these paths so that in the event of multiple failures, occuring with given probabilities, the total loss in routable demand due to both paths being intersected is small with high probability. We present a chance constraint model and solution approaches based on an explicit integer programming formulation, a robust formulation and a cutting plane approach that yield reasonably good solutions assuming that the failures are caused by at most two elementary events, which may each affect several network components.

Dual-feasible Functions and Some Applications

Jürgen Rietz

A *dual-feasible function (DFF)* yields feasible values for dual variables of the continuous relaxation of certain problems and can therefore yield valid non-trivial bounds.

Classical dual-feasible functions

A function $f : [0,1] \to [0,1]$ is a DFF, if for any finite index set I of numbers $x_i \in \mathbb{R}_+, i \in I$ the implication $\sum_{i \in I} x_i \leq 1 \Longrightarrow \sum_{i \in I} f(x_i) \leq 1$ holds. The DFF f is a *maximal dual-feasible function (MDFF)*, if no other DFF g with $g(x) \geq f(x)$ for all $x \in [0,1]$ exists.

Theorem 1. *Any MDFF f is monotone, superadditive, symmetric in the sense $f(x)+f(1-x) = 1$ for all x and obeys $f(0) = 0$. On the other hand, any function $f : [0,1] \to \mathbb{R}_+$ with $f(0) = 0$, $f(x) + f(1-x) = 1$ for all $x \in [0, \frac{1}{2}]$ and $f(x+y) \geq f(x) + f(y)$ for all x, y with $0 < x \leq y < \frac{1}{2}$ and $x + y \leq \frac{2}{3}$ is a MDFF.*

Given an instance $E := (m; L; \mathbf{l}; \mathbf{b})$ of the one-dimensional cutting stock problem (1D-CSP) and a DFF f, a lower bound for the optimal objective function value z_C of the continuous relaxation is $z[f] := \sum_{i=1}^{m} b_i * f(\ell_i/L)$. There is always a DFF $\hat{f}$ with $z_C = z[\hat{f}]$.

Some important parameter dependent MDFFs are the following with $0 < \lambda \leq 1/2$ and $C \geq 1$:

$$f_{MT,0}(x;\lambda) := \begin{cases} 0 & \text{if } x < \lambda \\ 1 & \text{if } x > 1-\lambda \\ x & \text{otherwise} \end{cases}$$

$$f_{CCM,1}(x;C) := \begin{cases} \lfloor Cx \rfloor / \lfloor C \rfloor & \text{if } x < 1/2 \\ 1/2 & \text{if } x = 1/2 \\ 1 - f_{CCM,1}(1-x;C) & \text{otherwise} \end{cases}$$

$$f_{BJ,1}(x;C) := \left(\lfloor Cx \rfloor + \max\{0, \frac{\mathsf{frac}(Cx) - \mathsf{frac}(C)}{1 - \mathsf{frac}(C)}\} \right) / \lfloor C \rfloor$$

Generalizations

The concept of (maximal) DFFs can be generalized in various ways (see e.g. [1]), for instance:

- functions with domain $[0,1]^m$ for the m-dimensional vector packing problem ($m \in \mathbb{N} \setminus \{0\}$)
- functions with domain $\mathbb{R}$ instead of an interval, yielding some theoretical challenges
- data-dependent DFFs, which are valid for a given instance only, but can consider the order demands to sharpen the bounds

Since the MDFFs are always superadditive and yield the function value zero at the point zero, they can sometimes be used to derive valid inequalities or cuts for general integer linear optimization problems.

References

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Solutions of Bilevel Optimization Problems being Polyhedral on the Lower Level

Alexandra Rittmann

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This talk discusses bilevel optimization problems being polyhedral on the lower level. Bilevel optimization problems are hierarchical optimizations problems, where the set of all variables is divided between upper and lower level. Fulop [1993] has shown, that linear bilevel optimization problems can be solved by optimizing the upper level objective function over the set of Pareto-optimal extremal points of a corresponding multiple objective linear program. By extending this result we present a more direct approach based on polyhedral projection. We consider the projection of the feasible set of the lower level onto a subspace, which is essentially determined by the set of the variables of the upper level. It is shown how solver software for both multiple objective linear programming and polyhedral projection in combination with branch-and-cut techniques can be used to solve bilevel optimization problems being polyhedral on the lower level.

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Secure Communication Networks and Graph Colourings

Ingo Schiermeyer

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Secure communication in networks can be formulated as a graph edge colouring problem, where edges correspond to channels and colours to passwords. For a secure communication between two agencies we require that consecutive channels (edges) receive distinct passwords (colours), i.e. any two consecutive edges obtain distinct colours (are properly coloured).

An edge-coloured graph G is called *properly connected* if any two vertices are connected by a path whose edges are properly coloured. The *proper connection number* of a connected graph G, denoted by $\mathrm{pc}(G)$, is the smallest number of colours that are needed in order to make G properly connected. Our main result is the following: Let G be a connected graph of order n and $k \geq 2$. If $|E(G)| \geq \binom{n-k-1}{2} + k + 2$, then $\mathrm{pc}(G) \leq k$ except when $k = 2$ and $G \in \{G_1, G_2\}$, where $G_1 = K_1 \vee (2K_1 + K_2)$ and $G_2 = K_1 \vee (K_1 + 2K_2)$.

Group Sparsity in Optimal Control

Christopher Schneider (Co-author: Gerd Wachsmuth)

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It is well known that optimal control problems with L^1-control costs produce sparse solutions, i.e., the optimal control is zero on whole intervals. In this talk, we study a general class of convex linear-quadratic optimal control problems with a sparsity functional that promotes a so-called *group sparsity* structure of the optimal controls. In this case, the components of the control function take the value zero on parts of the interval, simultaneously. These problems are both theoretically interesting and practically relevant. After obtaining results about the structure of the optimal controls, we derive stability estimates for the solution of the problem w.r.t. perturbations and L^2-regularization. These results are consequently applied to prove convergence of the Euler discretization. Finally, the usefulness of our approach is demonstrated by solving an illustrative example using a semismooth Newton method.

Newton's Method for the Primal Training of Support Vector Machines

Nico Strasdat

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Support Vector Machines (SVMs) are a basic tool for machine learning. They are used in many applications to handle classification and regression problems. In order to take advantage of large datasets, efficient training methods are necessary. In the original formulation of the training problem, the objective is to minimize a convex function over some (possibly infinite dimensional) inner product space. It is well-known that this optimization problem can be reformulated as a finite-dimensional convex problem. However, this approach involves ambiguity of the solution in general. We present a characterization of this non-uniqueness. Moreover, we apply the theoretical insight to construct an adapted version of Newton's method for the efficient solution of SVM training problems.

A Condorcet Jury Theorem for Couples

Raphael Thiele

Friedrich-Schiller-Universität Jena, Institut für Mathematik

The agents of a jury have to decide between a good and a bad option through simple majority voting. Our jury consists of N independent couples. Each couple consists of two correlated agents of the same competence level. Different couples may have different competence levels. In addition, each agent is assumed to be better than completely random guessing.
We prove tight lower and upper bounds for the quality of the majority decision. The lower bound is the same as the competence of majority voting of N independent agents. The upper bound cases for negatively correlated couples can be much better than the value for $2\,N$ independent agents.

Trust-Region Concept for Multiobjective Heterogeneous Optimization Problems

Jana Thomann

Technische Universität Ilmenau, Institut für Mathematik

In many applications of multiobjective optimization the objective functions are heterogeneous, for example regarding the evaluation time. But methods for multiobjective optimization problems do not distinguish between the objectives so far, but treat them all equally. In this talk the idea is introduced to use an iterative approach for such problems based on the trust region concept for scalar optimization problems.

Polyhedral Projection Problems and Vector Linear Programming

Benjamin Weißing

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Consider a vector-valued function $f : \mathbb{R}^d \to \mathbb{R}^q$ and the feasible set $X \subseteq \mathbb{R}^d$. A *Vector Optimisation Problem* consists in finding

$$\min f(x) \text{ subject to } x \in X. \tag{VOP}$$

Embedding (VOP) into a set-valued framework has proven to be a very fruitful approach, c.f. [1]. Especially, algorithms for solving Vector Linear Programmes (X is a polyhedral convex set and f is linear) have been developed and implemented based on this approach [2].
We consider another type of problem: Given a closed, convex polyhedron $S \subseteq \mathbb{R}^{d+q}$, one tries to

$$\text{compute } P = \left\{ y \in \mathbb{R}^q \mid \exists x \in \mathbb{R}^d : \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) \in S \right\}. \tag{PP}$$

Here, to "compute" means to obtain a description of the polyhedron P in terms of vertices/directions or in terms of intersecting half spaces. Problem (PP) is called *Polyhedral Projection Problem.* At first glance, (PP) is not related to optimisation. However, it has been shown that Vector Linear Programmes are equivalent to Polyhedral Projection Problems, see [3]. Hence for any application which can be expressed in terms of Vector Linear Programming a description in terms of (PP) is also possible. Moreover, this In many cases, a representation may be simpler or "more natural" than the conversion to a Vector Linear Programme.
In this talk, a solution concept for (PP) will be introduced. Algorithms to solve (PP) according to this solution concept will be presented. Furthermore, polarity between convex polyhedra will be used to establish a duality theory for (PP).

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