On characteristic matrices and spectral functions of first-order symmetric systems

Vadim Mogilevskii

Let $\mathbb{H}$ be a finite-dimensional Hilbert space, let $[\mathbb{H}]$ be the set of all operators in $\mathbb{H}$ and let $J \in [\mathbb{H}]$ satisfies $J^* = J^{-1} = -J$. We will discuss first-order symmetric system

$$Jy' - B(t)y = \lambda\Delta(t)y + \Delta(t)f(t) \tag{1}$$

with the $[\mathbb{H}]$-valued coefficients $B(t) = B^*(t)$ and $\Delta(t) \geq 0$ defined on an interval $\mathcal{I} = [a, b)$ with the regular endpoint a. Let $T_{\min}$ be the minimal relation and let $Y(\cdot, \lambda)$ be the $[\mathbb{H}]$-valued solution of the system (1) with $f = 0$ satisfying $Y(0, \lambda) = I$. A spectral (pseudospectral) function of such a system is defined as an $[\mathbb{H}]$-valued distribution function $\Sigma(s),\ s \in \mathbb{R}$, such that the Fourier transform

$$\widehat{f}(s) = \int_{\mathcal{I}} Y^*(t, s)\Delta(t)f(t)\, dt$$

is an isometry V (resp. partial isometry V with $\ker V = \operatorname{mul} T_{\min}$) from $L^2_\Delta(\mathcal{I})$ into $L^2(\Sigma)$. We describe all generalized resolvents $y = R(\lambda)f,\ f \in L^2_\Delta(\mathcal{I})$, of $T_{\min}$ in terms of λ-depending boundary conditions imposed on regular and singular boundary values of a function y at the endpoints a and b respectively. This enables us to parametrize all characteristic matrices $\Omega(\lambda)$ of the system (1) immediately in terms of boundary conditions. Such a parametrization is given both by the block-matrix representation of $\Omega(\lambda)$ and by the formula similar to the Krein formula for resolvents.

System (1) is called absolutely definite if the set $\{t \in \mathcal{I} : \Delta(t) \text{is invertible}\}$ has a nonzero Lebesgue measure. For an absolutely definite system the above parametrization of $\Omega(\lambda)$ gives rise to parametrization of all pseudospectral and spectral functions of this system by means of a Nevanlinna boundary parameter. Our results implies the parametrisations of pseudospectral and spectral functions obtained by Langer and Textorius [1] and Sakhnovich [3] for the particular case of the system (1) on a compact interval $\mathcal{I} = [a, b]$.

The results of the talk are partially specified in [2].

[1] H. Langer and B. Textorius: Spectral functions of a symmetric linear relation with a directing mapping, I, *Proc. Roy. Soc. Edinburgh Sect. A*, 97 (1984), 165-176.

[2] V.I. Mogilevskii: On generalized resolvents and characteristic matrices of first-order symmetric systems, arXiv:1403.3995v1 [math.FA] 16 Mar 2014.

[3] A.L. Sakhnovich: Spectral functions of a canonical system of order $2n$, *Math. USSR-Sb.*, 71 (1992), 355–369.