Hamiltonian Control Systems

Add Lecture 1

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Port-Based Modeling of Physical Systems

Each engineering domain consists of two sub-domains:

- Electrical $=$ electric $+$ magnetic
- Mechanical $=$ kinetic $+$ potential
- Hydraulic $=$ hydraulic kinetic $+$ hydraulic potential

Only chemical and thermal domain have no sub-domains.*

*Strongly related with irreversible transformation of energy).

Port-Based Modeling of Physical Systems

How to treat all domains on equal footing? $\Rightarrow$ Generalized Bond Graph (GBG) formalism [Breedveld 1982]. Main ideas:

- Decompose 'conventional' engineering domains, i.e., electrical, mechanical, hydraulical, into new domains.
- For each new domain introduce two variables, called **power conjugate variables**, whose product equals power (e.g., current $\times$ voltage, velocity $\times$ force, temperature $\times$ entropy flow).
- Label these variables as **efforts** $e \in \mathscr{E}$ and **flows** $f \in \mathscr{F}$.
- Each element defines a power port, with

$$P = ef.$$

Port-Based Modeling of Physical Systems

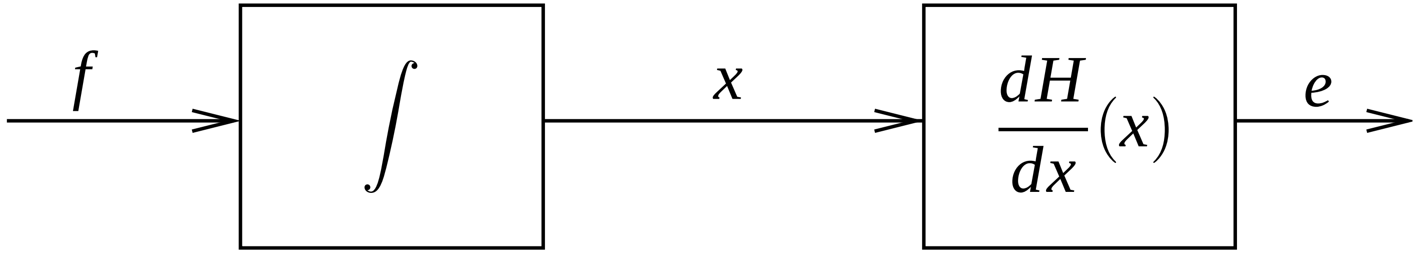
physical domain	flow $f \in \mathcal{F}$	effort $e \in \mathcal{E}$
electric	current	voltage
magnetic	voltage	current
potential translation	velocity	force
kinetic translation	force	velocity
potential rotation	angular velocity	torque
kinetic rotation	torque	angular velocity
potential hydraulic	volume flow	pressure
kinetic hydraulic	pressure	volume flow
chemical	molar flow	chemical potential
thermal	entropy flow	temperature

Energy Storage

The structure of any **storage element** is the following:

$$\begin{aligned} \dot{x}(t) &= f(t) \\ x(t) &= x(0) + \int_0^t f(\tau)d\tau \\ e(t) &= \frac{dH}{dx}\big(x(t)\big), \end{aligned}$$

where $H(x)$ is the stored energy.



Energy Storage

This completes the GBG table:

physical domain	flow $f \in \mathscr{F}$	effort $e \in \mathscr{E}$	state variable $x = \int f \mathrm{d}t$
electric	current	voltage	charge
magnetic	voltage	current	flux linkage
potential translation	velocity	force	displacement
kinetic translation	force	velocity	momentum
potential rotation	angular velocity	torque	angular displacement
kinetic rotation	torque	angular velocity	angular momentum
potential hydraulic	volume flow	pressure	volume
kinetic hydraulic	pressure	volume flow	flow tube momentum
chemical	molar flow	chemical potential	number of moles
thermal	entropy flow	temperature	entropy

Energy Storage

In this setting, we have that the

- constitutive relationships are of the form $e = \hat{e}(x)$, and
- dynamical relationships are $\dot{x} = f$.

Furthermore, observe that the change in energy given by $\dot{H}(x) = \frac{d}{dt}H(x(t))$ is now **always** the external power flow of the storage element, i.e.,

$$\dot{H}(x) = \frac{dH}{dx}(x)\dot{x} = ef.$$

Thus, by construction: **product of effort and flow**!

Energy Storage

How do we obtain $H(x)$?

- Integral of power $P = ef$ w.r.t. time yields energy

$$H(t) = \int e(t) f(t) dt. \qquad (\star)$$

- Recall dynamic relationship: $\dot{x} = f$, or equivalently,

$$dx = f dt.$$

- Substitution of the latter into ($\star$), and using $e = \hat{e}(x)$, yields:

$$H(x) = \int \hat{e}(x) dx.$$