

The Erdős–Szekeres Theorem and Related Results

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The Erdős–Szekeres k -gon theorem (1935) says that for any integer $k \geq 3$ there is an integer $n(k)$ such that any set of $n(k)$ points in the plane, no three on a line, contains k points which are vertices of a convex k -gon. It is a classical result both in combinatorial geometry and in Ramsey theory.

We will discuss various results and open problems related to the Erdős–Szekeres theorem. For example, it is still widely open if the minimum possible value of $n(k)$ is equal to $2^{k-2} + 1$, as conjectured by Erdős and Szekeres more than fifty years ago. It might be particularly interesting for this audience that some recent results related to the Erdős–Szekeres theorem are purely combinatorial, dealing with colored (hyper)graphs on linearly ordered vertex sets.