Sommersemester 2015

Technische Universität Ilmenau
Institut für Mathematik
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Mathematik mit Maple für IN

Lösungen der Aufgaben aus Übungsserie 6

Aufgabe 1:

a)

> $restart : with(plots) :$

> $g := x \rightarrow \frac{\sin^2(x)}{4}$

$$g := x \rightarrow \frac{1}{4} \sin(x)^2 \qquad \textbf{(1.1)}$$

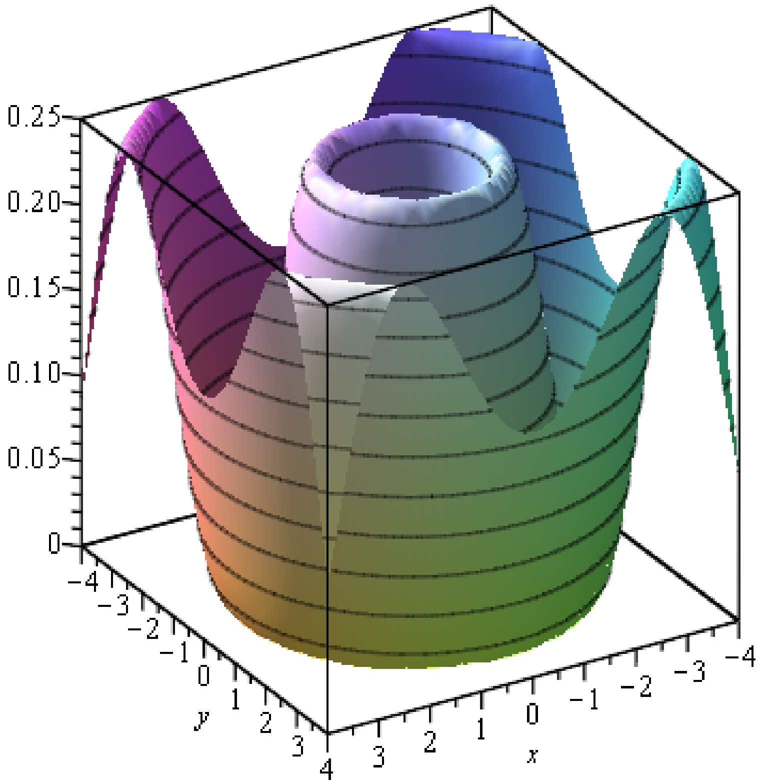
> $f := (x, y) \rightarrow g(\mathrm{sqrt}(x^2 + y^2))$

$$f := (x, y) \rightarrow g\left(\sqrt{x^2 + y^2}\right) \qquad \textbf{(1.2)}$$

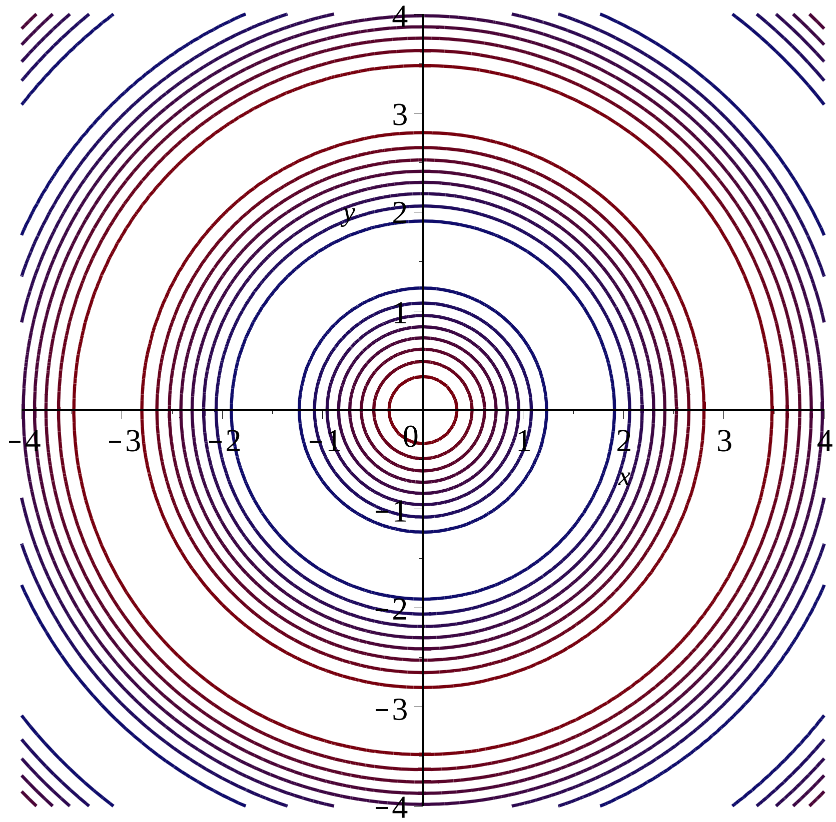
> $'f(x, y)' = f(x, y)$

$$f(x, y) = \frac{1}{4} \sin\left(\sqrt{x^2 + y^2}\right)^2 \qquad \textbf{(1.3)}$$

> $plot3d(f(x, y), x = -4..4, y = -4..4, axes = BOXED, style = PATCHCONTOUR, orientation = [59, 63])$



> $contourplot(f(x, y), x = -4..4, y = -4..4, numpoints = 10000)$



b)

> $restart : with(plots) :$

> $g := x \rightarrow \exp(-x) + 2$

$$g := x \rightarrow e^{-x} + 2 \tag{1.4}$$

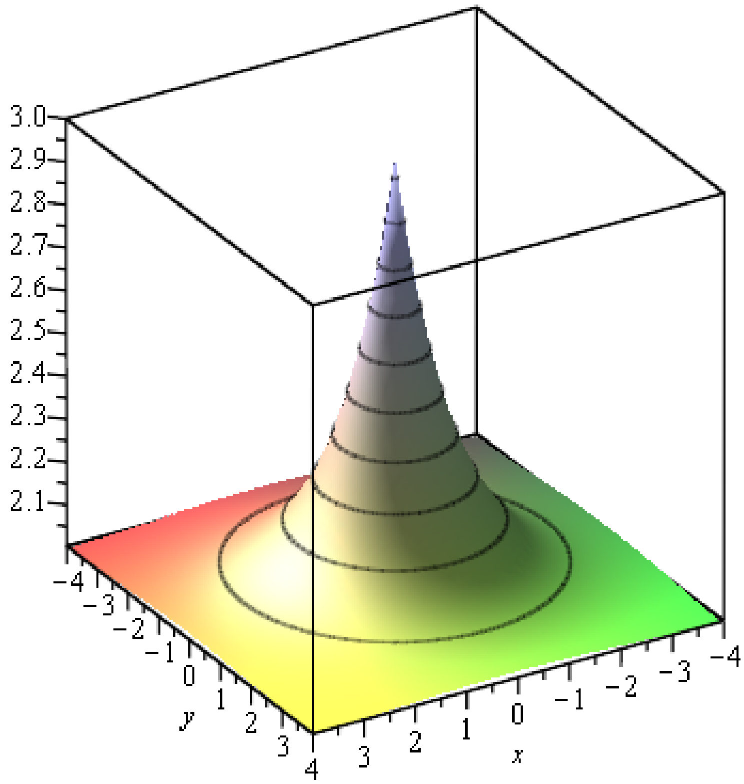
> $f := (x, y) \rightarrow g(\mathrm{sqrt}(x^2 + y^2))$

$$f := (x, y) \rightarrow g\left(\sqrt{x^2 + y^2}\right) \tag{1.5}$$

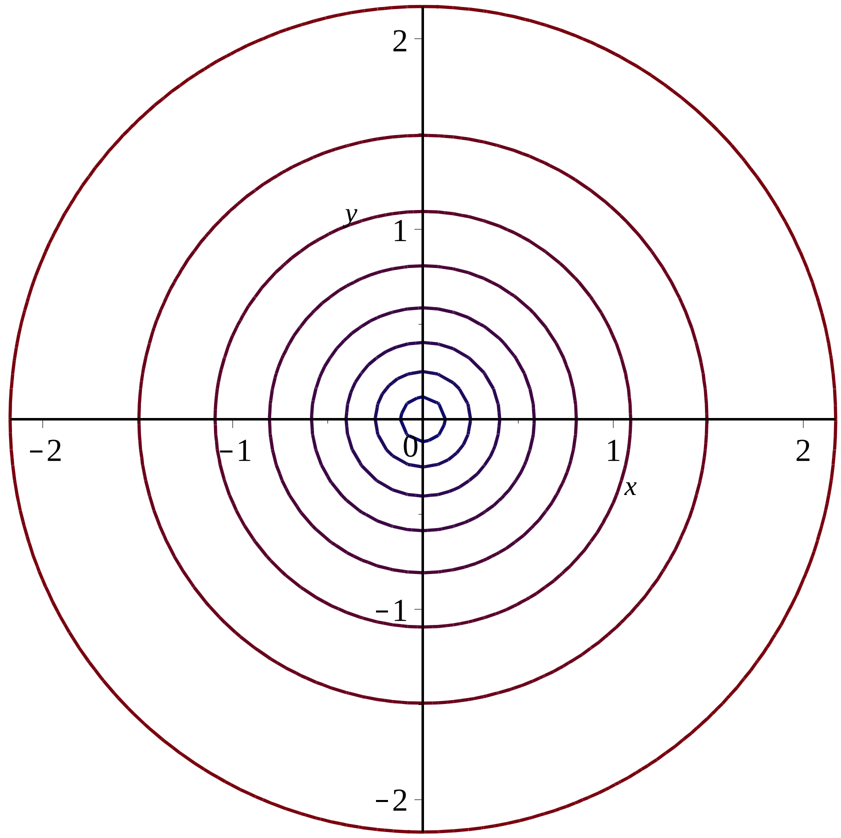
> $'f(x, y)' = f(x, y)$

$$f(x, y) = e^{-\sqrt{x^2 + y^2}} + 2 \tag{1.6}$$

> $plot3d(f(x, y), x = -4 ..4, y = -4 ..4, axes = BOXED, style = PATCHCONTOUR, orientation = [59, 63])$



> $contourplot(f(x, y), x = -4..4, y = -4..4, numpoints = 10000)$



Aufgabe 2:

> *restart*

> $f := (x, y) \rightarrow \frac{x^2 - y^2}{x^2 + y^2}$

$$f := (x, y) \rightarrow \frac{x^2 - y^2}{x^2 + y^2} \quad \textbf{(2.1)}$$

> $limit(f(x, y), \{x = 0, y = 1\})$

$$-1 \quad \textbf{(2.2)}$$

> $limit(f(x, y), \{x = 1, y = -1\})$

$$0 \quad \textbf{(2.3)}$$

Aufgabe 3:

> $restart : with(LinearAlgebra) :$

> $f := (x, y) \rightarrow \frac{\text{sqrt}(4 - x^2 + y^2)}{2}$

$$f := (x, y) \rightarrow \frac{1}{2} \sqrt{4 - x^2 + y^2} \qquad (3.1)$$

a)

> $fx := D[1](f)$

$$fx := (x, y) \rightarrow -\frac{1}{2} \frac{x}{\sqrt{4 - x^2 + y^2}} \qquad (3.2)$$

> $fy := D[2](f)$

$$fy := (x, y) \rightarrow \frac{1}{2} \frac{y}{\sqrt{4 - x^2 + y^2}} \qquad (3.3)$$

b)

> $gradf := (x, y) \rightarrow Vector([fx(x, y), fy(x, y)])$

$$gradf := (x, y) \rightarrow Vector([fx(x, y), fy(x, y)]) \qquad (3.4)$$

> $'gradf\left(\frac{1}{2}, -\frac{7}{2}\right)' = gradf\left(\frac{1}{2}, -\frac{7}{2}\right)$

$$gradf\left(\frac{1}{2}, -\frac{7}{2}\right) = \begin{bmatrix} -\frac{1}{16} \\ -\frac{7}{16} \end{bmatrix} \qquad (3.5)$$

c)

> $v := Vector([1, -1])$

$$v := \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad (3.6)$$

> $v0 := Normalize(v, 2)$

$$v0 := \begin{bmatrix} \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} \end{bmatrix} \qquad (3.7)$$

> $Diff(f, 'v')(a) = DotProduct\left(gradf\left(\frac{1}{2}, -\frac{7}{2}\right), v0\right)$

$$\frac{\partial}{\partial v} f(a) = \frac{3}{16}\sqrt{2} \qquad (3.8)$$

d)

> $d := Vector([dx, dy])$

$$d := \begin{bmatrix} dx \\ dy \end{bmatrix} \qquad (3.9)$$

> $df := unapply(DotProduct(gradf(x, y), d, conjugate = false), x, y, dx, dy)$

$$df := (x, y, dx, dy) \rightarrow -\frac{1}{2} \frac{x\,dx}{\sqrt{4 - x^2 + y^2}} + \frac{1}{2} \frac{y\,dy}{\sqrt{4 - x^2 + y^2}} \quad \textbf{(3.10)}$$

e)

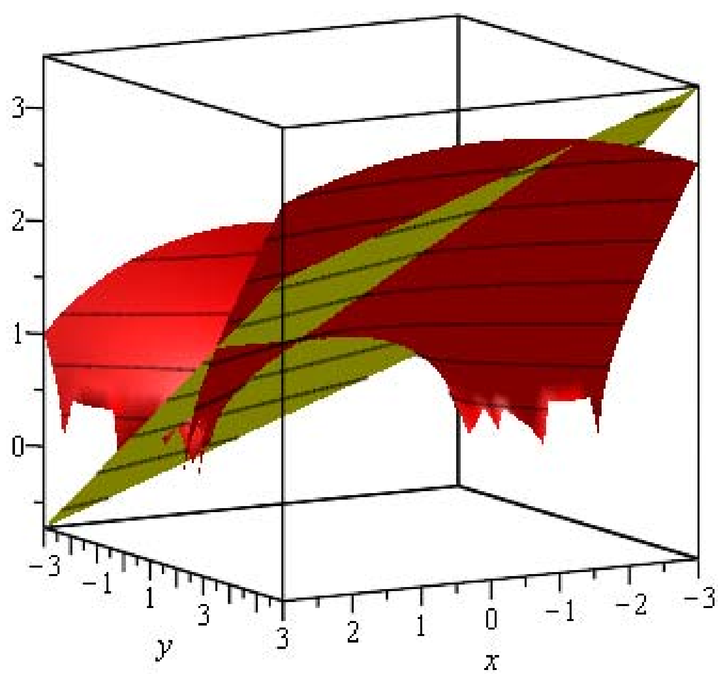
> $T := unapply(f(1, \mathrm{sqrt}(2)) + df(1, \mathrm{sqrt}(2), x - 1, y - \mathrm{sqrt}(2)), x, y)$

$$T := (x, y) \rightarrow \frac{1}{2}\sqrt{5} - \frac{1}{10}\sqrt{5}\,(x - 1) + \frac{1}{10}\sqrt{2}\,\sqrt{5}\,\left(y - \sqrt{2}\right) \quad \textbf{(3.11)}$$

f)

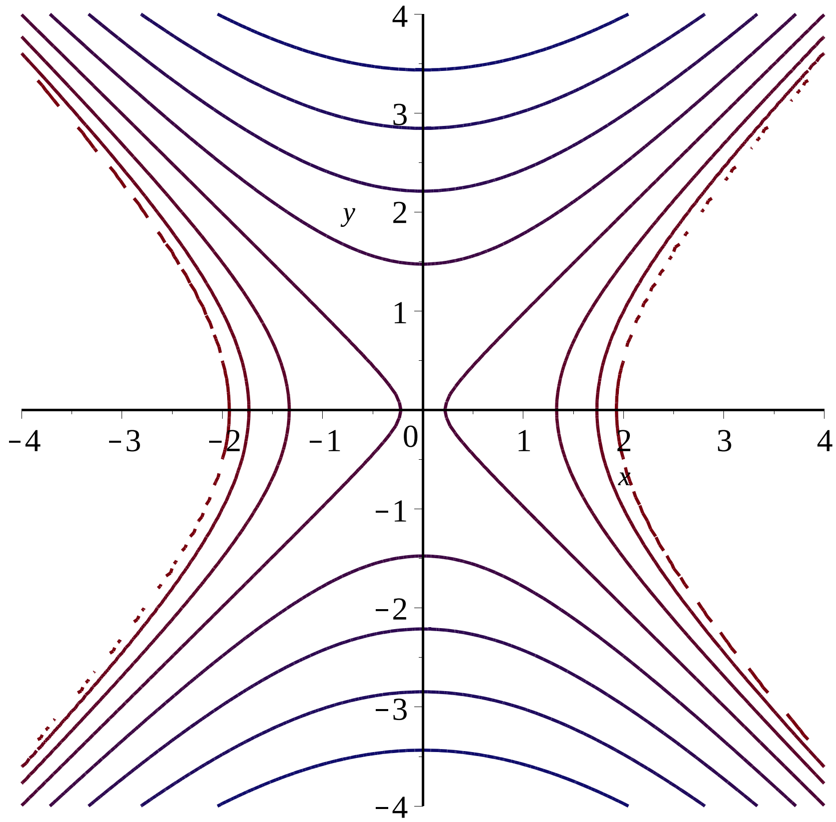
> *with*(*plots*) :

> *plot3d*([$f(x, y)$, $T(x, y)$], $x = -3..3$, $y = -3..6$, *axes* = *BOXED*, *style* = *PATCHCONTOUR*, *color* = [*red*, *yellow*], *orientation* = [60, 80])



g)

> *contourplot*($f(x, y)$, $x = -4..4$, $y = -4..4$, *numpoints* = 10000)



Aufgabe 4:

> *restart* : *with*(*LinearAlgebra*) :

> $f := (x, y, z) \rightarrow \ln\left(\mathrm{sqrt}\left(\left(2 \cdot x^2 + 2 \cdot y^2 + 2 \cdot z^2\right)^3\right)\right)$

$$f := (x, y, z) \rightarrow \ln\left(\sqrt{\left(2\,x^2 + 2\,y^2 + 2\,z^2\right)^3}\right) \quad \textbf{(4.1)}$$

> $gradf := Vector([diff(f(x, y, z), x), diff(f(x, y, z), y), diff(f(x, y, z), z)])$

$$gradf := \begin{bmatrix} \frac{6\,x}{2\,x^2 + 2\,y^2 + 2\,z^2} \\ \frac{6\,y}{2\,x^2 + 2\,y^2 + 2\,z^2} \\ \frac{6\,z}{2\,x^2 + 2\,y^2 + 2\,z^2} \end{bmatrix} \quad \textbf{(4.2)}$$

> $d := Vector([dx, dy, dz])$

$$d := \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \quad \textbf{(4.3)}$$

> $df := unapply(DotProduct(gradf, d, conjugate = false), x, y, z, dx, dy, dz)$

$$df := (x, y, z, dx, dy, dz) \rightarrow \frac{6\,x\,dx}{2\,x^2 + 2\,y^2 + 2\,z^2} + \frac{6\,y\,dy}{2\,x^2 + 2\,y^2 + 2\,z^2} + \frac{6\,z\,dz}{2\,x^2 + 2\,y^2 + 2\,z^2} \quad \textbf{(4.4)}$$

> $df(-1, 2, -2, dx, dy, dz)$

$$-\frac{1}{3}\,dx + \frac{2}{3}\,dy - \frac{2}{3}\,dz \quad \textbf{(4.5)}$$

> $dx := -0.9 - (-1);\ dy := 1.8 - 2;\ dz := -2.1 - (-2)$

$$dx := 0.1$$
$$dy := -0.2$$
$$dz := -0.1 \quad \textbf{(4.6)}$$

> $evalf(f(-1, 2, -2) + df(-1, 2, -2, dx, dy, dz))$

$$4.235557637 \quad \textbf{(4.7)}$$

Aufgabe 5:

> $restart$

> $f := (x, y) \rightarrow 89 \cdot x^2 - 96 \cdot x \cdot y + 61 \cdot y^2 - 260 \cdot x + 70 \cdot y + C$

$$f := (x, y) \rightarrow 89\,x^2 - 96\,x\,y + 61\,y^2 - 260\,x + 70\,y + C \quad \textbf{(5.1)}$$

Für die Gleichung der Tangentialebene von f(x,y) im Berührpunkt $P_0 = (x_0, y_0, 0)$ gilt z=T(x,y)=0, das heißt die partiellen Ableitungen von f(x,y) an der Stelle (x_0, y_0) sind gleich Null.

> $fx := D[1](f)$

$$fx := (x, y) \rightarrow 178\,x - 96\,y - 260 \quad \textbf{(5.2)}$$

> $fy := D[2](f)$

$$fy := (x, y) \rightarrow -96\,x + 122\,y + 70 \quad \textbf{(5.3)}$$

> $solve(\{fx(x, y) = 0, fy(x, y) = 0\}, \{x, y\})$

$$\{x = 2, y = 1\} \quad \textbf{(5.4)}$$

Damit gilt $P_0 = (2, 1, 0)$. Und die Lösung für C erhält man durch Einsetzen der Koordinaten von P_0 in f(x,y)=0:

> $'C' = solve(f(2, 1) = 0, C)$

$$C = 225 \quad \textbf{(5.5)}$$

Aufgabe 6:

> $restart : with(LinearAlgebra) :$

> $f := (x, y) \rightarrow (x - y) \cdot \sin(y)$

$$f := (x, y) \rightarrow (x - y)\,\sin(y) \quad \textbf{(6.1)}$$

> $gradf := unapply(Vector([diff(f(x, y), x), diff(f(x, y), y)]), x, y) :$

> $'gradf(x, y)' = gradf(x, y)$

$$gradf(x,y) = \begin{bmatrix} \sin(y) \\ -\sin(y) + (x-y)\cos(y) \end{bmatrix} \quad \textbf{(6.2)}$$

> $with(VectorCalculus):$

> $H := Hessian(f(x,y), [x,y])$

$$H := \begin{bmatrix} 0 & \cos(y) \\ \cos(y) & -2\cos(y) - (x-y)\sin(y) \end{bmatrix} \quad \textbf{(6.3)}$$

> $d := Vector([dx, dy])$

$$d := (dx)e_x + (dy)e_y \quad \textbf{(6.4)}$$

> $df := unapply(DotProduct(gradf(x,y), d), x, y, dx, dy):$

> $'df(x,y,dx,dy)' = df(x,y,dx,dy)$

$$df(x,y,dx,dy) = \sin(y)\, dx + (-\sin(y) + (x-y)\cos(y))\, dy \quad \textbf{(6.5)}$$

> $d2f := unapply(BilinearForm(d, d, H, conjugate = false), x, y, dx, dy):$

> $'d2f(x,y,dx,dy)' = d2f(x,y,dx,dy)$

$$d2f(x,y,dx,dy) = dx\cos(y)\, dy + dy\,(\cos(y)\, dx + (-2\cos(y) - (x-y)\sin(y))\, dy) \quad \textbf{(6.6)}$$

> $T2 := unapply\left(f(1,1) + df(1,1,x-1,y-1) + \frac{1}{2}\cdot d2f(1,1,x-1,y-1), x, y\right):$

> $'T2(x,y)' = simplify(T2(x,y))$

$$T2(x,y) = \cos(1)\, x\, y - \cos(1)\, y^2 + \sin(1)\, x - \sin(1)\, y - \cos(1)\, x + \cos(1)\, y \quad \textbf{(6.7)}$$

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