Sommersemester 2015

Technische Universität Ilmenau
Institut für Mathematik
PD Dr. Erhard Hexel
Mathematik mit Maple für IN

Lösungen der Aufgaben aus Übungsserie 7

Aufgabe 1:

> $restart : with(plots) :$

> $with(LinearAlgebra) :$

> $f := (x, y) \rightarrow \exp(-x^2) \cdot (4 \cdot y + x^2 - y^2)$

$$f := (x, y) \rightarrow e^{-x^2} (4y + x^2 - y^2) \quad \textbf{(1.1)}$$

> $fx := D[1](f); fy := D[2](f)$

$$fx := (x, y) \rightarrow -2x e^{-x^2} (4y + x^2 - y^2) + 2x e^{-x^2}$$
$$fy := (x, y) \rightarrow e^{-x^2} (-2y + 4) \quad \textbf{(1.2)}$$

> $gradf := (x, y) \rightarrow Vector([fx(x, y), fy(x, y)])$

$$gradf := (x, y) \rightarrow Vector([fx(x, y), fy(x, y)]) \quad \textbf{(1.3)}$$

> $g := gradf(x, y)$

$$g := \begin{bmatrix} -2x e^{-x^2} (x^2 - y^2 + 4y) + 2x e^{-x^2} \\ e^{-x^2} (-2y + 4) \end{bmatrix} \quad \textbf{(1.4)}$$

> **use** $RealDomain$ **in** $L := solve(\{g(1) = 0, g(2) = 0\})$ **end use**

$$L := \{x = 0, y = 2\} \quad \textbf{(1.5)}$$

> $P := [rhs(L[1]), rhs(L[2])]$ *# extremwertverdächtige Stelle*

$$P := [0, 2] \quad \textbf{(1.6)}$$

> $with(VectorCalculus) :$

> $H := Hessian(f(x, y), [x, y] = P)$

$$H := \begin{bmatrix} -6 & 0 \\ 0 & -2 \end{bmatrix} \quad \textbf{(1.7)}$$

> $Determinant(H)$

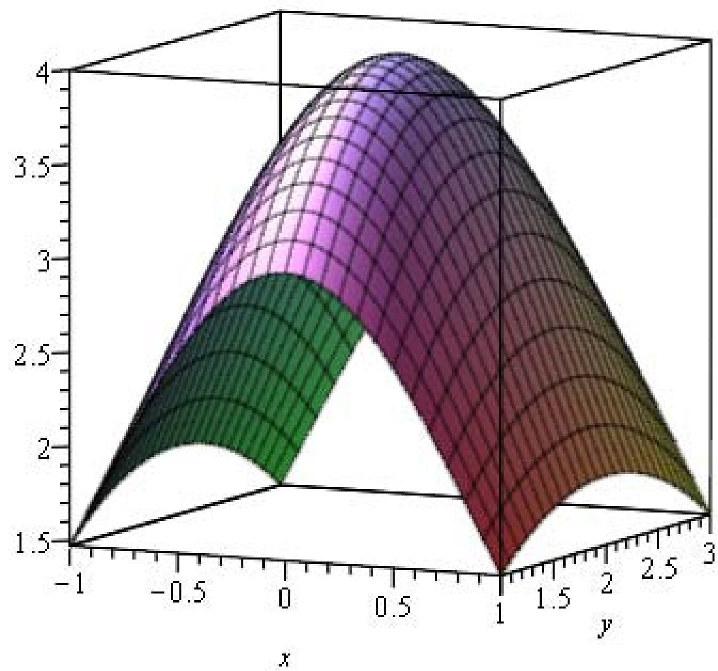
$$12 \quad \textbf{(1.8)}$$

> $H(1, 1)$

$$-6 \quad \textbf{(1.9)}$$

An der Stelle P=(0,2) besitzt die Funktion f(x,y) folglich ein lokales Maximum.

> $plot3d(f(x, y), x = -1..1, y = 1..3, axes = BOXED, orientation = [-64, 82])$



Aufgabe 2:

> $restart : with(plots) :$

> $with(LinearAlgebra) :$

> $f := (x, y) \rightarrow x^2 + y^2$

$$f := (x, y) \rightarrow x^2 + y^2 \quad \textbf{(2.1)}$$

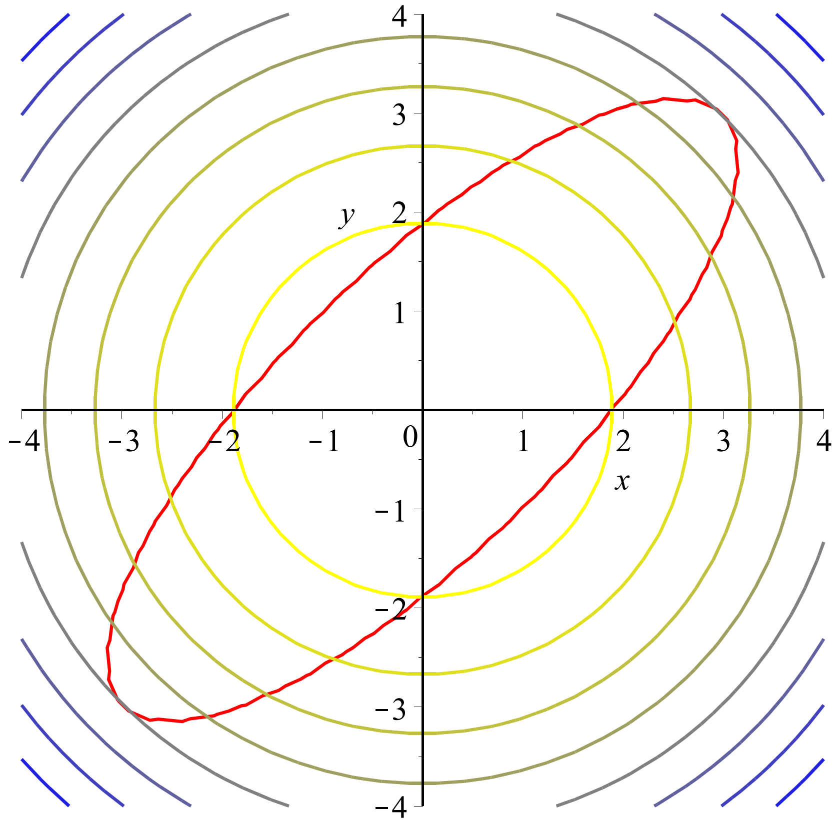
> $g := (x, y) \rightarrow 5 \cdot x^2 + 5 \cdot y^2 - 8 \cdot x \cdot y - 18$

$$g := (x, y) \rightarrow 5\, x^2 + 5\, y^2 - 8\, x\, y - 18 \quad \textbf{(2.2)}$$

> $p1 := contourplot(f(x, y), x = -4..4, y = -4..4, grid = [30, 30], coloring = [yellow, blue]) :$

> $p2 := implicitplot(g(x, y) = 0, x = -4..4, y = -4..4, color = red) :$

> $display(\{p1, p2\})$



Definition und Untersuchung der Ersatzfunktion L=L(x,y,λ) mit Lagrange-Multiplikator λ:

> $L := unapply(f(x,y) + \lambda \cdot g(x,y), x, y, \lambda)$

$$L := (x, y, \lambda) \rightarrow x^2 + y^2 + \lambda\,(5\,x^2 + 5\,y^2 - 8\,x\,y - 18) \tag{2.3}$$

> $Lx := \mathrm{D}[1](L);\ Ly := \mathrm{D}[2](L);\ L\lambda := \mathrm{D}[3](L)$

$$Lx := (x, y, \lambda) \rightarrow 2\,x + \lambda\,(10\,x - 8\,y)$$

$$Ly := (x, y, \lambda) \rightarrow 2\,y + \lambda\,(-8\,x + 10\,y)$$

$$L\lambda := (x, y, \lambda) \rightarrow 5\,x^2 + 5\,y^2 - 8\,x\,y - 18 \tag{2.4}$$

> $gradL := (x, y, \lambda) \rightarrow Vector([Lx(x, y, \lambda), Ly(x, y, \lambda), L\lambda(x, y, \lambda)])$

$$gradL := (x, y, \lambda) \rightarrow Vector([Lx(x, y, \lambda), Ly(x, y, \lambda), L\lambda(x, y, \lambda)]) \tag{2.5}$$

> $G := gradL(x, y, \lambda)$

$$G := \begin{bmatrix} 2\,x + \lambda\,(10\,x - 8\,y) \\ 2\,y + \lambda\,(-8\,x + 10\,y) \\ 5\,x^2 - 8\,x\,y + 5\,y^2 - 18 \end{bmatrix} \tag{2.6}$$

> **use** *RealDomain* **in** $LL := solve(\{G(1) = 0, G(2) = 0, G(3) = 0\})$ **end use**

$$LL := \left\{\lambda = -\frac{1}{9}, x = -1, y = 1\right\}, \left\{\lambda = -\frac{1}{9}, x = 1, y = -1\right\}, \{\lambda = -1, x = -3, y = -3\}, \{\lambda = -1, x = 3, y = 3\} \quad (2.7)$$

Die extremwertverdächtigen Stellen:

> **for** k **from** 1 **to** 4 **do** $P[k] := [rhs(LL[k][2]), rhs(LL[k][3])]; f(P[k][1], P[k][2])$ **end do**

$$P_1 := [-1, 1]$$
$$2$$
$$P_2 := [1, -1]$$
$$2$$
$$P_3 := [-3, -3]$$
$$18$$
$$P_4 := [3, 3]$$
$$18 \quad (2.8)$$

Aufgabe 3:

> $restart : with(plots) : with(DEtools) :$

a)

> $dgla := \mathrm{D}(y)(x) + y(x)\cdot\cos(x) + \sin(x) = 0$

$$dgla := \mathrm{D}(y)(x) + y(x)\cos(x) + \sin(x) = 0 \quad (3.1)$$

Selbst gewählte Anfangsbedingungen:

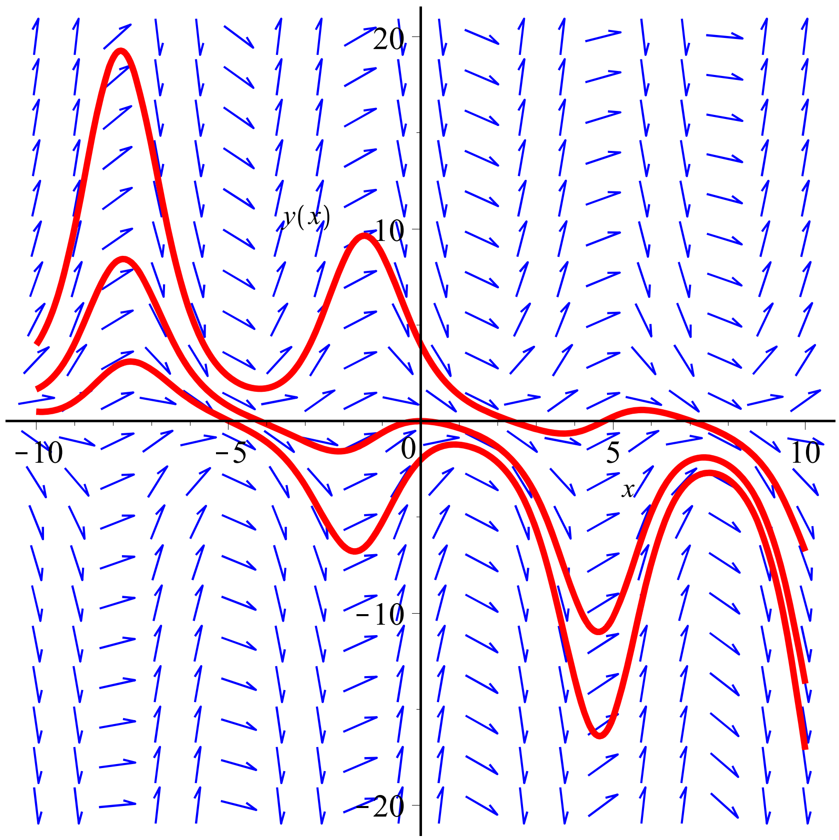
> $AnfBed := \{[y(0) = -2], [y(0) = 0], [y(0) = 4]\}$

$$AnfBed := \{[y(0) = -2], [y(0) = 0], [y(0) = 4]\} \quad (3.2)$$

> $Farbe := color = blue, linecolor = red$

$$Farbe := color = blue, linecolor = red \quad (3.3)$$

> $DEplot(dgla, \{y(x)\}, x = -10..10, y = -20..20, AnfBed, stepsize = 0.05, Farbe)$



> $dsolve(dgla, y(x))$

$$y(x) = e^{-\sin(x)} \left(\int \left(-\sin(x)\, e^{\sin(x)}\right) dx\right) + e^{-\sin(x)}\, \_C1 \quad \textbf{(3.4)}$$

b)

> $dglb := y(t) \cdot D(y)(t) = 1$

$$dglb := y(t)\ D(y)(t) = 1 \quad \textbf{(3.5)}$$

> $AnfBed := \{[y(0) = 1]\}$

$$AnfBed := \{[y(0) = 1]\} \quad \textbf{(3.6)}$$

> $DEplot(dglb, \{y(t)\}, t=-5..5, y=-4..4, AnfBed, stepsize=0.05, Farbe)$

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Warning, plot may be incomplete, the following errors(s) were
issued:
   cannot evaluate the solution further left of -.50000007,
probably a singularity
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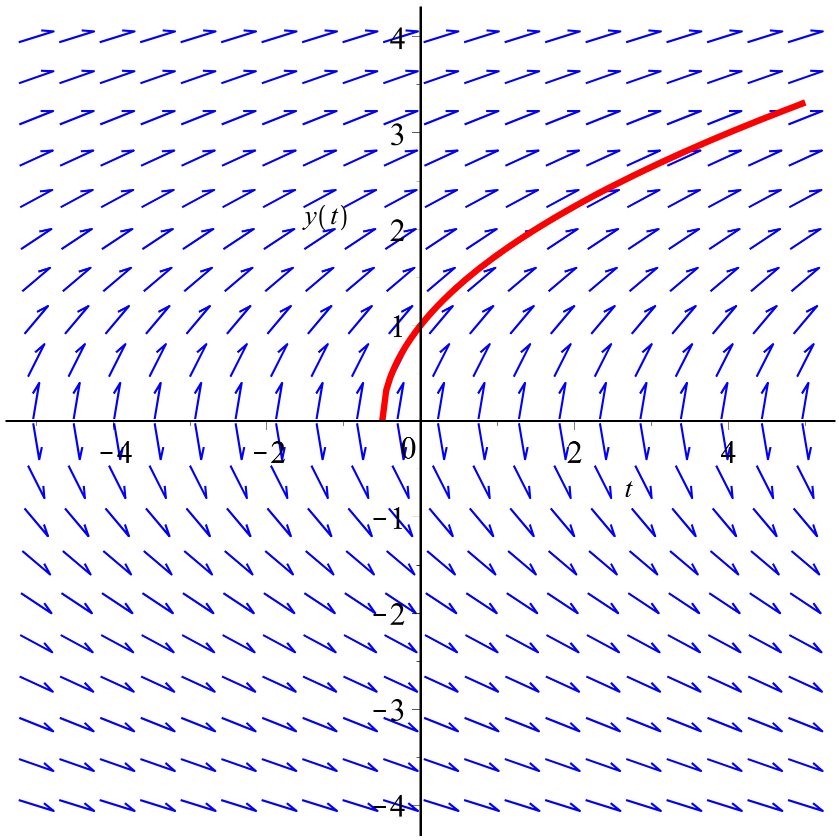


Wie man am Richtungsfeld bereit erkennt und die folgende Darstellung zeigt, so ist die mit dem DEplot-Befehl gefundene Lösungskurve für y(0)=1 unvollständig:

> $l := dsolve(dglb, y(t), implicit)$

$$l := y(t)^2 - \_C1 - 2\,t = 0 \tag{3.7}$$

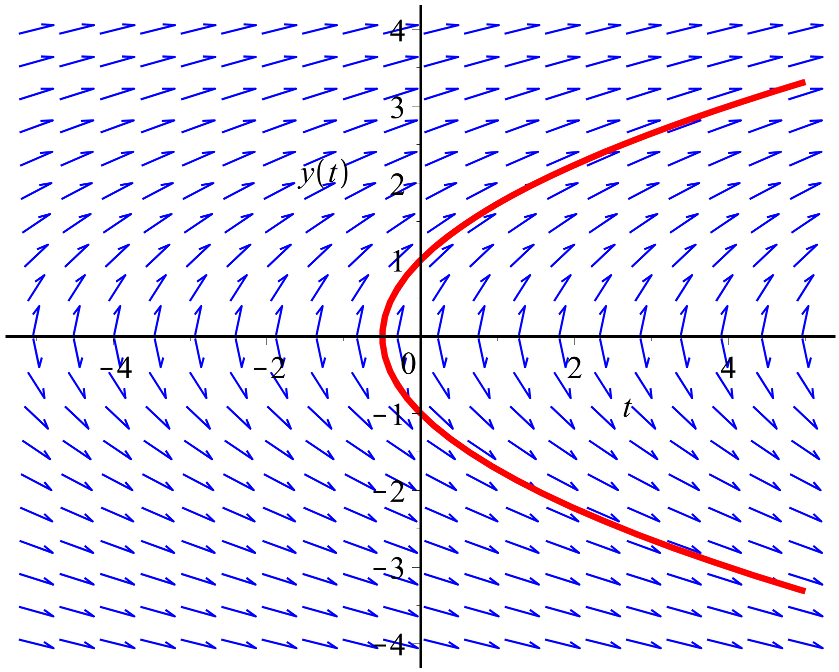
> $P1 := implicitplot\big(subs(\_C1 = 1, l\big), t = -5..5, y = -4..4, numpoints = 2000, scaling = CONSTRAINED, color = red, thickness = 3\big)$

$$P1 := PLOT(\ldots) \tag{3.8}$$

> $P2 := DEplot(dglb, \{y(t)\}, t = -5..5, y = -4..4, stepsize = 0.05, color = blue)$

$$P2 := PLOT(\ldots) \tag{3.9}$$

> $display(\{P1, P2\})$



Aufgabe 4:

> $restart : with(DEtools) :$

a)

> $dgla := t \cdot \mathrm{D}(y)(t) + y(t)^2 = 0$

$$dgla := t\,\mathrm{D}(y)(t) + y(t)^2 = 0 \quad \textbf{(4.1)}$$

> $dsolve(dgla, y(t))$

$$y(t) = \frac{1}{\ln(t) + \_C1} \quad \textbf{(4.2)}$$

b)

> $dglb := x \cdot \mathrm{D}(y)(x) = y(x) \cdot (\ln(y(x)) - \ln(x))$

$$dglb := x\,\mathrm{D}(y)(x) = y(x)\,(\ln(y(x)) - \ln(x)) \quad \textbf{(4.3)}$$

> $dsolve(dglb, y(x))$

$$y(x) = \frac{x\,\mathrm{e}^{\_C1\,x}}{\mathrm{e}^{-1}} \quad \textbf{(4.4)}$$

c)

> $dglc := (x^2+1)\cdot D(y)(x) + + x\cdot y(x) - x\cdot (x^2+1) = 0$

$$dglc := (x^2+1)\, D(y)(x) + x\, y(x) - x\,(x^2+1) = 0 \quad \textbf{(4.5)}$$

> $dsolve(dglc, y(x))$

$$y(x) = \frac{1}{3}\, x^2 + \frac{1}{3} + \frac{\_C1}{\sqrt{x^2+1}} \quad \textbf{(4.6)}$$

Aufgabe 5:

> $restart : with(DEtools) :$

a)

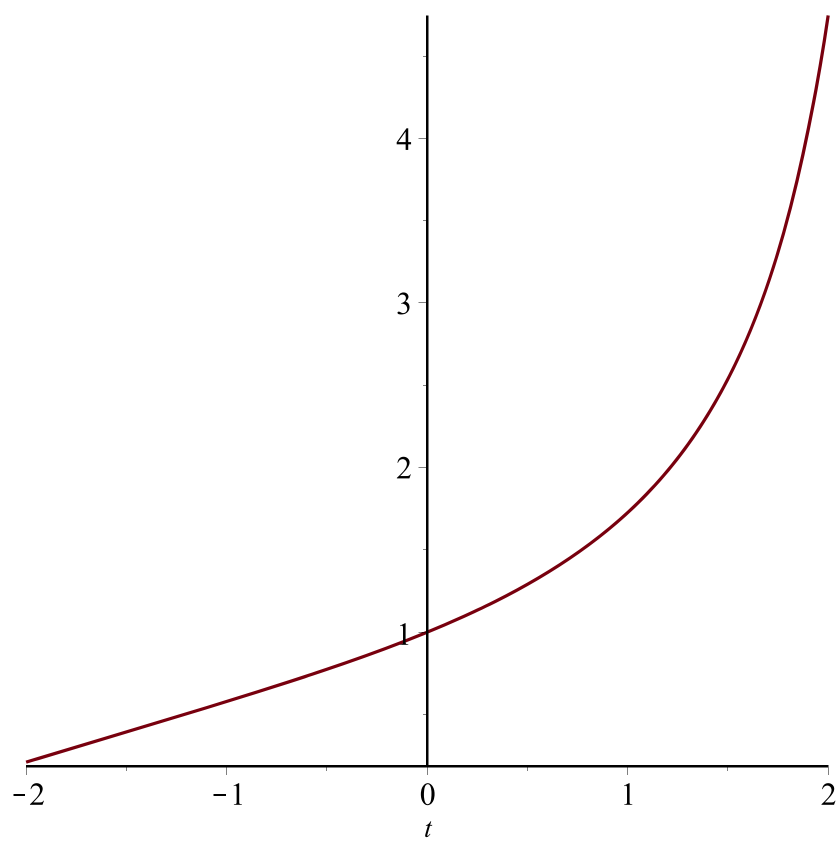
> $dgla := D(y)(t)\cdot \sin(t) = y(t)\cdot \ln(y(t))$

$$dgla := D(y)(t)\, \sin(t) = y(t)\, \ln(y(t)) \quad \textbf{(5.1)}$$

> $la := unapply\left(rhs\left(dsolve\left(\left\{dgla, y\left(\frac{Pi}{2}\right) = \exp(1)\right\}, y(t)\right)\right), t\right)$

$$la := t \rightarrow e^{-\frac{-1+\cos(t)}{\sin(t)}} \quad \textbf{(5.2)}$$

> $plot(la(t), t=-2..2)$



b)

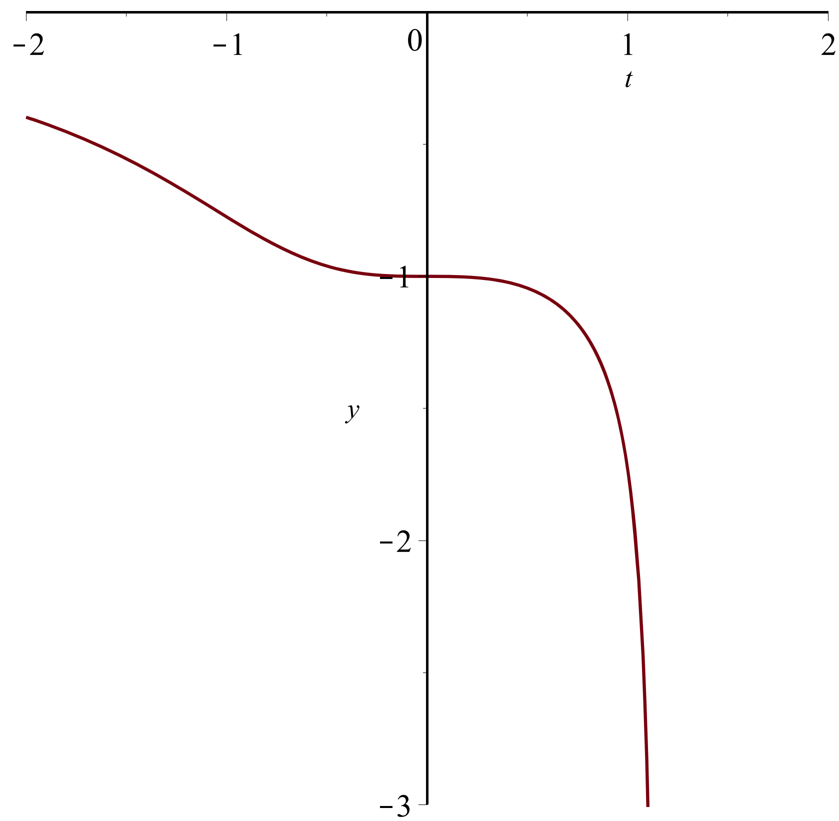
> $dglb := \mathrm{D}(y)(t) = t^2 \cdot y(t)^3$

$$dglb := \mathrm{D}(y)(t) = t^2\, y(t)^3 \tag{5.3}$$

> $lb := unapply(rhs(dsolve(\{dglb, y(0) = -1\}, y(t))), t)$

$$lb := t \rightarrow -\frac{3}{\sqrt{-6\,t^3 + 9}} \tag{5.4}$$

> $plot(lb(t), t = -2..2, y = -3..0)$



Aufgabe 6:

> $restart : with(DEtools) :$

> $dgl := b \rightarrow (\mathrm{D}@@4)(y)(x) - y(x) = b(x)$

$$dgl := b \rightarrow \mathrm{D}^{(4)}(y)(x) - y(x) = b(x) \tag{6.1}$$

> $constcoeffsols(dgl(0), y(x))$

$$[\mathrm{e}^{-x}, \mathrm{e}^{x}, \sin(x), \cos(x)] \tag{6.2}$$

> $b := x \rightarrow x^3 + 1 + 5 \cdot \cos(x)$

(6.3)

$$b := x \rightarrow x^3 + 1 + 5\cos(x) \quad \textbf{(6.3)}$$

> $dsolve(dgl(b), y(x))$

$$y(x) = -x^3 - \frac{5}{4}\, x \sin(x) - \frac{5}{2} \cos(x) - 1 + \_C1 \cos(x) + \_C2\, e^{x} + \_C3 \sin(x) + \_C4\, e^{-x} \quad \textbf{(6.4)}$$