Group Symmetric Covariance Estimation

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We consider the problem of Gaussian and robust covariance estimation under the group symmetry constraints. We assume the estimated matrix to be invariant under the conjugation action of a unitary matrix group, referred to as a group symmetry property. Examples of group symmetric structures include circulant, perHermitian, proper quaternion and other structures. We develop group symmetric versions of the sample covariance and Tyler's scatter M-estimators and investigate their performance gains. In particular, using the finite group representation theory, we derive the exact number of samples needed to guarantee the existence and uniqueness of these estimates and derive their high probability error bounds. The classical results claim that at least $n = p$ and $n = p+1$ sample points in general position are necessary to ensure the existence and uniqueness of the sample covariance and Tyler's estimator respectively, where p is the ambient dimension. We significantly weaken this requirement for both estimates in group symmetric scenario and show that in many cases even two samples are enough to guarantee existence and uniqueness regardless of the ambient dimension.