Spectral analysis for three coupled strips quantum graph

A. I. Popov, I. V. Blinova, A. N. Skorynina, I. Yu. Popov
St. Petersburg National Research University
of Information Technologies, Mechanics and Optics,
49 Kronverkskiy, St. Petersburg, 197101, Russia,
popov239@gmail.com, irin-a@yandex.ru, popov1955@gmail.com

Infinite quantum graph Γ formed by three coupled identical strips with honeycomb lattice of edges E and vertices V is considered. The Schrödinger operator is constructed in the framewor of the theory of self-adjoint extensions of symmetric operators in the Hilbert spaces. It is defined at each edge by the following differential expression

$$H = -\frac{d^2}{dx^2}. \tag{1}$$

The domain of H is as follows (we assume δ-coupling at the vertices):

$$\begin{cases} f \in W_2^1(\Gamma) \bigcap W_2^2(\Gamma \setminus V), \\ \sum_{e \in E_v} \frac{df}{dx_e}(v) = \alpha f(v). \end{cases} \tag{2}$$

Here α is some fixed number, $E_v := \{e \in E \,|v \in e\}$ is the set of edges adjacent to the vertex $v, v \in V$. The sum is taken over all edges e incident to the vertex v and the derivatives are taken along e in the directions away from the vertex v (outgoing direction).

The spectral analysis of the Hamiltonian is made for different values of the parameter α. The method of the transfer-matrices is used. The spectral equation is obtained in an explicit form, The essential spectrum has bent structure. The condition on the operator parameter α ensuring the existence of eigenvalues in the gaps is obtained. For positive α the operator is positive. For negative α the negative lower bound of the essential spectrum is found. We specify the restrictions on the model parameters ensuring the existence of the eigenvalue below the threshold.