5th Anniversary WORKSHOP
of the
GAMM Activity Group
Applied Operator Theory

May 30-31, 2013, TU Berlin

1 Program

Thursday, May 30, 2013

13:15	Opening
13:30- 13:55	**Karl-Heinz Förster:** *5 Years Activity Group Applied Operator Theory: PAST and PRESENT*
14:00-14:25	**Bernhard Gramsch:** *Division of distibutions and small ideals of operators*
14:30-14:55	**Markus Haase:** *The Operator Theory of Abstract Square Functions*
15:00-15:25	**Hendrik Vogt:** *Perturbation theory for accretive operators in L_p and elliptic operators with complex coefficients*

15:30 - 16:00	Coffee break

16:00 - 16:25	**Hafida Laasri:** *Robustness of L2-maximal regularity under non-autonomous multiplicative perturbations*
16:30 - 16:55	**Birgit Jacob:** *Semigroups related to Hamiltonian systems*
17:00 - 17:25	**Sascha Trostorff:** *Evolutionary Equations with Frictional Boundary Conditions*
17:30 - 17:55	**Junjie Huang:** *Left invertibility of formal Hamiltonian operators*

18:15	*General Assembly*
19:30	*Dinner at Satyam (Goethestraße 5)*

Friday, May 31, 2013

9:00-9:25	**Johannes Brasche:** *Large coupling convergence for Schrödinger operators with negative potentials*
9:30-9:55	**Olaf Post:** *Boundary triples associated with quadratic forms*
10:00-10:25	**Jonathan Rohleder:** *Titchmarsh-Weyl theory for Schrödinger operators on non-compact trees*
10:30	*Workshop photo*

10:35 - 11:00	Coffee break

11:00-11:25	**Leslie Leben:** *Eigenspaces and spectra of nonnegative matrices under rank one perturbations*
11:30-11:55	**Annemarie Luger:** *A new result on generalized poles*
12:00-12:25	**Friederich Philipp:** *Relatively Symmetric Perturbations of Selfadjoint Operators*

12:30	Lunch break

14:00	*Operator Theory within the GAMM: FUTURE developments, planned activities.*
14:30-14:55	Coffee break

15:00 - 15:25	**Uwe Günther:** *PT-symmetries in physics and some operator-theoretic challenges*
15:30 - 15:55	**Karl-Heinz Förster:** *The block numerical range of operators in Banach spaces*
16:00 - 16:25	**Nadezda Rautian and Victor Vlasov:** *Spectral analysis of the abstract integro-differential equations in Hilbert space*
16:30 - 16:55	**Olivier Sete:** *Expansion of f(T) in a series of Faber-Walsh polynomials*
17:00	Closing

2 Abstracts

Controllability of a string submitted to unilateral constraint

Johannes Brasche

Abstract: Let $\mathscr{E}$ and $\mathscr{P}$ be nonnegative quadratic forms in a Hilbert space $\mathscr{H}$ such that $\mathscr{E} - b\mathscr{P}$ is lower semibounded, closed and densely defined for every $b \geq 0$. We present results on strong, uniform and trace norm convergence of the resolvents of the selfadjoint operators H_{-b} associated with $\mathscr{E} - b\mathscr{P}$, as $b \longrightarrow \infty$. In particular, we discuss the case when H_{-b} equals a one-dimensional Schrödinger operator and its potential equals $-b$ times a discrete measure and an equilibrium measure, respectively.

The block numerical range of operators in Banach spaces

K.-H. Förster

Abstract: The block numerical range of (linear) operators in Hilbert spaces is a generalization of the well-known numerical range for operators in Hilbert spaces introduced in the beginning of the 20th century. Recently the concept of the block numerical range was introduced, see Ch. Tretter, Spectral Theory of Block Operator Matrices and Applications, 2008.
The numerical range of operators in normed spaces and normed algebras was introduced by G. Lumer (1961) and F.L. Bauer (1962), see F.F Bonsall, J. Duncan, Numerical Ranges I, II, 1971.
In this talk we generalize both concepts and introduce the blocknumerical range of operators in products of Banach spaces. The essential block numerical range is characterized with help of the block numerical range in normed para-algebras, see D. Przeworska-Rolewicz, S. Rolewicz, Equations in linear spaces 1968.
This is a joint work with P. Kallus and D. Kretschmann.

Division of distibutions and small ideals of operators

Bernhard Gramsch

Abstract: The complex homotopy principle of Oka - Grauert - Gromov (cf., e.g. Ann. of Math.175 (2012) 45 - 69) in the version of Bungart and Leiterer (cf.: Banach algebras, de Gruyter 1998 , 189 - 204) is applied to the parameter dependent factorisation for operator valued distributions:

$$B(t) = D(t)T(t) \tag{1}$$

where B and D are distributions and T an (analytic) Fredholm function depending continuously on a parameter t (cf. e.g.: Kaballo : Indag Math. 23 (2012) 970 - 994). Despite classical counterexamples in the scalar valued case, some sufficient conditions can be given for (1) with $T(t,z)$, z one complex variable. For several variables the situation is much more complicated. In some cases the "quotient D" of B and the multiplication with T has the form:

$$D(t) = B(t)A(t) + S(t) \tag{2}$$

with a multiplication operator $A(t)$ and an operator distribution $S(t)$ with values in small ideals with fast decreasing approximation numbers, respectively eignvalues. These small ideals have been considered by Grothendieck, Rotfeld and Rosenberger (cf. Bull. Lond. M. S. 44 (2012) 1085 - 1102; Math. Z. 133 (1973) 219 - 242). (1) and (2) can be applied to the Hörmander classes of pseudodifferential operators as Fréchet algebras with submultiplicativity and spectral invariance (cf. Lauter, Nistor, e.g. : J. Funct. Anal. 169 (1999) 81 - 120 ; J. Inst. Math. Jussieu 4 (2005) 65 - 82 ; ibid. 114 (2011) 255 - 283 ; Schrohem, e.g. Math. Nachr. 199 (1999), 145 - 185 ; cf. Adv. in PDE, vol 5 (1994) 235 - 258). The presence of topological conditions for $T(t,z)$ in the equation (1) can be explained by the Arens - Royden theorem, a version of the h - principle for (commutative) Banach algebras. There are some possibly new results for scalar distributions on regions in the complex plane.

PT-symmetries in physics and some operator-theoretic challenges

Uwe Günther

Abstract: In the first half of the talk, a brief overview is given about basic results obtained in PT-symmetry related physics since 1998: early numerical indications on the reality of the spectrum of certain non-Hermitian effective Hamiltonians in quantum mechanics, the concrete reality condition for the spectrum, PT phase transitions, self-adjointness in Krein spaces, hidden PT-symmetry in some hydrodynamic problems as well as most recent physical applications in optical waveguide systems, threshold lasers, coherent perfect absorbers and PT-symmetric electronic LRC circuits. In the second half of the presentation, we comment on some operator-theoretic challenges like the structure of the so-called C-operator for model Hamiltonians with ix^3 potential, indications on the unboundedness of this C-operator and the possible deep relationships to phase transitions in Ising models and models with quartic coupling terms. Finally, some links to quantum field theories are briefly sketched.

The Operator Theory of Abstract Square Functions

Markus Haase

Abstract: Square function estimates are a classical tool in Harmonic Analysis. Building on work of McIntosh [5], Cowling et al. [1] established a link between certain square functions and the boundedness of H^∞-calculus of sectorial operators on certain Banach spaces. In a famous (yet unfortunately uncompleted) preprint [3], Kalton and Weis showed how to use the concept of *γ-radonifying* operators to model square functions for sectorial operators on general Banach spaces and to recover the connection between (so defined) square function estimates and a bounded H^∞-calculus.

In my talk I will report on recent work [2] of Bernhard Haak (Bordeaux) and myself where we develop this approach further. We give a general *definition* of an abstract square function (in terms of γ-radonifying operators), show how *every* functional calculus gives rise to such square functions, and demonstrate that the boundedness of the classical square functions (in the case of bounded H^∞-calculus) has its root in a new boundedness concept for subsets of Hilbert spaces, the so-called *ℓ^1-boundedness*.

It turns out that for a sectorial operator with a bounded H^∞-calculus on a Banach space of finite cotype *essentially all* square functions associated with the functional calculus are bounded, a result obtained independently (and in a different language) by Le Merdy in [4].

References

[1] COWLING, M., DOUST, I., MCINTOSH, A., AND YAGI, A. Banach space operators with a bounded H^∞ functional calculus. *J. Austral. Math. Soc. Ser. A 60*, 1 (1996), 51–89.

[2] HAAK, B. and HAASE, M., Square function estimates and functional calculus. Preprint.

[3] KALTON, N., AND WEIS, L. The H^∞-functional calculus and square function estimates. unpublished manuscript, 2004.

[4] LE MERDY, C. H^∞-functional calculus and square function estimates for Ritt operators. arXiv:1202.0768v2.

[5] MCINTOSH, A. Operators which have an H^∞ functional calculus. In *Miniconference on operator theory and partial differential equations (North Ryde, 1986)*, vol. 14 of *Proc. Centre Math. Anal. Austral. Nat. Univ.* Austral. Nat. Univ., Canberra, 1986, pp. 210–231.

Left invertibility of formal Hamiltonian operators

Junjie Huang

Abstract: In this talk, we investigate the left invertible completion of the partial formal Hamiltonian operators $\begin{pmatrix} A & ? \\ 0 & -A^* \end{pmatrix}$ and $\begin{pmatrix} A & ? \\ C & -A^* \end{pmatrix}$ with unbounded entries. In particular, the left invertible completion of the partial Hamiltonian operators are also given. Based on the space decomposition technique, the alternative sufficient and necessary conditions are given according to whether the dimension of $\mathscr{R}(A)^{\perp}$ is finite or infinite.
This work were completed jointly with Alatancang and Yaru Qi.

Semigroups related to Hamiltonian systems

Birgit Jacob

Abstract: Many physical systems can be modeled using a Hamiltonian framework. In this talk we study infinite-dimensional Hamiltonian systems which can be modeled by a linear system operator $A : D(A) \subset X \to X$, $X = L^2(a,b;\mathbb{R}^n)$, of the form

$$\begin{aligned} Ax &:= P_1(\mathcal{H}x)' + P_0(\mathcal{H}x), \quad x \in D(A), \\ D(A) &:= \left\{ x \in X \mid \mathcal{H}x \in H^1(a,b;\mathbb{R}^n) \text{ and } W_B \begin{bmatrix} (\mathcal{H}x)(a) \\ (\mathcal{H}x)(b) \end{bmatrix} = 0 \right\}. \end{aligned}$$

We characterize the property that A generates a C_0-semigroup in terms of the matrices P_0, P_1, $\mathcal{H}$ and W_B. Further, we characterize contraction and unitary semigroups.
This is joint work with Kirsten Morris (Waterloo) and Hans Zwart (Twente).

Robustness of L^2-maximal regularity under non-autonomous multiplicative perturbation

Hafida Laasri

Abstract: The purpose of this talk is to study maximal regularity for a class of non-autonomous linear evolution equations in Hilbert spaces. Starting from an autonomous evolution equation, the non-autonomous evolution equation is constructed by introducing a time-dependent multiplicative perturbation.
Let $A \in \mathscr{L}(D,H)$, where H and D are two separable Hilbert spaces such that D is continuously and densely embedded in H. We assume that A has L^2-maximal regularity, that is for every $f \in L^2(0,\tau;H)$ there exists a unique function u belonging to the space $L^2(0,\tau;D) \cap W^{1,2}(0,\tau;H)$ such that

$$(1) \quad \dot{u}(t) + Au(t) = f(t) \quad t\text{-a.e. on } [0,\tau] \quad u(0) = 0.$$

We consider a function $P : [0,\tau] \longrightarrow \mathscr{L}(H)$ which is selfadjoint, bounded below and Lipschitz continuous. Moreover, if we assume that $-A$ generates an analytic semigroup of contractions, then we show that the non-autonomous Cauchy problem

$$(2) \quad \dot{u}(t) + AP(t)u(t) = f(t) \quad t\text{-a.e. on } [0,\tau] \quad u(0) = 0.$$

has the L^2-maximal regularity, which means in this case that for each $f \in L^2(0,\tau;H)$ there is a unique solution u of (2) satisfying $\dot{u} \in L^2(0,\tau;H)$ and $u(t) \in D(AP(t))$ for a.e. $t \in (0,\tau)$ and $t \mapsto AP(t)u(t) \in L^2(0,\tau;H)$.

Eigenspaces and spectra of nonnegative matrices under rank one perturbations

Leslie Leben

Abstract: In this talk, rank one perturbations of matrices in the space $(\mathbb{C}^n, [\cdot,\cdot])$ are studied. Here, $[\cdot,\cdot]$ is an indefinite inner product with an invertible symmetric matrix G as its Gramian, i.e. $[x, y] = \langle Gx, y\rangle$. Let A be a nonnegative matrix, i.e. $[Ax, x] \geq 0$, and let B be a symmetric matrix with respect to $[\cdot,\cdot]$, such that $B - A$ is of rank one. We will give a full spectral description of B (depending on the spectrum of A), including all possible structures of the algebraic eigenspaces of B. In particular, we show the estimate

$$|\dim \mathfrak{L}_0(A) - \dim \mathfrak{L}_0(B)| \leq 2,$$

where $\mathfrak{L}_0(A)$ is the algebraic eigenspaces of A at 0. As an illustration, the case of (3×3)-matrices will be discussed and all possible B will be described.

The talk is based on a joint work with F. Martinez Peria and C. Trunk.

A new result on generalized poles

Annemarie Luger

Abstract: In this talk we are revisiting generalized poles of a matrix valued generalized Nevanlinna function Q.
By definition these are eigenvalues of a (minimally) representing relation of Q. Hence the question appears how can these points (and its spectral characteristics) be characterized analytically in terms of the function. Partial results have been obtained during the last 30 years. However, the question of the algebraic multiplicity for matrix valued functions was been open. In the talk we present a short answer.

Relatively Symmetric Perturbations of Selfadjoint Operators

Friedrich Philipp

Abstract: We consider non-selfadjoint perturbations of a selfadjoint operator with relative bound less than one. Under the assumption that the perturbation is relatively symmetric with respect to the selfadjoint operator, we prove that outside a certain compact region the spectrum of the perturbed operator is real and separable in the sense of Lubic and Macaev.

Boundary triples associated with quadratic forms

Olaf Post

Abstract:There is a natural extension of the classical concept of boundary triples to quadratic forms, including the main example, the Dirichlet form on a manifold with boundary. We provide main results of the theory (a Krein-type resolvent formula) and give many examples, showing the usefulness of such a concept. Moreover, we indicate the relations to existing concepts.

Spectral analysis of the abstract integro-differential equations in Hilbert space

V. V. Vlasov and N. A. Rautian

Abstract: We study the spectra of the operator-valued functions which are the symbols of the abstract integro-differential equations in Hilbert space. We analyze the integro-differential equations arising in applications (Gurtin-Pipkin type equations describing the process of heat propagation in media with memory, integro-differential equations arising in the theory of viscoelasticity). We study the representations of the solutions of these equations as a series, obtained on the base of structure and asymptotic of spectra of above mentioned operator-valued functions.

Titchmarsh-Weyl theory for Schrödinger operators on noncompact trees

Jonathan Rohleder

Abstract: The spectral properties of Schrödinger operators on noncompact trees are discussed in terms of a Titchmarsh-Weyl function.

Expansion of $f(T)$ in a series of Faber–Walsh polynomials

Olivier Sète

Abstract: Let T be a bounded linear operator on a complex Banach space. Denote its spectrum by $\sigma(T)$. Let $E \supseteq \sigma(T)$ be a compact set composed of $\nu \geq 1$ components (none a single point), such that $K = \widehat{\mathbb{C}} \backslash E$ is (ν-times) connected. For E, there exist monic polynomials b_n, $n = 0, 1, 2, \ldots$, of respective degrees n, such that any analytic function f on E can be expanded in a series $f(z) = \sum_{n=0}^{\infty} a_n b_n(z)$, which is uniformly convergent on E. The polynomials b_n are Faber–Walsh polynomials associated with E.
Now, let f be analytic on E, and thus on $\sigma(T)$. Then

$$f(T) = \frac{1}{2\pi i} \int_{\Gamma} f(z)(z - T)^{-1} \, dz,$$

where Γ is any closed Jordan curve in the domain of analyticity of f and surrounding $\sigma(T)$. Let $f(z) = \sum_{n=0}^{\infty} a_n b_n(z)$ be the Faber–Walsh series for f. We show that

$$f(T) = \sum_{n=0}^{\infty} a_n b_n(T),$$

where the series converges in the operator norm.

Evolutionary Equations with Frictional Boundary Conditions

Sascha Trostorff

Abstract: We present a solution theory for a class of evolutionary inclusions, involving maximal montone operators,in a suitable Hilbert space setting. As an example of this class we consider evolutionary equations with nonlinear boundary conditions, which occur in frictional contact problems in the theory of elasticity.

Perturbation theory for accretive operators in L_p and elliptic operators with complex coefficients.

Hendrik Vogt

Abstract: We are going to look at second order elliptic operators

$$\mathscr{L} = \nabla \cdot (a\nabla) - b_1 \cdot \nabla - \nabla \cdot b_2 - Q$$

on an open set $\Omega \subseteq \mathbb{R}^N$ that do *not* act in all the L_p-spaces. In the case of real coefficients it is well known that the operator may generate a C_0-semigroup on L_p only for p from a certain subinterval of $[1,\infty)$ if the lower order terms are singular. In the case of complex coefficients, the same can happen for non-singular coefficients, even if one assumes uniform ellipticity.

We are particularly interested in the case that the sesquilinear form associated with $\mathscr{L}$ is not sectorial. Then it can happen that one does not obtain a C_0-semigroup on L_2, but only on L_p for certain $p \neq 2$, and it requires some work to associate the generator of a C_0-semigroup on L_p with $\mathscr{L}$. We will accomplish this with a new perturbation theorem for accretive operators in L_p.

(joint work with T. ter Elst, Z. Sobol and V. Liskevich)