DAEs - Control and Numerics

Exercise Sheet 3 - Input-Output systems

Solutions

Exercise 9 (Re-interpretation of variables)
Writing down the equations yields

$$E(t) = \begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&C&0&0&0&0\\ 0&0&0&0&L&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix}, \quad A(t) = \begin{bmatrix} R&-1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&R_L(t)&-1\\ 1&0&0&0&0&0&0&0\\ 1&0&-1&0&-1&0&0&0\\ 0&0&0&0&1&0&-1&0\\ 0&1&0&1&0&0&0&0\\ 0&0&0&-1&0&1&0&1\\ 0&0&0&0&0&0&0&1 \end{bmatrix}$$

and

$$B(t) = \begin{bmatrix} 0&0\\ 0&0\\ 0&0\\ 0&0\\ -1&0\\ 0&0\\ 0&0\\ 0&1\\ 0&0\\ 0&0 \end{bmatrix}, \quad f(t) = \begin{pmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\ -R_0 e^{\alpha t} \end{pmatrix}$$

Let $\mathcal{E}(t) := [E(t)\ 0]$, $\mathcal{A}(t) := [A(t)\ B(t)]$ and $w = (x^\top, u^\top)^\top$. Assume $\mu = 0$. Then the matrices to construct the condensed form for $\mathcal{E}(t)\dot{w} = \mathcal{A}(t)w + f$ are as follows

$$Z_{2,3} = Z_2 = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}, \quad Z_3 = [\,],$$

and

$$T_2 = \ker Z_2^\top \mathcal{A} = \begin{bmatrix} 1 & 0 \\ R & 0 \\ 1 & 0 \\ \text{-}R & \text{-}1 \\ 0 & 0 \\ \text{-}R & \text{-}1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{with } \operatorname{rank} \mathcal{E} T_2 = 1.$$

This contradicts the assumptions for the condensed form because $\mathcal{E}T_2$ must have full rank. Therefore we have to repeat the procedure with $\mu = 1$ or even higher. This is carried out on the next exercise sheet.

Exercise 10 (Consistency and regularity of control problems)
The matrix pair (E, A) is regular, hence the control system is regular and consistent. Therefore the trivial zero-feedback makes the system regular and since the system is already of index one the zero feedback fulfills this, too.

Exercise 11 (Stabilizability and detectability)
Since the rank of $[\lambda E - A, B]$ does not depend on λ, the system is finite dynamics stabilizable. It is also impulse controllable because $\operatorname{rank}[E, AS_\infty, B] = 4 = n$, where

$$S_\infty = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The matrix $[\lambda E^\top - A^\top, C^\top]$ has rank four independent of λ, hence the system is finite dynamics detectable. Finally, $\operatorname{rank}[E^\top, A^\top T_\infty, C^\top] = 4 = n$, where

$$S_\infty = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

hence the system is impulse observable.