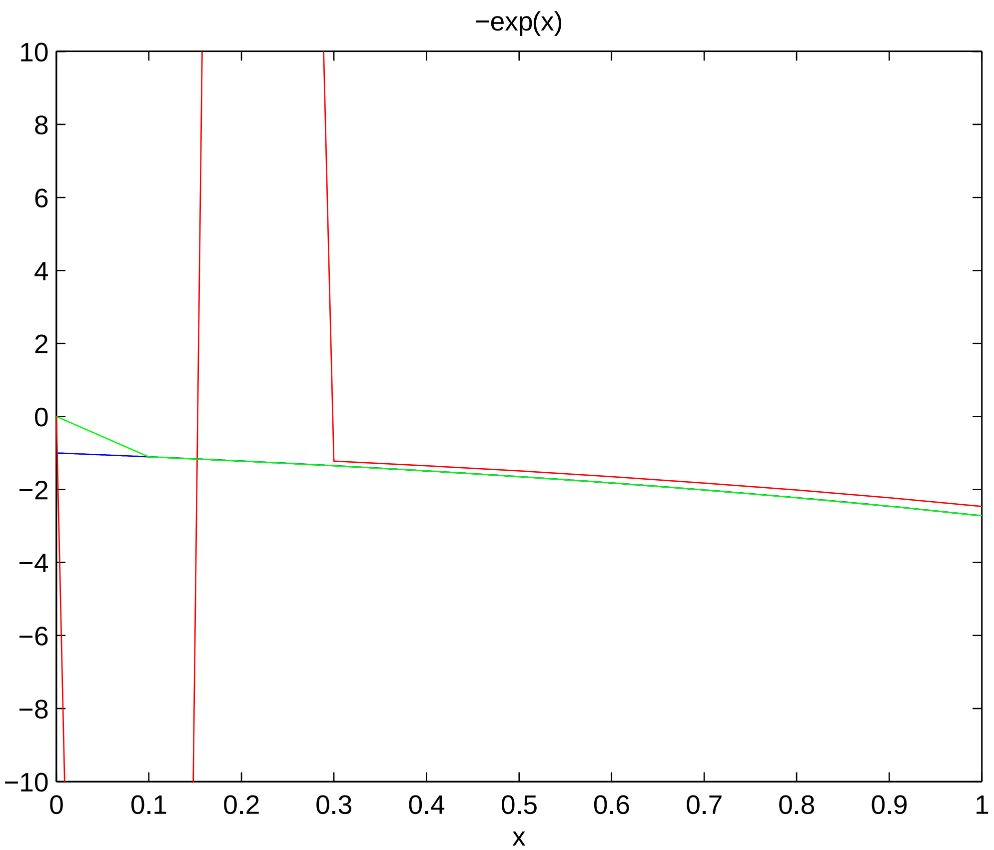
DAEs - Control and Numerics
Exercise Sheet 2 - Numerical solutions
Solutions

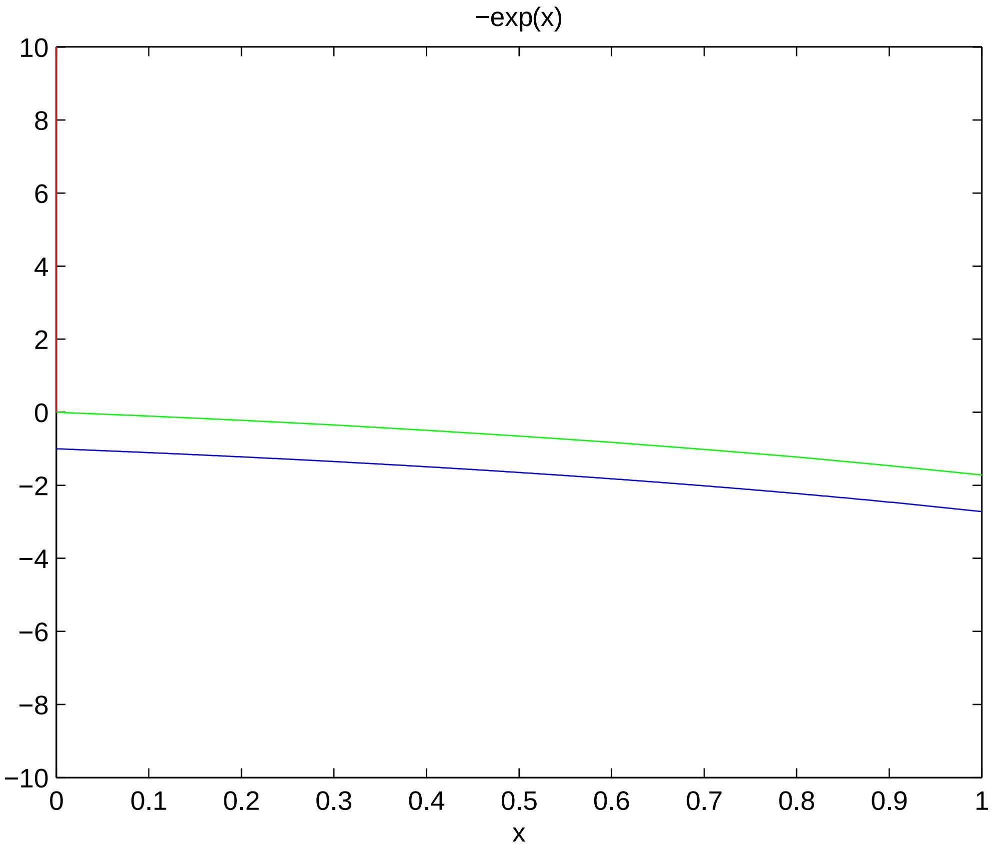
Exercise 5 (Classical solvers and higher index - Matlab experiment)
First observe that the exact solution is given by

$$x(t) = \begin{pmatrix} -e^t \\ -e^t \\ -e^t \end{pmatrix}.$$

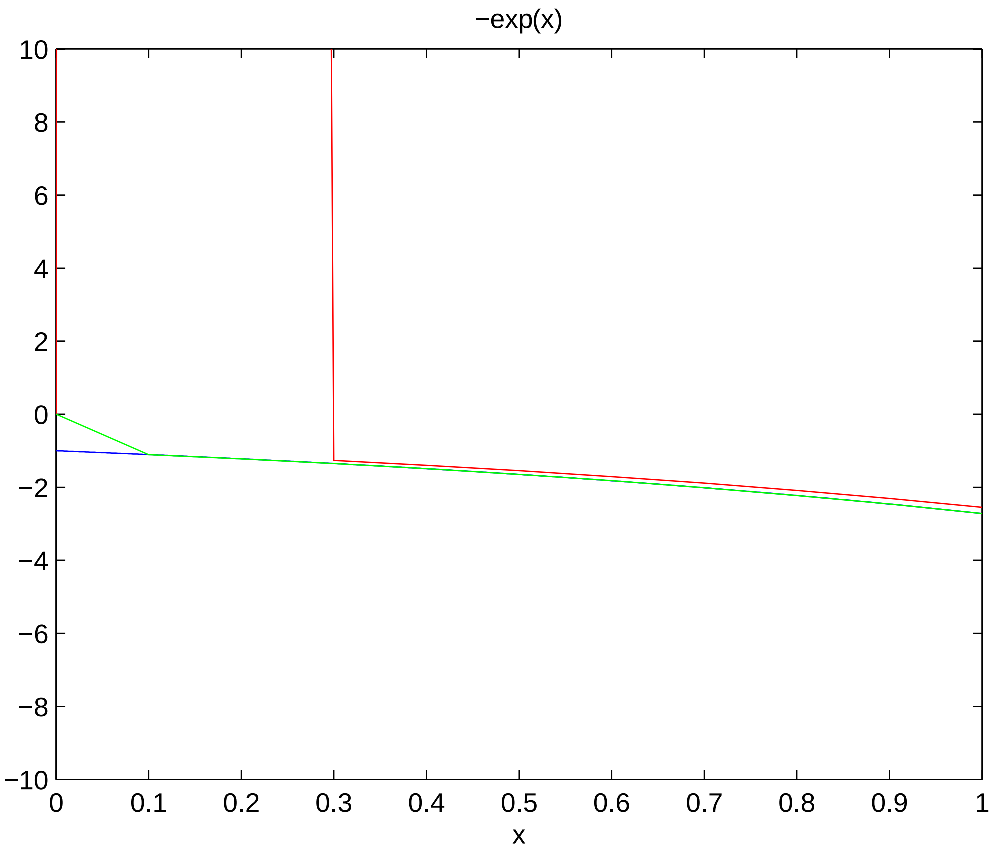
Implicit Euler (with initial value $x = (0,0,0)^\top$, h=0.1), blue is exact solution, green is numerical solution for $x_1(t)$ and red is numerical solution for $x_2(t)$:



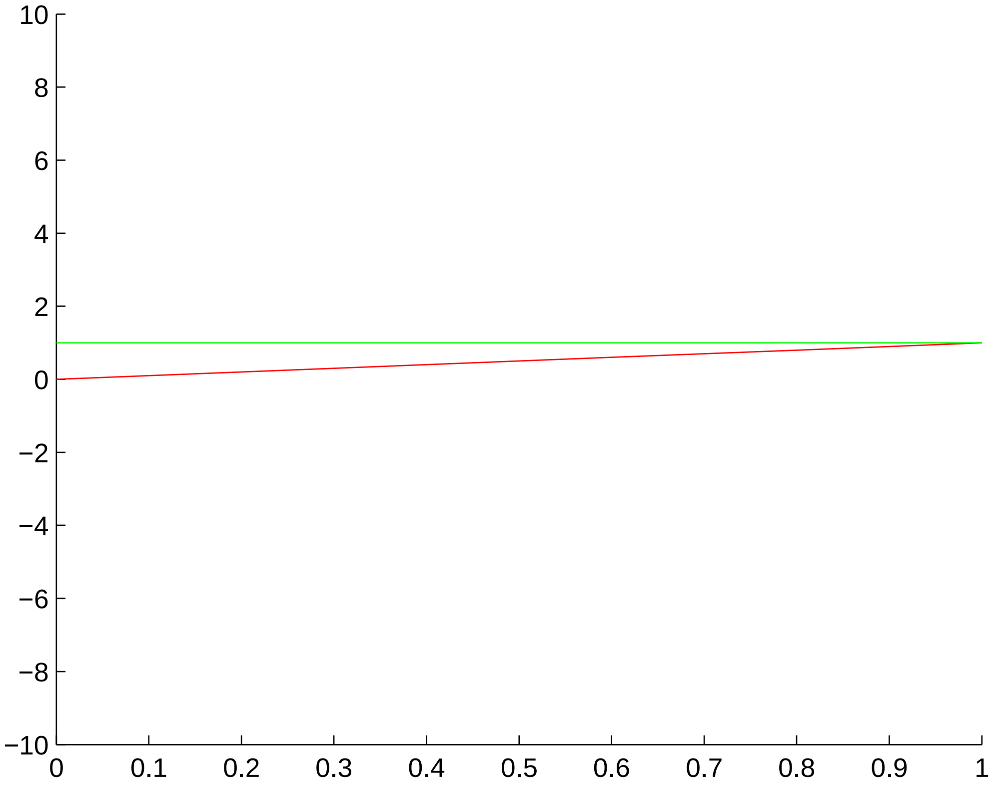
Gauss with $s = 2$ (with initial value $x = (0,0,0)^\top$):



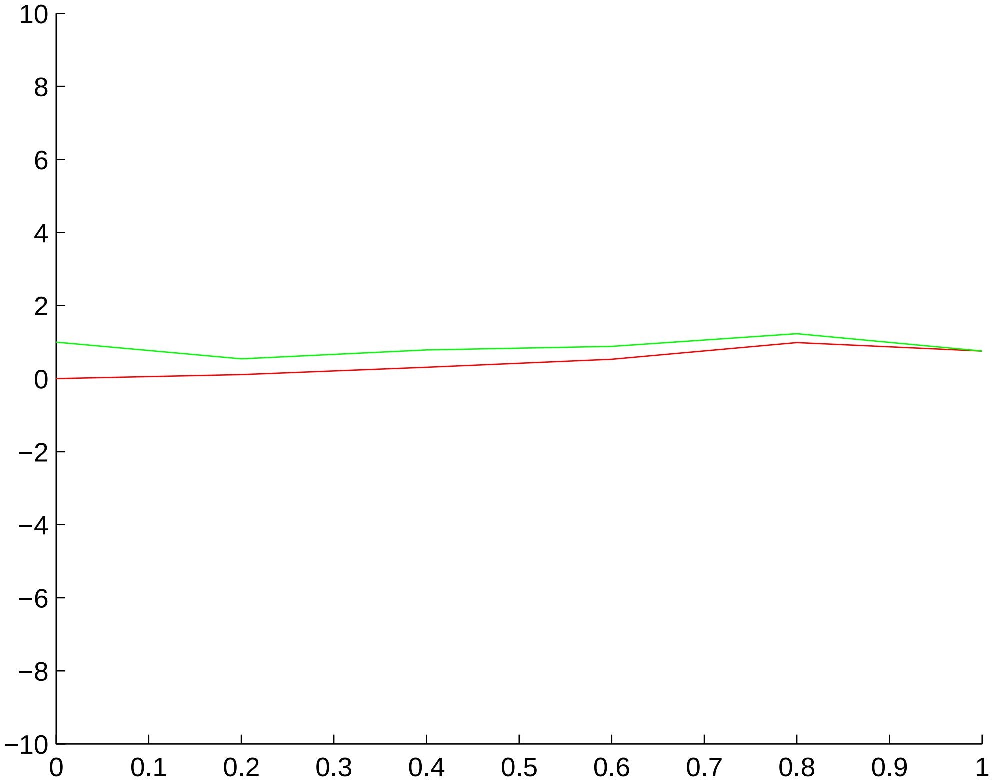
Radau with $s = 2$ (with initial value $x = (0, 0, 0)^\top$):



Exercise 6 (Time varying DAEs with classical solvers - Matlab experiment)
Implicit Euler:



Gauss with $s = 2$:



Radau with $s = 2$: no solution.

Exercise 7 (Parameter depended solutions)
Example 5.15 in [1].

Exercise 8 (Normal form and derivative array)
Example 3.54 in [1].

$$M_2(t) = \begin{bmatrix} 0 & t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & t & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & t \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad N_2(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

hence $\operatorname{corank} M_2(t) = 2 = \operatorname{corank} M_1(t)$, i.e. $\mu = 1$ and $v = 0$. Since $\operatorname{rank} M_1(t) = 2$ it follows that $a = 2$ and

$$Z_{2,3}^\top = \begin{bmatrix} 1 & 0 & 0 & t \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Therefore

$$\hat{A}_2 = Z_{2,3} N_1(t) \begin{bmatrix} I_2 \\ 0_{2\times 2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Since $\hat{A}_2$ also has full column rank the matrix T_2 is an empty 2×0 matrix, hence $\hat{E}_1$ does not exists, i.e. $d = 0$. The condensed form is then given by

$$0 = x(t) + \hat{f}_2(t),$$

where $\hat{f}_2(t) = \begin{bmatrix} f_1(t) + t\dot{f}_2(t) \\ f_2(t) \end{bmatrix} = Z_{2,3}^\top (f_1, f_2, \dot{f}_1, \dot{f}_2)^\top$.

References

[1] Peter Kunkel and Volker Mehrmann. *Differential-Algebraic Equations. Analysis and Numerical Solution.* EMS Publishing House, Zürich, Switzerland, 2006.