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The Eigenvalue Problem for the Cavity Maxwell Operator

Abstract. In this talk we discuss the propagation problem of the time harmonic electromagnetic field $\mathsf{e} := (\mathbf{E}, \mathbf{H})^T$ (time convention is taken to be $\mathrm{e}^{-\mathrm{i}\omega t}$) inside a Lipschitz bounded domain $\Omega \subset \mathbb{R}^3$. Ω represents a cavity, which is filled with a general bianisotropic medium and is perfect conducting, that is, the boundary condition $\hat{\mathbf{n}} \times \mathbf{E} = \mathbf{0}$ on $\partial\Omega$ is satisfied. $\hat{\mathbf{n}}$ denotes the outward normal, which is defined almost everywhere on $\partial\Omega$.

This problem is formally formulated as an eigenvalue problem for a linear operator pencil

$$\mathcal{Q}\mathsf{e} = \omega \mathsf{M}(\omega)\mathsf{e}. \tag{$\star$}$$

Here $\mathcal{Q}$ stands for the self-adjoint Maxwell operator

$$\mathcal{Q} := \mathrm{i} \begin{bmatrix} 0 & -\operatorname{curl} \\ \operatorname{curl} & 0 \end{bmatrix},$$

and $\mathsf{M} = \mathsf{M}(\omega)$ for the material matrix

$$\mathsf{M} := \begin{bmatrix} \boldsymbol{\varepsilon} & \boldsymbol{\xi} \\ \boldsymbol{\zeta} & \boldsymbol{\mu} \end{bmatrix},$$

which models the medium properties inside Ω. The entries of M are $L^\infty(\Omega)$ functions that depend on the eigenvalue ω.

We realize ($\star$) in a Hilbert space setting and our analysis breaks down into two steps:

1. We first study the hollow cavity, modeled by the simplest case $\mathsf{M} = I$. Then ($\star$) becomes a standard eigenvalue problem for which we employ a variant of the spectral theorem tfor discrete self-adjoint operators to prove existence for the eigenelements.

2. For the general matrix operator M, we use perturbation arguments to prove existence for the eigenelements of the general cavity and characterize them with aim of their hollow cavity counterparts.