Stokes graph and non-oscillating solutions

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Stokes flows with varying viscosity η and density ρ are considered. We deal with the case when the flow is concentrated in narrow neighborhood of a network. The main term of the asymptotic expansion (in respect to the width of these narrow channels) for the flow velocity satisfies the 1D Schrödinger equation with a specific potential:

$$v'' - \frac{\eta'}{\eta}\frac{\rho'}{\rho}v = 0.$$

Here $v = v(x)$ is the flow velocity. It allow one to use the metric graph with the Scrödinger operator $L = -\frac{d^2}{dx^2} + \frac{\eta'}{\eta}\frac{\rho'}{\rho}$ on edges for the description of the flow. We call it the Stokes graph. It is analogous to the corresponding quantum graph. It is necessary to determine boundary conditions at the graph vertices. Consider a vertex (let it be zero point) with n output edges. From physical conditions one has $\rho_1(0) = \rho_2(0) = ...\rho_n(0) = \rho(0)$ and $v_1'(+0) = v_2'(+0) = ...v_n'(+0) = v'(0)$. The last condition is related with the pressure continuity. Here $v_j'(+0)$ is the derivative in the outgoing direction at the vertex 0. The continuity equation gives us for this vertex:

$$\sum_{j=1}^{n} v_j = -\left(\frac{\rho(0)}{\sum\limits_{j=1}^{n}\rho_j'(+0)}\right)v'(0).$$

It is analogous to well-known δ'-coupling condition for quantum graph. The coupling constant is related with the density derivative.

Non-oscillating solutions on the Stokes graph are studied. Estimates analogous to the Harnack's inequality for an elliptic operator on a manifold are obtained. The relation between the spectral properties of the graph Hamiltonian and the parameters of the corresponding Stokes flow is discussed.