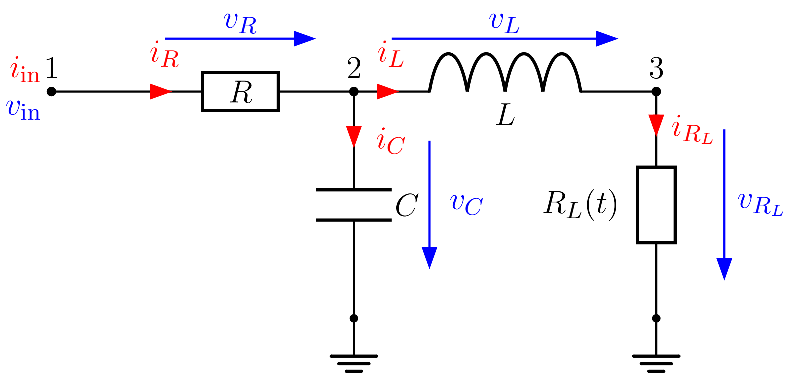
DAEs - Control and Numerics

Exercise Sheet 3 - Input-Output Systems

Exercise 9 (Re-interpretation of variables)
Consider the following circuit



where the load resistor

$$R_L(t) = R_0 e^{\alpha t}, \quad R_0 > 0,\ \alpha > 0,$$

models the exponentially growing power consumption, e.g. of a city, and the rest of the circuit models the transmission line. This transmission line is connected at node 1 to the grid which provides the current i_{in} and voltage v_{in}. Additionally assume that the voltage at the consumption is fixed by the following value

$$v_{R_L}(t) = V_0 \sin(\omega t), \quad V_0 > 0 \text{ (e.g. 220V)},\ \omega > 0 \text{ (e.g. 50Hz)}.$$

Find the (time-varying) matrices $E(t), A(t) \in \mathbb{R}^{10\times 8}$, $B(t) \in \mathbb{R}^{10\times 2}$ such that this circuit is modeled as a DAE $E\dot{x} = Ax + Bu + f$ where $x = (i_R, v_R, i_C, v_C, i_L, v_L, i_{R_L}, v_{R_L})^\top$ and $u = (i_{\text{in}}, v_{\text{in}})$. Note that you obtain four equations for each element (resistors, inductor, capacitors), three current-equations for each node, two voltage-equations for each loop and one equation for the condition on v_{R_L}.
Obtain the condensed form for this DAE and re-interprate the states and the inputs such that the resulting system is strangeness-free.

Exercise 10 (Consistency and regularity of control problem)
Consider the control problem

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x.$$

Check consistency and regularity for this system. Does there exist a proportional output feedback that makes the closed loop system regular and of index at most one?

Exercise 11 (Stabilizability and detectability)
Consider the DAE from Exercise 10. Check for finite dynamics stabilizability, impulse controllability, finite dynamics detectability and impulse observability.