Trajektorienregelung mechanischer Systeme mittels Servobindungen

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1 Introduction

Trajektorienregelung mechanischer Systeme mittels Servobindungen

1. Introduction to Light-Weight Robotics
2. Modeling using Flexible Multibody Systems
3. Inversion based Control using Servo-Constraints
4. Passive Joint Manipulator
5. Control of Flexible Multibody Systems
6. Conclusions

Why Light-Weight Robotics?

traditional robots and machine tools
+ stiff design
+ can achieve high accuracy
+ easy to control
but
- heavy
- high energy consumption

solution: light-weight robotics (mass reduction)
reduce mass of bodies (decreased mass = decreased stiffness)
motor 1
motor 2
flexible body
both types of systems possess very similar properties and problems (underactuated)

Why Light-Weight Robotics?

originally used in aerospace, now interest in broader applications such as medical robotics, service robotics, industrial robotics and machine tools
but not yet widely used due to many unsolved challenges in control

challenges:
- less control inputs than degrees of freedom (underactuated)
- difficult to control
- vibrations are easily excited

Why Light-Weight Robotics?

passive joint manipulator (Fukuoka University)
serial flexible manipulator (IST)

Why Light-Weight Robotics?

passive joint manipulators
flexible manipulators
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Floating Frame of Reference
Formulation
- basic idea: reference coordinates \( q_e \)
- describe the global nonlinear motion
- elastic coordinates \( q_k \) describe the elastic deformation
- global shape functions \( \Phi \) and \( \Psi \)
- equations of motion (minimal coordinates)
- \( \Phi(R,R') \) is key issue in the modeling process (use e.g. eigenmodes from FE analysis)

Modeling of Flexible Multibody Systems
- Trajectory control of a serial elastic two-arm manipulator
- control goal: tracking of an end-effector point trajectory
- \( \mathbf{r}^{\text{eff}}(\mathbf{q}) - \mathbf{r}^{*}(t) \)

Underactuated multibody systems have less control inputs \( u \in \mathbb{R}^m \) than generalized coordinates \( q \in \mathbb{R}^n \)

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Floating Frame of Reference

Modeling of Flexible Multibody Systems

Rigid Multibody System

Modeling of Flexible Multibody Systems

Elastic Body

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Floating Frame of Reference

Modeling of Flexible Multibody Systems

Rigid Multibody System

Modeling of Flexible Multibody Systems

Elastic Body
Differentially Flat System: the states and inputs can be computed algebraically using the desired outputs and their time derivatives up to a certain order

\[ x = f(x,u), \quad y = h(x) \]

\[ x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \]

• the states and inputs are completely specified by the output,
• the inverse model does not contain any unspecified/internal dynamics

Non-Flat System: non-flat systems contain an internal dynamics

\[ x = g(q,y,...,y^{(m)}), \quad u = g(q,y,...,y^{(m)}) \]

• the states of the internal dynamics, and thus \( x, u \) are not completely specified by the output,
• the inverse model itself is a dynamic system

These are system properties, independent of the used states \( x \) (coordinates) but dependent on the choice of system output \( y \) and system dynamics \( f \)

MBS with servo-constraints

End-Effector Trajectory Tracking Control

Feed-Forward Control by Model Inversion

Ideal realization

Orthogonal realization

Passive constrained-manifold

Tangential subspace

Orthogonal subspace

Skew-orthogonal realization

MBS with passive constraints (joints)

Exact Model Inversion Using Servo-Constraints

MBS with servo-constraints

Analysis of Servo-Constraints
...
Simple changes can alter system properties

Input force $F$ and output mass position $x$.

**Constrained manifold**
- Orthogonal realization (index 3, non-flat)
- Constraint manifold
- Tangent realization without damping: index 5 (differential flat)

**Invertible model**
- Invertible model does not contain any dynamics

Schematic representation of inverted model:
- **Tangent realization**
- **Constraint manifold**
- **Orthogonal realization** (index 3, non-flat)

## Mathematical Formulations

1. $0 = \frac{k_x}{m_x} - \dot{y}_d$ \quad independent of $F$; algebraic equation for $s$
2. $m_k \ddot{x} - m_x \ddot{y}_d - F = m_y \dddot{y}_d - m_x \dddot{s}$ \quad unknown two derivatives of (1): $k_s = m_y \dddot{y}_d = 0$

**Pure algebraic equation for $F$**

3. $F = (m_y + m_x) \dddot{y}_d - \left( \frac{m_k}{k_s} \dddot{y}_d \right)$

**Relation independent of $F$**

4. $\dddot{s} = \dddot{y}_d$

## Input-Output Representation

**Non-flat system**

- Input $u$
- Output $y$

**Flat system**

- Input $u$
- Output $y$

**Zero Dynamics**

- Input $u^*$
- Output $y^*$ (e.g., $y^* = 0$)

**Stability of Linearized Zero Dynamics**

- Real ($\alpha$) vs. Imaginary ($\beta$)
- Stability analysis

### Summary
- Changes in the system can alter its properties.
- Algebraic equations for $s$ and $F$.
- Non-flat and flat systems.
- Zero dynamics crucial for control design.

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**Table of Maneuvers**

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 45^\circ$</td>
<td>$t = 0.000800$ s</td>
<td>$t = 9.000000$ s</td>
</tr>
<tr>
<td>$\alpha = 0^\circ$</td>
<td>$t = 0.000800$ s</td>
<td>$t = 9.000000$ s</td>
</tr>
</tbody>
</table>

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**Schematic Diagrams**

- Constraints and manifolds.
- Algebraic and differential equations.
- Input-output relationships.

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**Notes**

- Invertible model does not contain dynamics.
- Zero dynamics: remaining dynamics when output is kept constant by control input $u^*$.
(Non) Minimum phase

unstable output \( y \)
non-minimum phase system
stable output \( y \)
minimum phase system

stability of zero dynamics depends on system dynamics (geometry, mass distribution and choice of output)

for non-minimum phase systems it yields un-bounded inputs \( w_0 \), thus un-bounded inputs \( w_0 \)
can only be used for minimum phase systems (e.g.: MBS with collocated output)

Feed-Forward Control by Model Inversion

stable model inversion (Devasia, Chen, Paden 1996) yields bounded but non-causal solution

- output trajectory \( y_0 \) starts and ends in stationary points
- the corresponding equilibrium points of the internal dynamics are hyperbolic
- solution formulated as two-sided boundary value problem (BVP)
- boundary condition: unstable manifold \( W^- \) at starting point \( q_{0\text{f}} \)
- stable manifold \( W^+ \) at final point \( q_{0\text{f}} \)

• numerical solution, e.g. Matlab BVP4c/BVP5c
• alternative solution using nonlinear programming (Bastos/Seifried/Brüls)
• multiple shooting (Brida/Bastos/Seifried)

Multibody system with servo-constraints (inverse model as DAE)
\[
M(q)q + B(q, q)u + g(q) = 0
\]

1.) extract explicitly internal dynamics as ODE (see cart problem)
\( \rightarrow \) corresponds to nonlinear control approach (diffeomorphic coordinate transformation to Byrnes/Isidori input/output normal form)
Problem: only possible for small systems (2-3 degrees of freedom) or special choice of system output (e.g. collocated output, linear in generalized coordinates)

2.) solve inverse model as DAE
+ simple formulation, also for large systems
+ forward dynamic model might include additional constraints (loops, contact)
\( \rightarrow \) index 3 or higher occurs, numerically difficult to solve

use projected equation (index reduced)
\[
0 = HM^r Bu + HM^r (g - \tilde{k}) + \Pi - y_d
\]
\[
0 = \tilde{M}q - \tilde{D}(g - \tilde{k}) + \tilde{B}u
\]

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Passive-Joint Manipulator

- passive joint
- active joint

- numerical solution, e.g. Matlab BVP4c/BVP5c
- alternative solution using nonlinear programming (Bastos/Seifried/Brüls)
- multiple shooting (Brida/Bastos/Seifried)
In this example the internal dynamics is described by the unactuated coordinate $\gamma$. Non-minimum phase systems require solving the boundary value problem (BVP) to obtain the solution of internal dynamics. The adaptation of mass distribution improves the system's stability and performance. The experimental results of the inverse kinematics for a rigid system show good agreement with the desired trajectory, while the non-minimum phase design introduces deviations from the simulation results due to limitations of the experimental setup (motors operate in velocity mode, i.e., control inputs are motor angular velocities). The control equation for the passive joint manipulator is given by $u_{vel} = \alpha_g + K_p(\alpha_g - \alpha) + K_d(\dot{\alpha} - \dot{\alpha})$. The control of flexible multibody systems is an active area of research with applications in lightweight robotics and robotics systems. Trajectory control of mechanical systems using servobindungen (servo constraints) is a key component in achieving precise and efficient movement.
Serial Flexible Manipulator

- Floating frame of reference approach with tangent system
- Using 2-10 eigenmodes as shape function per arm
- Equation of motion derived with Neweul-M

Rigid body inverse dynamics

Stable inversion

<50 mm

<0.1 mm

\[
q_d(t) - \chi(t) = \chi(t) - \chi(t)
\]

Feed-forward with joint control

Nominal system

Increase of end-effector mass by 20 %

Simple joint control not enough

Point Mass Model

Active Vibration Damping by Curvature Feedback

Direct use of strain-measurement (curvature) in feedback control

No observer necessary

Main idea:

Connection between strain \( \varepsilon(x,t) \) and curvature \( \kappa(x,t) \)

Assumptions:
- Linear elastic material: \( \sigma(x,t) = E \varepsilon(x,t) \)
- Euler-Bernoulli-Beam theory and pure bending:
  - Stress due to bending: \( \sigma(x,t) = \frac{M(x,t)}{I} \)
  - Differential equation of bending deflection:
    \( w''(x,t) = \frac{M(x,t)}{EI} \)
  - Curvature of neutral axis:
    \( \kappa(x,t) = \frac{w''(x,t)}{l} = \kappa_0(x,t) \)
  - Strain due to curvature:
    \( \varepsilon(x,t) = -\kappa_0(x,t) \kappa_0(x,t) \)

Computation of desired curvature \( \kappa_d(x,t) \)

Curvature Feedback

- Elastic coordinates \( \chi \) obtained as solution of inverse model
- Second spatial derivative of shape function \( \phi'' \)

Feed-forward with PID-joint control and curvature feedback

With \( \kappa = \left[ \kappa_1(x,t), \kappa_2(x,t), \ldots, \kappa_k(x,t) \right] \)}
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robustness against parameter uncertainty
- increase of end-effector mass by 20 %

Simulation Results

route control + PID-joint control

feed-forward
- PID-joint control
- PID-curvature feedback

Simulation Results

trajectory control of a parallel elastic manipulator for verification of control strategies

Special ITM Test Bed

• 3 flexible arms (up to 96 elastic shape functions)
• 1 kinematics loop
• 2 linear direct drives with force (current) as control input

Parallel Kinematics (Non-Minimum Phase)

free end-effector:

\[
M \ddot{q} + k \dot{q} + g + C \ddot{q} + B u + w^T F
\]

\( c_1(q) = 0 \) loop closing constraint
\( c_2(q) = h(q) - y_2 = 0 \) servo constraint
\( c_3(q) = 0 \) permanent contact
\( F = F_2 \) desired contact force

Parallel Kinematics (Non-Minimum Phase)

end-effector in contact:

\[
M \ddot{q} + k \dot{q} + g + C \ddot{q} + B u + w^T F
\]

\( c_1(q) = 0 \) loop closing constraint
\( c_2(q) = h(q) - y_2 = 0 \) servo constraint
\( c_3(q) = 0 \) permanent contact
\( F = F_2 \) desired contact force

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Conclusions

• structural flexibilities in light-weight designs might cause large tracking errors
• accurate feedforward control (inverse model) necessary
• exact model inversion using servo-constraints is a straight forward approach, allows the inclusion of different types of constraints
• combination of accurate models and efficient solvers necessary
• active vibration damping by curvature feedback is a simple and efficient approach