Stabilisierung kontroll-affiner Systeme durch Lie-Klammer Approximationen mit Anwendungen zur abstandsbasierten Formationssteuerung

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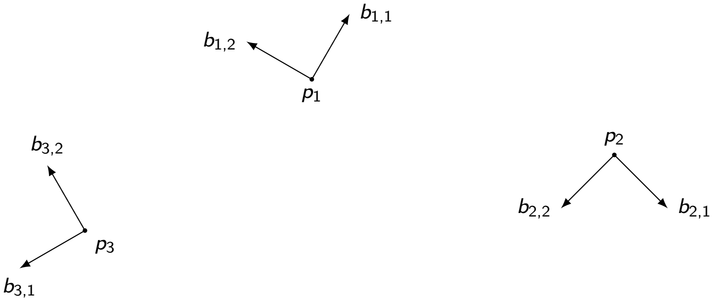
Motivation: distanced-based formation control

Model: N kinematic points in $\mathbb{R}^n$:

$$\dot{p}_i \;=\; \sum_{k=1}^{n} u_{i,k}\, b_{i,k}, \qquad i = 1, \ldots, N,$$

where

- $p_i = (p_{i,1}, \ldots, p_{i,n}) \in \mathbb{R}^n$ position of agent i,
- $b_{i,1}, \ldots, b_{i,n} \in \mathbb{R}^n$ orthonormal basis of velocity direction,
- $u_{i,1}, \ldots, u_{i,n}$ input channels for velocities.

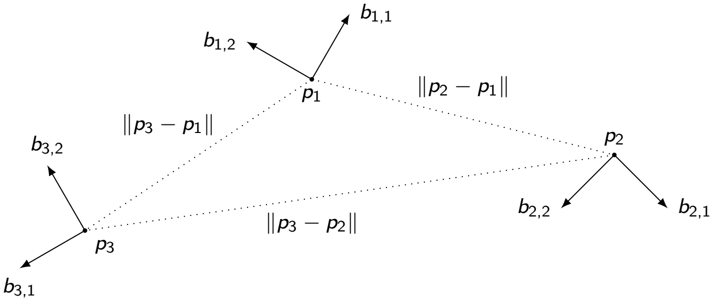


Motivation: distanced-based formation control

Measurements: described by an undirected graph $G = (V, E)$, where

- $V = \{1, \ldots, N\}$ set of nodes,
- $E = \{\{i,j\} \subseteq V \mid |\{i,j\}| = 2\}$ set of edges, we write $ij := \{i,j\}$.

If $ij \in E$, then agent i can measure *distance* ($\neq$relative position) to agent j and vice versa.



Motivation: distanced-based formation control

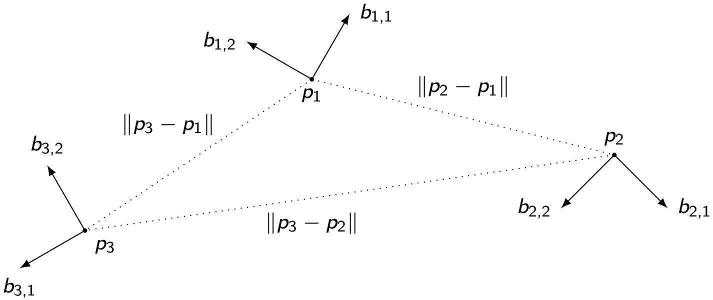
Objective: steer agents to state $p = (p_1, \dots, p_N) \in \mathbb{R}^{nN}$ such that

$$\forall ij \in E: \qquad \|p_j - p_i\| = d_{ij},$$

where $d_{ij} > 0$ *desired distance* between agents i, j.

Constraints:

- autonomous agents $\Rightarrow$ decentralized, distributed control law,
- each agent can only use its own distance measurements,
- no shared clock, no storage of data.



Motivation: distanced-based formation control

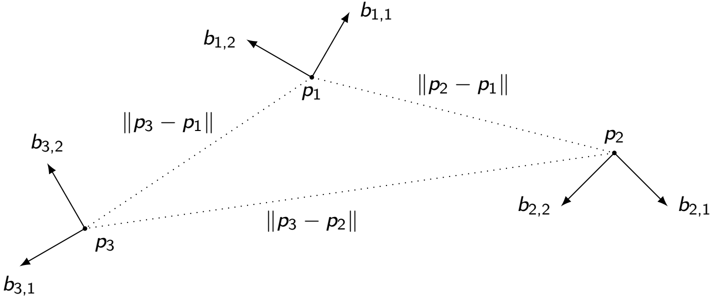
Standard approach: gradient-based control law.

Local potential function for agent i:

$$\psi_i(p) := \frac{1}{4} \sum_{j \in V:\, ij \in E} \left(\|p_j - p_i\|^2 - d_{ij}^2 \right)^2$$

Control law for agent i:

$$u_i = (u_{i,1}, \ldots, u_{i,n}) = -\nabla_{p_i} \psi_i(p) = \frac{1}{2} \sum_{j \in V:\, ij \in E} (p_j - p_i)(\|p_j - p_i\|^2 - d_{ij}^2)$$



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Then, set of *target formations*

$$F \;:=\; \{ p = (p_1, \ldots, p_n) \in \mathbb{R}^{nN} \mid \forall ij \in E \colon \|p_j - p_i\| = d_{ij} \}$$

- is locally asymptotically stable (Krick et al., 2009; Oh, Ahn, 2014)
- is locally exponentially stable if frameworks (G, p) with $p \in F$ are *infinitesimally rigid* (Mou et al, 2016, Sun et al., 2016)

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- is locally exponentially stable if frameworks (G, p) in F are *infinitesimally rigid* (Mou et al, 2016, Sun et al., 2016)

Obstacle: implementation requires measurements of relative positions

actively controlled variable $\neq$ sensed variable

Approach: approximate the gradient by Lie brackets.

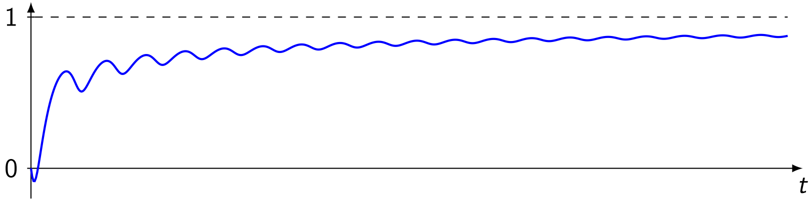
Toy model: $N = 1$ agent in $n = 1$ dimension, i.e., $\dot{p} = u \in \mathbb{R}$, with potential $\psi(p) := \frac{1}{2}|p - 1|^2$.

Define $u_1, u_2, h_1, h_2 \colon \mathbb{R} \to \mathbb{R}$ by

$$u_1(t) := \cos t, \qquad u_2(t) := \sin t,$$

$$h_1(y) := \begin{cases} y \sin(\log y) & y > 0 \\ 0 & y \leq 0 \end{cases}, \quad h_2(y) := \begin{cases} y \cos(\log y) & y > 0 \\ 0 & y \leq 0 \end{cases}.$$

Control law: $u := u_1(t)\, h_1(\psi(p)) + u_2(t)\, h_2(\psi(p))$



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Control law: $u := u_1(t)\, h_1(\psi(p)) + u_2(t)\, h_2(\psi(p))$

"Lie bracket" of $f_1 := (h_1 \circ \psi)$ and $f_2 := (h_2 \circ \psi)$:

$$[f_1, f_2](p) \; := \; f_2'(p)\, f_1(p) - f_1'(p)\, f_2(p) \; = \; -\underbrace{\psi(p)}_{> 0 \text{ for } p \neq 1}\, \psi'(p)$$

Closed-loop system: $\dot{p} = u_1(t)\, f_1(p) + u_2(t)\, f_2(p) = \sum_i u_i(t) f_i(p)$.

Integral equation, averaging procedure:

$$p(t) = p(t_0) + \sum_i \int_{t_0}^{t} \underbrace{u_i(s)}_{\uparrow} \, \underbrace{f_i(p(s))}_{\downarrow} \, \mathrm{d}s$$

Approximation of Lie Brackets

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$$= p(t_0) + \sum_i \big[U_i(s) f_i(p(s))\big]_{t_0}^{t} - \sum_{i,j} \int_{t_0}^{t} U_i(s)\, u_j(s)\, f_i'(p(s))\, f_j(p(s))\,\mathrm{d}s$$

$$= p(t_0) + \sum_i \big[U_i(s) f_i(p(s))\big]_{t_0}^{t} - \int_{t_0}^{t} \frac{1}{2}[f_1, f_2](p(s))\,\mathrm{d}s$$

$$- \sum_{i,j} \int_{t_0}^{t} \underbrace{\widetilde{UV}_{j,i}(s)}_{\uparrow}\ \underbrace{f_i'(p(s))\, f_j(p(s))}_{\downarrow}\,\mathrm{d}s$$

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$$- \sum_{i,j} \int_{t_0}^{t} \underbrace{\widetilde{UV}_{j,i}(s)}_{\uparrow}\ \underbrace{f_i'(p(s))\, f_j(p(s))}_{\downarrow}\,\mathrm{d}s$$

$$= p(t_0) + \big[D_1(s, p(s))\big]_{t_0}^{t} + \int_{t_0}^{t} \Big(\frac{1}{2}[f_1, f_2](p(s)) + D_2(s, p(s))\Big)\,\mathrm{d}s$$

with certain time-varying remainder vector fields D_1, D_2.

Approximation of Lie Brackets

Closed-loop system: $\dot{p} = u_1(t)\, f_1(p) + u_2(t)\, f_2(p)$

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A detailed analysis shows: D_1, D_2 vanish sufficiently fast compared to $[f_1, f_2]$ when $\psi(p) \to 0$. Thus, for $\psi(p)$ close to 0:

$$p(t) \approx p(t_0) + \int_{t_0}^{t} \frac{1}{2}[f_1, f_2](p(s))\, \mathrm{d}s = p(t_0) - \frac{1}{2} \int_{t_0}^{t} \psi(p(s))\, \psi'(p(s))\, \mathrm{d}s$$

$\Rightarrow$ trajectories follow approximately $[f_1, f_2]$.

Approximation of Lie Brackets

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Toy model: system $\dot{p} = u_1(t)\, f_1(p) + u_2(t)\, f_2(p)$ on $\mathbb{R}$.

More general: averaging analysis by Kurzweil, Jarník (1988); Sussmann, Liu (1991). Control-affine system

$$\dot{p} \;=\; f_0(p) + \sum_{i=1}^{m} u_i^{\omega}(t)\, f_i(p),$$

with

- a drift vector field $f_0 \colon \mathbb{R}^n \to \mathbb{R}^n$,
- suff. smooth control vector fields $f_1, \ldots, f_m \colon \mathbb{R}^n \to \mathbb{R}^n$,
- time-varying functions $u_1^{\omega}, \ldots, u_m^{\omega} \colon \mathbb{R} \to \mathbb{R}$ of the form

$$u_i^{\omega}(t) = \Lambda_i(t)\, \omega^{e_i}\, \sin(\omega\, \Omega_i\, t + \Phi_i),$$

where $\omega > 0$ is a frequency parameter

Approximation of Lie Brackets

Control-affine system

$$\dot{p} \;=\; f_0(p) + \sum_{i=1}^{m} u_i^{\omega}(t)\, f_i(p),$$

Integral equation, averaging procedure:

$$\begin{aligned} p(t) \;=\;& p(t_0) + \big[D_1^{\omega}(s, p(s))\big]_{t_0}^{t} \\ &+ \int_{t_0}^{t} \Big(f_0(p(s)) + \sum_{0<|I|\le r} \frac{v_I(s)}{|I|} [f_I](p(s)) \;+\; D_2^{\omega}(s, p(s)) \Big)\, \mathrm{d}s, \end{aligned}$$

where

- $D_1^{\omega}, D_2^{\omega}$ certain time-varying remainder vector fields,
- $I = (i_1, \ldots, i_k)$ multi-index of length $|I| = k$,
- $v_I \colon \mathbb{R} \to \mathbb{R}$ time-varying function (depending on u_i^{ω}),
- $[f_I] := [f_{i_1}, [f_{i_2}, \cdots, [f_{i_{k-1}}, f_{i_k}] \cdots]]$ iterated Lie bracket.

One can show that $D_1^{\omega}, D_2^{\omega} \to 0$ as $\omega \to \infty$ on compact sets.

Approximation of Lie Brackets

ω-dependent system $\qquad \Sigma^\omega: \dot{p} \;=\; f_0(p) + \sum_{i=1}^{m} u_i^\omega(t)\, f_i(p)$

Limit system $\qquad \Sigma^\infty: \dot{p} \;=\; f_0(p) + \sum_{0<|I|\leq r} \frac{v_I(t)}{|I|}[f_I](p)$

Theorem (Kurzweil, Jarník (1988); Sussmann, Liu (1991))

For every initial condition (t_0, p_0)*, every* $T > 0$*, and every* $\varepsilon > 0$*, there exists* $\omega_0 > 0$ *such that for every* $\omega \geq \omega_0$*, and every* $t \in [t_0, t_0 + T]$*:*

$$\|x^\infty(t) - x^\omega(t)\| \;\leq\; \varepsilon.$$

Application: motion planning for nonholonomic systems.

Approximation of Lie Brackets

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Theorem (Dürr et al. (2013))

Case $r = 2$ and $u_i^\omega(t) = \Lambda_i(t)\sqrt{\omega}\sin(\omega\,\Omega_i\, t + \Phi_i)$, Λ_i bounded.

For every compact set $K \subseteq \mathbb{R}^n$, every $T > 0$, and every $\varepsilon > 0$, there exists $\omega_0 > 0$ such that for every $\omega \geq \omega_0$, every $(t_0, p_0) \in \mathbb{R} \times K$, and every $t \in [t_0, t_0 + T]$:

$$\|x^\infty(t) - x^\omega(t)\| \;\leq\; \varepsilon$$

as long as x^∞ stays in K.

Application: stabilization of control-affine systems.

ω-dependent system $\qquad \Sigma^\omega : \dot{p} \;=\; f_0(p) + \sum_{i=1}^{m} u_i^\omega(t)\, f_i(p)$

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Theorem (Dürr et al. (2013))

Case $r = 2$ *and* $u_i^\omega(t) = \Lambda_i(t)\sqrt{\omega}\sin(\omega\,\Omega_i\, t + \Phi_i)$, Λ_i *bounded.*

For a compact $K \subseteq \mathbb{R}^n$, *the following implication holds:*

$$K \text{ is LUAS for } \Sigma^\infty \qquad \Longrightarrow \qquad K \text{ is } \omega\text{-PLUAS for } \Sigma^\omega$$

*LUAS=**l**ocally **u**niformly **a**symptotically **s**table.*
*ω-PLUAS=**p**ractically LUAS if ω sufficiently large.*

ω-dependent system $\quad \Sigma^\omega : \dot{p} = f_0(p) + \sum_{i=1}^{m} u_i^\omega(t)\, f_i(p)$

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Theorem (S. (2017))

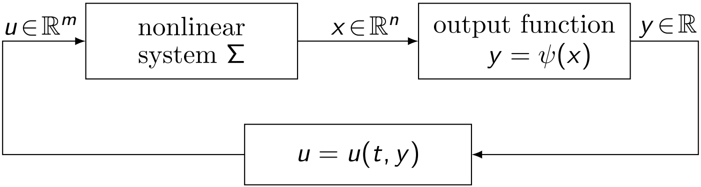
General $r \in \mathbb{N}$, and $K \subseteq \mathbb{R}^n$ compact.
Suppose that u_i^ω satisfy certain averaging conditions as $\omega \to 0$.

(a) *K is LUAS for Σ^∞ $\implies$ K is ω-PLUAS for Σ^ω*
(b) *If vector fields satisfy certain boundedness conditions, then:*

$$K \text{ is LUES for } \Sigma^\infty \implies K \text{ is } \omega\text{-LUES for } \Sigma^\omega$$

*LUES=**l**ocally **u**niformly **e**xponentially **s**table.*
ω-LUES=LUES if ω sufficiently large.

Application to extremum seeking control.

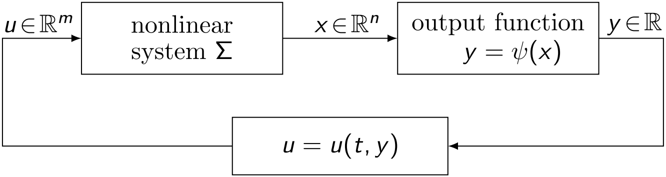


Objective: find a control law u that steers the system to an extremum of ψ (w.l.o.g. minimum of ψ)

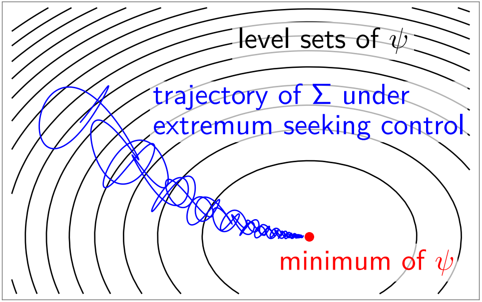
Restrictions:

- no information about the current system state x,
- no explicit knowledge about the output function $x \mapsto \psi(x)$,
- only real-time measurements of $y = \psi(x)$ are available

Application to extremum seeking control.



Idea by Dürr et al.(2013): approximate descent directions of ψ by Lie brackets (feed in sinusoidal perturbations).



Application to extremum seeking control

Applicable to control-affine systems

$$\dot{p} \;=\; g_0(p) + \sum_{k=1}^{m} u_k \, g_k(p), \qquad y = \psi(p).$$

Dürr et al. (2013):

- Define $u_{2k-1}^{\omega}(t) := \sqrt{\omega\,\Omega_k}\cos(\omega\,\Omega_k\,t)$, $u_{2k}^{\omega}(t) := \sqrt{\omega\,\Omega_k}\sin(\omega\,\Omega_k\,t)$.
- Define $h_1(y) := y$ and $h_2(y) := 1$ for every $y \in \mathbb{R}$.

Control law: $u_k = u_{2k-1}^{\omega}(t)\,h_1(\psi(p)) + u_{2k}^{\omega}(t)\,h_2(\psi(p))$.

Application to extremum seeking control

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$$\dot{p} \;=\; g_0(p) + \sum_{k=1}^{m} u_k\, g_k(p), \qquad y = \psi(p).$$

Dürr et al. (2013):

- Define $u^{\omega}_{2k-1}(t) := \sqrt{\omega\,\Omega_k}\cos(\omega\,\Omega_k\,t)$, $u^{\omega}_{2k}(t) := \sqrt{\omega\,\Omega_k}\sin(\omega\,\Omega_k\,t)$.
- Define $h_1(y) := y$ and $h_2(y) := 1$ for every $y \in \mathbb{R}$.

Control law: $u_k = u^{\omega}_{2k-1}(t)\,h_1(\psi(p)) + u^{\omega}_{2k}(t)\,h_2(\psi(p))$.

New vector fields $f_{2k-1} := (h_1 \circ \psi)g_k$ and $f_{2k} := (h_2 \circ \psi)g_k$.

ω-dependent system $\quad \Sigma^{\omega}\colon \dot{p} \;=\; g_0(p) + \sum_{i=1}^{2m} u^{\omega}_i(t)\,f_i(p),$

Limit system $\quad \Sigma^{\infty}\colon \dot{p} \;=\; g_0(p) + \frac{1}{2}\sum_{k=1}^{m} [f_{2k-1}, f_{2k}](p),$

with descent directions $[f_{2k-1}, f_{2k}](p) = -(\mathcal{L}_{g_k}\psi)(p)g_k(p)$.

Application to extremum seeking control

$h_1(y) := y, \quad h_2(y) := 1.$

ω-dependent system $\quad \Sigma^\omega : \dot{p} \;=\; g_0(p) + \sum_{k=1}^{m} u_k \, g_k(p)$

under control law $\quad u_k = u^\omega_{2k-1}(t) \, h_1(\psi(p)) + u^\omega_{2k}(t) \, h_2(\psi(p)),$

Limit system $\quad \Sigma^\infty : \dot{p} \;=\; g_0(p) - \frac{1}{2} \sum_{k=1}^{m} (\mathcal{L}_{g_k} \psi)(p) g_k(p),$

Theorem (Dürr et al. (2013))

Suppose that $K \subseteq \mathbb{R}^n$ is a compact set of minima of ψ. Then, the following implication holds:

$$K \text{ is LUAS for } \Sigma^\infty \quad \Longrightarrow \quad K \text{ is } \omega\text{-PLUAS for } \Sigma^\omega .$$

$$h_1(y) := \sqrt{y}\,\sin(\log y), \quad h_2(y) := \sqrt{y}\,\cos(\log y).$$

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under control law $\quad u_k = u^\omega_{2k-1}(t)\, h_1(\psi(p)) + u^\omega_{2k}(t)\, h_2(\psi(p)),$

Limit system $\quad \Sigma^\infty\colon \dot{p} \;=\; g_0(p) - \frac{1}{2}\sum_{k=1}^{m}(\mathcal{L}_{g_k}\psi)(p) g_k(p),$

Theorem (S. (2017))

Suppose that $\psi \geq 0$ and that $K := \psi^{-1}(\{0\})$ is compact.

(a) *K is LUAS for Σ^∞ $\quad\Longrightarrow\quad$ K is ω-PLUAS for Σ^ω*

(b) *If the drift g_0 satisfies certain boundedness conditions, then:*

K is LUES for Σ^∞ $\quad\Longrightarrow\quad$ K is ω-LUES for Σ^ω

$$h_1(y) := \sqrt{y}\,\sin(\log y), \quad h_2(y) := \sqrt{y}\,\cos(\log y) \text{ for } y > 0.$$

Example

Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times n}$. Consider the system

$$\Sigma^\omega: \qquad \dot{x} = Ax + Bu, \qquad y = \psi(x) := \|x\|^2$$

with $u = (u_1, \ldots, u_m)^\top$ under the control law

$$u_k = \lambda\, u^\omega_{2k-1}(t)\, h_1(\psi(x)) + \lambda\, u^\omega_{2k}(t)\, h_2(\psi(x)).$$

with $\lambda, \omega > 0$. Assume that rank $B = n$. Limit system

$$\Sigma^\infty: \qquad \dot{x} = (A - \lambda^2 B^\top B)x$$

If $\lambda, \omega > 0$ are sufficiently large, the $\{0\}$ is LUES for Σ^ω.

Remark: choice of functions h_1, h_2 in

$$u_k \;=\; u^\omega_{2k-1}(t)\, h_1(\psi(p)) + u^\omega_{2k}(t)\, h_2(\psi(p))$$

- Dürr et al. (2013): $h_1(y) = y$ and $h_2(y) = 1$
  $\to$ ω-PLUAS
- Scheinker et al. (2014): $h_1(y) = \sin y$ and $h_2(y) = \cos y$
  $\to$ ω-PLUAS
- S. (2017): $h_1(y) = \sqrt{y}\,\sin(\log y)$ and $h_2(y) = \sqrt{y}\,\cos(\log y)$
  $\to$ ω-PLUAS and also ω-LUES
- S., Sun (2018): $h_1(y) = y\,\sin(\log y)$ and $h_2(y) = y\,\cos(\log y)$
  $\to$ LUAS (for every $\omega > 0$), but not LUES

Integral equation, averaging procedure:

$$p(t) \;=\; p(t_0) + \big[D^\omega_1(s, p(s))\big]^t_{t_0} + \int_{t_0}^{t} \Big(f^\infty(p(s)) \,+\, D^\omega_2(s, p(s))\Big)\,\mathrm{d}s,$$

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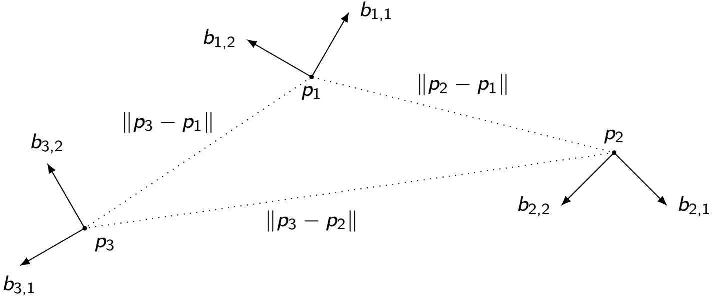


"Team" of control-affine systems

$$\dot{p}_i \;=\; \sum_{k=1}^{n} u_{i,k}\, b_{i,k}, \qquad y_i = \psi_i(p), \qquad i = 1, \ldots, N,$$

output functions = local potential functions

$$\psi_i(p) \;:=\; \frac{1}{4} \sum_{j\in V:\; ij\in E} \left(\|p_j - p_i\|^2 - d_{ij}^2\right)^2$$

can be computed by individual distance measurements.

Application to distanced-based formation control

$h_1(y) := y \sin(\log y), \quad h_2(y) := y \cos(\log y)$ for $y > 0$.

ω-dependent system $\quad \Sigma^\omega \colon \dot{p}_i \;=\; \sum_{k=1}^{n} u_{i,k}\, b_{i,k}, \qquad i = 1, \ldots, N$

under control law $\quad u_{i,k} = u^\omega_{i,k,1}(t)\, h_1(\psi_i(p)) + u^\omega_{i,k,2}(t)\, h_2(\psi_i(p)),$

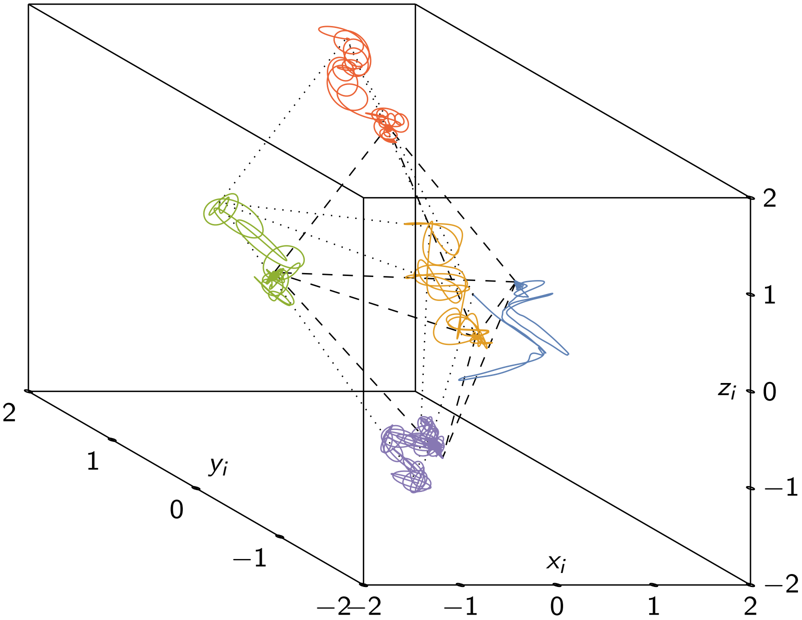
Limit system $\quad \Sigma^\infty \colon \dot{p}_i \;=\; -\frac{1}{2}\psi_i(p)\, \nabla_{p_i}\psi_i(p), \qquad i = 1, \ldots, N.$

Theorem (S., Sun(2018))

Suppose that the frameworks (G, p) with $p \in F$ are infinitesimally rigid. Then, ***for every*** $\boldsymbol{\omega > 0}$*, the set F is LUAS for Σ^ω.*

Remark: the domain of attraction increases when ω increases.

Application to distanced-based formation control



Thank you for your attention!