Ökonomische prädiktive Regelung

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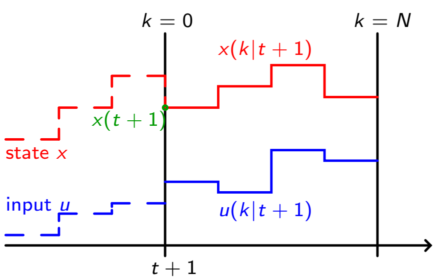
Outline

- Model predictive control (MPC) and economic MPC
- The role of dissipativity
  - Classifying optimal operating behaviors
  - Closed-loop convergence
- Average constraints
- Applications
  - Economic dispatch in power systems
  - Cooperative control of self-interested agents
- Conclusions

Introduction - Model predictive control

Model predictive control (MPC)

- Modern, optimization-based control technique
- Successful applications in many industrial fields



Basic MPC scheme

At each time t,

- solve finite horizon optimal control problem
- apply first part of optimal solution

Main advantages of MPC

- Can handle hard **constraints** on states and inputs
- Optimization of some **performance criterion**
- Applicable to **nonlinear, MIMO** systems

Model predictive control

- Nonlinear discrete time system $x(t+1) = f(x(t), u(t))$
- State and input constraints $x(t) \in \mathbb{X}$, $u(t) \in \mathbb{U}$

Standard MPC problem formulation

$$J_N^*(x(t)) = \min_{u(\cdot|t)} \sum_{k=0}^{N-1} \ell(x(k|t), u(k|t)) + V^f(x(N|t))$$

$$\text{s.t.} \quad x(0|t) = x(t), \quad x(k+1|t) = f(x(k|t), u(k|t)), \quad k = 0, \dots, N-1$$

$$x(k|t) \in \mathbb{X}, \; u(k|t) \in \mathbb{U}, \quad k = 0, \dots, N-1$$

$$x(N|t) \in \mathbb{X}^f$$

- Most results in MPC literature: classical control objective of **setpoint stabilization** is considered
- MPC controller design: determine V^f and $\mathbb{X}^f$ s.t. closed loop is stable [Chen & Allgöwer '98, Mayne et al. '00, Grüne '09, ...]
- Basic assumption: stage cost ℓ is **positive definite** w.r.t. setpoint to be stabilized
- However: **different** control objective is of interest in many applications

Economic model predictive control

- Maximization of product in process industry
- Minimization of energy consumption in building climate control
- Manufacturing industry: cost efficient scheduling of production process

$\Rightarrow$ Setpoint stabilization is **not** primary control objective

Economic MPC

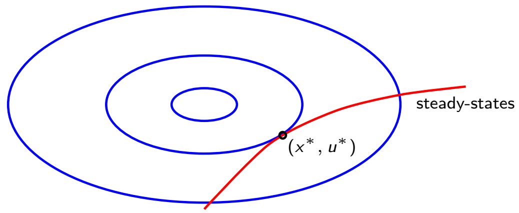
- Stage cost ℓ can be **general cost function**, need not be positive definite
  $\Rightarrow$ Closed-loop system does not necessarily converge to steady-state
- Resulting questions:
  - What is the **optimal operating regime** (steady-state, periodic, ...)?
  - Does the **closed-loop system** "find" optimal operating behavior?

Dissipativity in economic MPC

Definition - optimal operation at steady-state

- Optimal steady-state: $(x^*, u^*) = \arg\min_{x\in\mathbb{X}, u\in\mathbb{U}, x=f(x,u)} \ell(x,u)$
- A system is **optimally operated at steady-state** if for each feasible state and input sequences $x(\cdot)$ and $u(\cdot)$ the following holds:

$$\liminf_{T\to\infty} \sum_{t=0}^{T-1} \frac{\ell(x(t),u(t))}{T} \geq \ell(x^*,u^*).$$



Dissipativity in economic MPC

Definition - Dissipativity [Willems '72, Byrnes & Lin '94]

A system is strictly **dissipative** with respect to the **supply rate** s if there exists a **storage function** λ such that for all $x \in \mathbb{X}$ and $u \in \mathbb{U}$ it holds that

$$\lambda(f(x,u)) - \lambda(x) \leq s(x,u) - \alpha(\|x - x^*\|), \qquad \alpha \in \mathcal{K}_\infty.$$

Dissipativity and optimal steady-state operation

additional controllability condition [Müller et al. '13,15]

Optimal operation at steady-state $\Longrightarrow$ Dissipativity w.r.t. supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$

$\Longleftarrow$ [Angeli et al. '12]

Dissipativity in economic MPC

If steady-state operation is optimal, does closed-loop system converge to x^*?

- Main idea for stability proof in stabilizing MPC: use optimal value function as Lyapunov function

$$J_N^*(x(t+1)) - J^*(x(t)) \leq -\ell(x(t),u(t)) - \ell(x^*,u^*) \leq -\alpha(\|x(t)\|)$$

- In economic MPC: second inequality **not** satisfied!
- Define rotated cost function

$$L(x,u) = \ell(x,u) - \ell(x^*,u^*) + \lambda(x) - \lambda(f(x,u))$$

- If system is strictly dissipative: $L(x,u) \geq \alpha(\|x - x^*\|)$

Modified optimization problem

$$J_N^*(x(t)) = \min_{u(\cdot|t)} \sum_{k=0}^{N-1} \ell(x(k|t),u(k|t)) + V^f(x(N|t))$$

s.t. $x(0|t) = x(t), \quad x(k+1|t) = f(x(k|t),u(k|t)), \quad k = 0,\dots,N-1$

$x(k|t) \in \mathbb{X}, \quad u(k|t) \in \mathbb{U}, \quad k = 0,\dots,N-1$

$x(N|t) \in \mathbb{X}^f$

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Modified optimization problem

$$\widetilde{J}_N^*(x(t)) = \min_{u(\cdot|t)} \sum_{k=0}^{N-1} L(x(k|t), u(k|t)) + \widetilde{V}^f(x(N|t))$$

$$\text{s.t.} \quad x(0|t) = x(t), \quad x(k+1|t) = f(x(k|t), u(k|t)), \quad k = 0, \dots, N-1$$

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- If system is strictly dissipative: $L(x,u) \geq \alpha(\|x - x^*\|)$

- Key step: original and modified optimization problem have **same** solution
- Can use $\widetilde{J}_N^*$ as Lyapunov function:

$$\widetilde{J}_N^*(x(t+1)) - \widetilde{J}^*(x(t)) \leq -L(x(t), u(t)) \leq -\alpha(\|x(t)\|)$$

Dissipativity in economic MPC

Strict dissipativity and optimal steady-state operation

optimal operation at steady-state (strict form) ⟹ (local controllability at x^* [Müller et al. 15]) Strict dissipativity w.r.t. supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$

Strict dissipativity ⟹ ([Diehl et al. '11, Angeli et al. '12, Grüne '13, Müller et al. '14]) Closed-loop convergence to x^*

Strict dissipativity ⟹ optimal operation at steady-state (strict form)

Discussion

- Closed-loop system "does the right thing", i.e., "finds" optimal operating behavior
- Can be concluded **without** having to compute storage function λ

Dissipativity in economic MPC

Results can be extended to optimal periodic behavior:

Dissipativity and optimal periodic operation

Optimal periodic operation ⟹ (local controllability at Π^* [Müller et al. 15]) Strict dissipativity of P-step system w.r.t. $s(\tilde{x},\tilde{u}) = \tilde{\ell}(\tilde{x},\tilde{u}) - \tilde{\ell}(\tilde{x}^*,\tilde{u}^*)$

Strict dissipativity ⟹ ([Müller&Grüne'15, Zanon et al. '15]) Closed-loop convergence to Π^*

Strict dissipativity ⟹ Optimal periodic operation

Discussion

- Dissipativity plays central role in economic MPC
- Closed-loop system "does the right thing", i.e., "finds" optimal operating behavior
- Can be concluded **without** having to compute storage function
- Results hold for both optimal steady-state and periodic behavior

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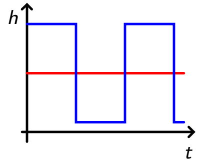
Average constraints

- In economic MPC: closed-loop system does not necessarily converge to some steady-state
- $\rightarrow$ **Constraints on averages** of state/input variables are important and have to be considered **online**
- Examples:
  - Process industry: average amount of inflow or heat flux in chemical reactor
  - Building climate control: small average temperature deviations
  - Manufacturing industry: average amount of parts which can be processed



- **Transient** average constraints: $\frac{\sum_{k=t}^{t+\tau-1} h(x(k),u(k))}{\tau} \in \mathbb{Y}$ for some $\tau \geq 1$ and all $t \geq 0$
- **Asymptotic** average constraints: $Av[h(x,u)] \in \mathbb{Y}$, with $Av[h(x,u)] := \{\xi : \exists \{t_n\} \rightarrow +\infty : \lim_{n\rightarrow\infty} \frac{\sum_{k=0}^{t_n} h(x(k),u(k))}{t_n+1} = \xi\}$

Average constraints

$$\min_{\mathbf{u}(t)} \sum_{k=0}^{N-1} \ell(x(k|t),u(k|t)) + V^f(x(N|t))$$

$$\text{s.t.} \quad x(0|t) = x(t), \quad x(k+1|t) = f(x(k|t),u(k|t)), \quad k = 0,\dots,N-1$$

$$x(k|t) \in \mathbb{X}, \quad u(k|t) \in \mathbb{U}, \quad k = 0,\dots,N-1$$

$$x(N|t) \in \mathbb{X}^f(t)$$

$$\sum_{k=0}^{N-1} h(x(k|t),u(k|t)) \in \mathbb{Y}_t \qquad \mathbb{Y}_{t+1} := \mathbb{Y}_t \oplus \mathbb{Y} \oplus \overline{\mathbb{Y}}(t) \oplus \{-h(x(t),u(t))\}$$

- Additional (time-varying) constraint is needed to ensure satisfaction of average constraints
- Systematic procedure available how $\mathbb{X}^f(t)$ and $\overline{\mathbb{Y}}(t)$ can be computed

Theorem [Müller et al. '14]

Suppose that $\mathbb{X}^f(t)$ and $\overline{\mathbb{Y}}(t)$ are appropriately defined. Then

- Initial feasibility implies recursive feasibility.
- Closed-loop system satisfies asymptotic average constraints $Av[h(x,u)] \subseteq \mathbb{Y}$.

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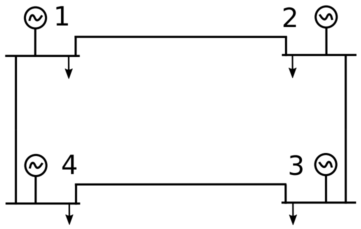
Applications

Application 1: Real Time Economic Dispatch for Power Systems



Power Dynamics:
$x_i = [P_i^M, \omega_i, \delta_i]$, $u_i = P_i^C$, $d_i = P_i^L$
$x_i^+ = A_{\mathcal{N}_i} x_{\mathcal{N}_i} + B_i u_i + E_i d_i$

Economic Dispatch: Power load P_i^L changes
⇒ drive system to (new) economic steady state

$$\min \sum_{i=1}^{M} \left(P_i^M\right)^2 \quad \text{st.} \quad \sum_{i=1}^{M} P_i^M = \sum_{i=1}^{M} P_i^L$$

Applications

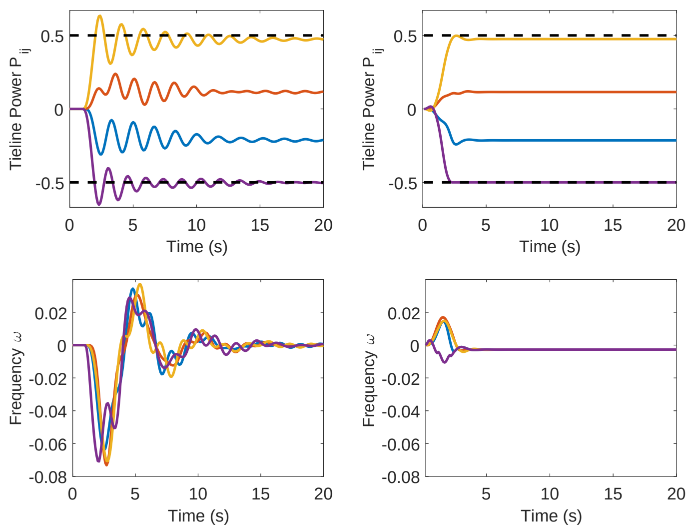
State of the art solution:
Economic Automatic Generation Control (EAGC) [Li et al. '14]
⇒ converges to (new) economic optimal steady state

Distributed Economic MPC solution:

- Economic stage cost: $l_i = (P_i^M)^2 + c\omega_i^2$
- Constraints: $P_i^M \in [P_M^{\min}, P_M^{\max}]$, $P_{ij} \in [P_{ij}^{\min}, P_{ij}^{\max}]$
- Average constraints: $Av[\omega_i] = 0$
- Strict dissipativity: system is optimally operated at steady-state

Applications

Simulation Result
EAGC DEMPC



Applications

Application 2: Cooperative control of self-interested agents

- Several independent systems pursuing a cooperative task; e.g. manufacturing robots, vehicle platoons, wind farms
- Each system has its own individual objective (self-interest), possibly conflicting with cooperative task
- Availability of information and communication between the systems is limited and requires time
- Centralized control not possible / not scalable / not desirable

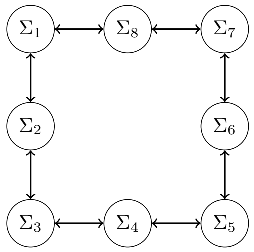
⇒ **Distributed economic MPC**

Applications

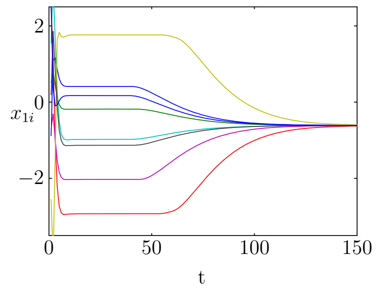
Example: Agreement of self-interested agents

- Double integrator systems: $A_i = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B_i = [0\ 1]^T$
- **Cooperative goal:** systems have to reach consensus asymptotically ($x_1 = x_2 = \cdots = x_n$)
- **Individual goal:** each system has preferred state
  $\rightarrow$ Cost functions $\ell_i(x_i, u_i) = (x_{1i} - a_i)^2 + (x_{2i} - b_i)^2 + (u_i - c_i)^2$
- Individual objectives **conflicting** with cooperative task of agreement
- Cooperative goal achieved via **average constraints**

Applications



Communication topology: cycle



Interpretation of simulation results

- During transient phase, each system converges to its preferred state $x_{1i} = a_i$
- Systems reach consensus asymptotically $\rightarrow$ ensured via average constraint
- Speed of convergence depends on different parameters

$\Rightarrow$ Cooperative requirement is satisfied while systems can act according to individual objective during transient phase

Conclusions

Conclusions

- **Model predictive control**: optimization-based control technique with many successful applications
- **Economic MPC**: new approaches to consider general control objectives
- **Dissipativity** plays a crucial role in economic MPC:
  - Classify optimal operating behavior
  - Convergence analysis of closed-loop system
- **Average constraints** are of importance in economic MPC and have to be dealt with online
- **Applications** in distributed control

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