DAEs - Control and Numerics

Exercise Sheet 4 - Stability and optimal control

Exercise 12 (Re-interpretation of variables - continued)
Finish Exercise 9.

Exercise 13 (Stability of DAEs and higher index)
Consider the DAE

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u.$$

What are the solutions of the homogeneous DAE (i.e. $u = 0$). What is the solution for the DAE with $u = \sin(t^2)$. Note that u is bounded. Is this DAE stable?

Exercise 14 (Optimal control)
Consider for $\varepsilon > 0$ the ODE

$$\begin{bmatrix} -\varepsilon & 1 \\ 0 & -\varepsilon \end{bmatrix} \dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad x(0) = x_0 := \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$
$$y = [0\ 1]x,$$

on the interval $[0, 1]$ with the cost function

$$J(x, u) = \int_0^1 (y + u)^2.$$

Formulate and solve the necessary condition $L(x, \mu, u) = 0$ with $x(0) = x_0$ and $\mu(1) = 0$.
Do the same for the system with $\varepsilon = 0$.

Exercise 15 (Lyapunov regularity)
Show that the underlying ODE of the DAE

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & -e^{-t+t\sin(t)} \end{bmatrix} x$$

is Lyapunov regular but the DAE itself is not.
Obtain its adjoint system and show that it is Lyapunov regular.