

# Mathematical Biology – Matrix Population Projection Models

Stuart Townley

University of Exeter, UK

March 12, 2014



European Regional  
Development Fund  
Investing in your Future



*convergence*  
for economic  
transformation

UNIVERSITY OF  
**EXETER**

**EPSRC**

Pioneering research  
and skills

- The second lecture focuses on matrix (P)opulation (P)rojection (M)odels–

- The second lecture focuses on matrix (P)opulation (P)rojection (M)odels–
- that is, linear, discrete–time, time–invariant, finite–dimensional linear models of the form

$$x(t + 1) = Ax(t), \quad x(0) = x^0, \quad t \in \mathbb{N}_0. \quad (1)$$

- The second lecture focuses on matrix (P)opulation (P)rojection (M)odels–
- that is, linear, discrete–time, time–invariant, finite–dimensional linear models of the form

$$x(t + 1) = Ax(t), \quad x(0) = x^0, \quad t \in \mathbb{N}_0. \quad (1)$$

- The object  $A$  in (1) denotes an  $n \times n$  (where  $n \in \mathbb{N}$ ), componentwise nonnegative matrix, the set of which we denote by  $\mathbb{R}_+^{n \times n}$ .

- The second lecture focuses on matrix (P)opulation (P)rojection (M)odels–
- that is, linear, discrete–time, time–invariant, finite–dimensional linear models of the form

$$x(t + 1) = Ax(t), \quad x(0) = x^0, \quad t \in \mathbb{N}_0. \quad (1)$$

- The object  $A$  in (1) denotes an  $n \times n$  (where  $n \in \mathbb{N}$ ), componentwise nonnegative matrix, the set of which we denote by  $\mathbb{R}_+^{n \times n}$ .
- The componentwise nonnegative vector  $x(t) \in \mathbb{R}_+^n$  in (1) denotes the population of a species at time step  $t \in \mathbb{N}_0$ .

$$x(t+1) = Ax(t), \quad x(0) = x^0, \quad t \in \mathbb{N}_0. \quad (1)$$

$$x(t+1) = Ax(t), \quad x(0) = x^0, \quad t \in \mathbb{N}_0. \quad (1)$$

- The model (1) is very simple mathematically, although there is some extra structure that follows from the componentwise nonnegativity of  $A$  and  $x$ .

$$x(t+1) = Ax(t), \quad x(0) = x^0, \quad t \in \mathbb{N}_0. \quad (1)$$

- The model (1) is very simple mathematically, although there is some extra structure that follows from the componentwise nonnegativity of  $A$  and  $x$ .
- Sykes (1969) writes (of modelling human populations) in his introduction “In demographic applications, the [matrix PPM] has been found to give predictions of future populations which might most charitably be described as poor.”



$$x(t+1) = Ax(t), \quad x(0) = x^0, \quad t \in \mathbb{N}_0. \quad (1)$$

- The model (1) is very simple mathematically, although there is some extra structure that follows from the componentwise nonnegativity of  $A$  and  $x$ .
- Sykes (1969) writes (of modelling human populations) in his introduction “In demographic applications, the [matrix PPM] has been found to give predictions of future populations which might most charitably be described as poor.”
- That said, the model (1) is used extensively in ecological modelling.



MAX-PLANCK-INSTITUT  
FÜR DEMOGRAFISCHE  
FORSCHUNG

MAX PLANCK INSTITUTE  
FOR DEMOGRAPHIC  
RESEARCH

[Institute](#)[Projects & Publications](#)[Laboratories](#)[Education & Career](#)[News & Press](#)

» Start » Laboratories » Evolutionary Biodemography » Projects » COMPADRE Plant Matrix Database & COMADRE Animal Matrix Database

MAX-PLANCK-GESELLSCHAFT

#### LABORATORIES

Demographic Data

**Evolutionary  
Biodemography**

Historical Demography

Statistical Demography

Survival and Longevity

Max Planck Research Group:

Lifecourse Dynamics and

Demographic Change

Max Planck Research Group:

#### LABORATORY

## Evolutionary Biodemography

[At a Glance](#)[Projects](#)[Publications](#)[Team](#)

[« Back to project list](#)

#### PROJECT

### COMPADRE Plant Matrix Database & COMADRE Animal Matrix Database

The COMPADRE Plant Matrix Database and COMADRE Animal Matrix Database contain demographic information in the form of projection matrices for **ca. 900 plants and over 1,200** animal species, derived from published and personally communicated research. The databases also contain taxonomic, phylogenetic, ecological, and biogeographic covariates.

[Detailed description](#)

- Therefore, the format of the second lecture is different to that of the rest.

- Therefore, the format of the second lecture is different to that of the rest.
- We would like you to put together material for a lecture on PPMs.

- Therefore, the format of the second lecture is different to that of the rest.
- We would like you to put together material for a lecture on PPMs.
- You have the following tasks:

- Therefore, the format of the second lecture is different to that of the rest.
- We would like you to put together material for a lecture on PPMs.
- You have the following tasks:
  - (i) Form five groups.

- Therefore, the format of the second lecture is different to that of the rest.
- We would like you to put together material for a lecture on PPMs.
- You have the following tasks:
  - (i) Form five groups.
  - (ii) Imagine that you have been asked to give a lecture on matrix projection modelling. Prompted by the list of questions, and others you think relevant, find and collate answers.

- Therefore, the format of the second lecture is different to that of the rest.
- We would like you to put together material for a lecture on PPMs.
- You have the following tasks:
  - (i) Form five groups.
  - (ii) Imagine that you have been asked to give a lecture on matrix projection modelling. Prompted by the list of questions, and others you think relevant, find and collate answers.
  - (iii) Start Exercise Sheet 2.



- Therefore, the format of the second lecture is different to that of the rest.
- We would like you to put together material for a lecture on PPMs.
- You have the following tasks:
  - (i) Form five groups.
  - (ii) Imagine that you have been asked to give a lecture on matrix projection modelling. Prompted by the list of questions, and others you think relevant, find and collate answers.
  - (iii) Start Exercise Sheet 2.
  - (iv) Start the Group Project. See the Group Project Sheet for more details. Each group shall have 20 minutes to present their solution to the rest of the group in the final MB problems class on Friday.

- Therefore, the format of the second lecture is different to that of the rest.
- We would like you to put together material for a lecture on PPMs.
- You have the following tasks:
  - (i) Form five groups.
  - (ii) Imagine that you have been asked to give a lecture on matrix projection modelling. Prompted by the list of questions, and others you think relevant, find and collate answers.
  - (iii) Start Exercise Sheet 2.
  - (iv) Start the Group Project. See the Group Project Sheet for more details. Each group shall have 20 minutes to present their solution to the rest of the group in the final MB problems class on Friday.
- Your solutions to parts (iii) and (iv) may help with part (ii).