

Elgersburg Summer School, March 2014
Mathematical Biology
Question prompts for matrix population projection models

Use a selection of the following questions, plus any of your own, to help direct your research into projection matrix modelling.

(1) **Overview:**

- (i) What is a projection matrix? What species are modelled by these objects? In what sense are they “projection” matrices? What can we use PPM models for?
- (ii) What is an age-based model? What is a stage-based model? What is special about an age-based PPM as compared to a stage-based PPM.
- (iii) What do the terms *fecundity*, *recruitment*, *stasis* and *growth* mean in the context of PPMs? How is a PPM parameterised by these terms?
- (iv) What structural properties do matrix PPMs have?
- (v) What other (physical) systems can be described by PPMs?

(2) **Positive matrices:**

- (i) Why are projection matrices positive? What is the associated directed graph with a nonnegative matrix?
- (ii) What is a life cycle and how is a life cycle represented by using a life cycle graph?
- (iii) What is the connection between a PPM and its life cycle graph?
- (iv) What do the terms *irreducible* and *primitive* mean? What is the *period* of a nonnegative matrix A ?
- (v) What is the Perron–Frobenius Theorem? When does it apply? Why is it useful?
- (vi) What are the *reproductive vector*, *stable-stage structure* and *asymptotic growth rate* associated with a PPM?

(3) **Dynamics:**

- (i) What are *asymptotic* dynamics? What are *transient* dynamics? How can these be described by matrix PPMs? What differences are introduced by matrix models as opposed to scalar population models?
- (ii) What do disturbances and perturbations mean in the context of populations modelled by PPMs? How can these be accommodated?
- (iii) What are the advantages and disadvantages of matrix PPMs? How can these disadvantages be overcome?
- (iv) What additional features could be added to PPMs? How else might the existing models be extended?

(4) **Perturbation analysis and sensitivities:**

- (i) What sources of uncertainty are present in PPMs and can these sources be modelled?
- (ii) What do the terms *sensitivity*, *elasticity* and *structured perturbation* refer to for PPMs? How are they calculated and what are they useful for? [Hint: Look at Exercise Sheet 2.]

The following references may (or may not) be useful. In chronological order:

References

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- [13] I. Stott, S. Townley, D. Carslake, and D. Hodgson, “On reducibility and ergodicity of population projection matrix models,” *Methods in Ecology and Evolution*, vol. 1, no. 3, pp. 242–252, 2010.
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