Spectral Theory and Differential Operators

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On a class of Sturm-Liouville operators which are connected to $\mathcal{PT}$ quantum mechanics

We consider so-called $\mathcal{PT}$ symmetric operators in the space $(L_2(\mathbb{R}), [\cdot,\cdot])$, with an indefinite inner product $[\cdot,\cdot]$ given via the fundamental symmetry $\mathcal{P}f(x) = f(-x)$ such that $[f,g] = (\mathcal{P}f, g)_{L_2}$. The space $(L_2(\mathbb{R}), [\cdot,\cdot])$ is an example of a Krein space. The action of the anti-linear operator $\mathcal{T}$ on a function of a real spatial variable x is defined by $\mathcal{T}f(x) = \overline{f(x)}$.

An operator A is said to be $\mathcal{PT}$-symmetric if it commutes with $\mathcal{PT}$.

In the last decade a generalization of the harmonic oscillator using a complex deformation was investigated. This operator is defined via the differential expression

$$(\tau y)(x) := -y''(x) + x^2(ix)^\epsilon y(x), \quad \epsilon > 0.$$

We will start our investigations with the discussion of some simple cases (like ϵ even) and we will concentrate on a describtion of self-adjoint, $\mathcal{P}$-selfadjoint and $\mathcal{PT}$-symmetric operators related to such a differential expression and their spectral properties. The talk is based on joint works with T.Ya. Azizov (Voronezh).