On characteristic matrices and spectral functions of first-order symmetric systems

Vadim Mogilevskii

Let $\mathbb{H}$ be a finite-dimensional Hilbert space, let $[\mathbb{H}]$ be the set of all operators in $\mathbb{H}$ and let $J \in [\mathbb{H}]$ satisfies $J^* = J^{-1} = -J$. We will discuss first-order symmetric system

$$Jy' - B(t)y = \lambda \Delta(t)y + \Delta(t)f(t)$$

(1)

with the $[\mathbb{H}]$-valued coefficients $B(t) = B^*(t)$ and $\Delta(t) \geq 0$ defined on an interval $I = [a, b]$ with the regular endpoint $a$. Let $T_{\text{min}}$ be the minimal relation and let $Y(\cdot, \lambda)$ be the $[\mathbb{H}]$-valued solution of the system (1) with $f = 0$ satisfying $Y(0, \lambda) = I$. A spectral (pseudospectral) function of such a system is defined as an $[\mathbb{H}]$-valued distribution function $\Sigma(s)$, $s \in \mathbb{R}$, such that the Fourier transform

$$\hat{f}(s) = \int_I Y^*(t, s)\Delta(t)f(t) \, dt$$

is an isometry $V$ (resp. partial isometry $V$ with ker $V = \text{mul}T_{\text{min}}$) from $L^2(\Delta(I))$ into $L^2(\Sigma)$. We describe all generalized resolvents $y = R(\lambda)f$, $f \in L^2(I)$, of $T_{\text{min}}$ in terms of $\lambda$-depending boundary conditions imposed on regular and singular boundary values of a function $y$ at the endpoints $a$ and $b$ respectively. This enables us to parametrize all characteristic matrices $\Omega(\lambda)$ of the system (1) immediately in terms of boundary conditions. Such a parametrization is given both by the block-matrix representation of $\Omega(\lambda)$ and by the formula similar to the Krein formula for resolvents.

System (1) is called absolutely definite if the set $\{ t \in I : \Delta(t) \text{is invertible} \}$ has a nonzero Lebesgue measure. For an absolutely definite system the above parametrization of $\Omega(\lambda)$ gives rise to parametrization of all pseudospectral and spectral functions of this system by means of a Nevanlinna boundary parameter. Our results implies the parametrisations of pseudospectral and spectral functions obtained by Langer and Textorius [1] and Sakhnovich [3] for the particular case of the system (1) on a compact interval $I = [a, b]$.

The results of the talk are partially specified in [2].

