Path-Following for Nonlinear Systems Subject to Constraints

Timm Faulwasser
Lehrstuhl für Systemtheorie und Regelungstechnik (Prof. Rolf Findeisen),
Otto-von-Guericke Universität Magdeburg

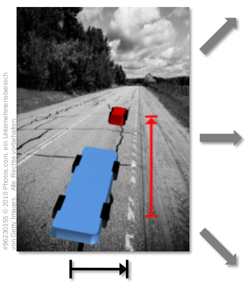
Path-Following for Nonlinear Systems Subject to Constraints

Timm Faulwasser

Chair for Systems Theory and Control
Institute for Automation Engineering
Otto von Guericke University Magdeburg

Elgersburg Workshop, 01.03.2010

Steering a Car as a Control Problem



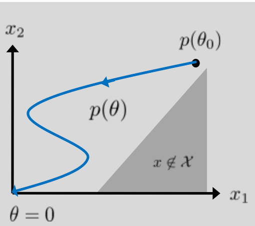
Stabilisation
- control distance
- velocity?

Trajectory Tracking
- determine reference offline
- design trajectory tracking controller
- robustness?

Path-Following
- geometric reference
- determine reference velocity online

Timm Faulwasser, Path-Following for Nonlinear Systems

Path-Following Problem



$\dot{x} = f(x,u), \quad x(0) = x_0$

- path $\subset$ regular, 1D curve

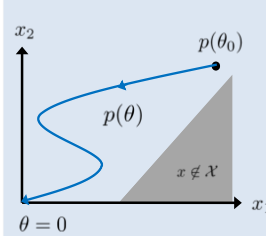
  $\mathcal{P} = \{p(\theta) \in \mathbb{R}^n : \theta \in [\theta_0, 0] \mapsto p(\theta)\}$
- final point =origin $\quad \lim_{\theta \to 0} \|p(\theta)\| = 0$
- given initial point $\quad p(\theta_0)$

Control Task
- path convergence $\quad \lim_{t \to \infty} \|x(t) - p(\theta(t))\| = 0$
- forward motion $\quad \forall \theta \in [\theta_0, 0): \quad \dot{\theta}(t) > 0$
- satisfaction of constraints $\quad x \in \mathcal{X}, \quad u \in \mathcal{U}$

Timm Faulwasser, Path-Following for Nonlinear Systems

Existing Results

- linear systems *[Aguiar et al. `05; Dacic & Kokotovic `06]*
- nonlinear systems *[Aguiar et al. `08]*
- robust path-following *•[Skjetne, Kokotovic et al. `05; Do et al. `06]*
- passive systems *[El-Hawwary `08]*
- feedforward control of robots *[Shin & McKay `85, Verscheure et al. `09]*

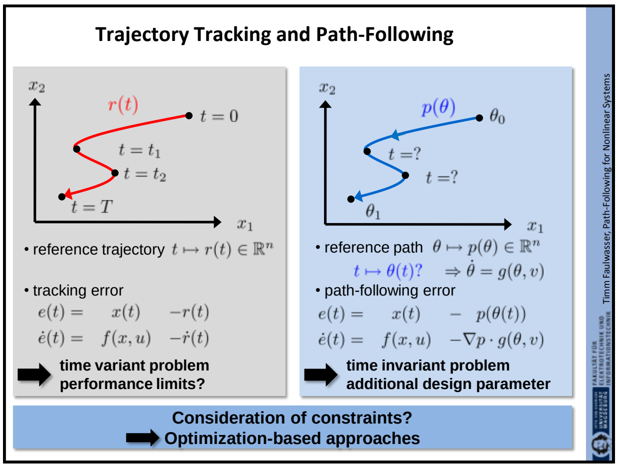


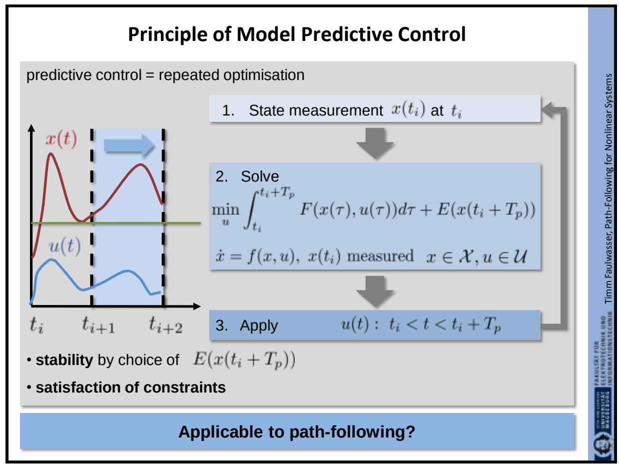
Usually, no consideration of constraints

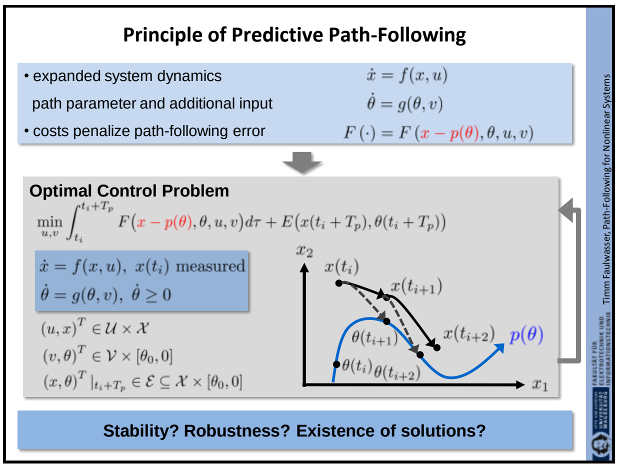
Timm Faulwasser, Path-Following for Nonlinear Systems

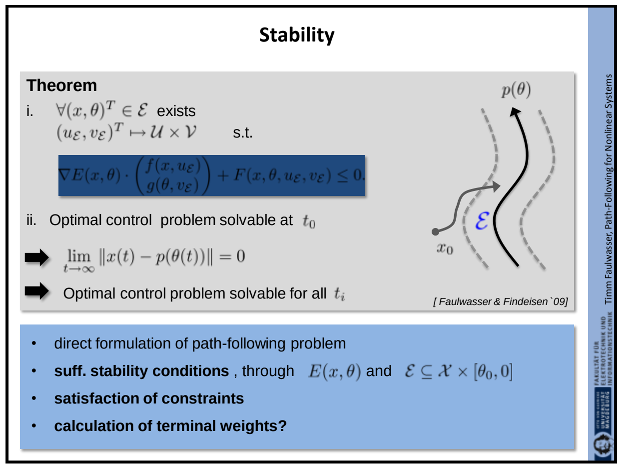
Path-Following for Nonlinear Systems Subject to Constraints

Timm Faulwasser
Lehrstuhl für Systemtheorie und Regelungstechnik (Prof. Rolf Findeisen),
Otto-von-Guericke Universität Magdeburg









Path-Following for Nonlinear Systems Subject to Constraints

Timm Faulwasser
Lehrstuhl für Systemtheorie und Regelungstechnik (Prof. Rolf Findeisen),
Otto-von-Guericke Universität Magdeburg

Calculation of Stabilising Terminal Weights

Idea

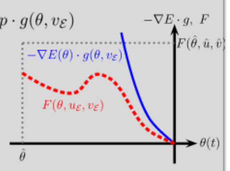
- path as terminal region $\mathcal{E} = \mathcal{P} \times [\theta_0, 0]$
- path exactly followable $\forall\theta \quad \exists u, v: \quad f(x,u) = \nabla p \cdot g(\theta, v)$
- terminal weight = costs along path $E(x,\theta) = E(\theta)$

Corollary

- quadratic cost $F(\cdot) = \left\| \begin{matrix} x - p(\theta) \\ \theta \end{matrix} \right\|_Q + \left\| \begin{matrix} u \\ v \end{matrix} \right\|_R$
- input signals $\forall\theta\ \exists u_\mathcal{E}, v_\mathcal{E}: \ f(x, u_\mathcal{E}) = \nabla p \cdot g(\theta, v_\mathcal{E})$
- if an $\epsilon \in \mathbb{R}$ exists, s.t. $\forall\theta \in [\theta_0, 0]$

$$\epsilon > \frac{\hat{q}\theta^2 + \left\| \begin{matrix} u_\mathcal{E} \\ v_\mathcal{E} \end{matrix} \right\|_R^2}{-g(\theta, v_\mathcal{E}) \cdot \theta} \geq 0.$$

Then $E(\theta) = \frac{1}{2}\epsilon \cdot \theta^2$ guarantees stability and $\lim_{t\to\infty} \|x(t) - p(\theta(t))\| = 0$

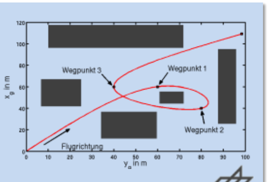


[Faulwasser & Findeisen `09]

Timm Faulwasser, Path-Following for Nonlinear Systems

Example: Path-Following for an Autonomous Helicopter

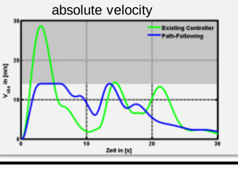
Prototype system ARTIS
- on-board image processing
- I-/O-linearising flight stabilisation
- task: **accurate path follwoing**



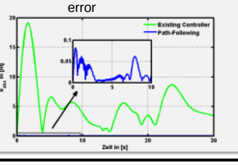
joint project with Institute for Flight Systems, DLR (S. Lorenz)

Comparision existing controller and path-following

absolute velocity



error



Timm Faulwasser, Path-Following for Nonlinear Systems

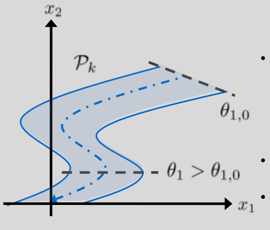
Steering a Car as a Control Problem

Predictive Path-Following
- 1D paths
- consideration of constraints

Extensions
- multi-dimensional path corridors?
- path-following for output paths?

Timm Faulwasser, Path-Following for Nonlinear Systems

Corridor Path-Following



$\dot{x} = f(x,u), \quad x(0) = x_0$

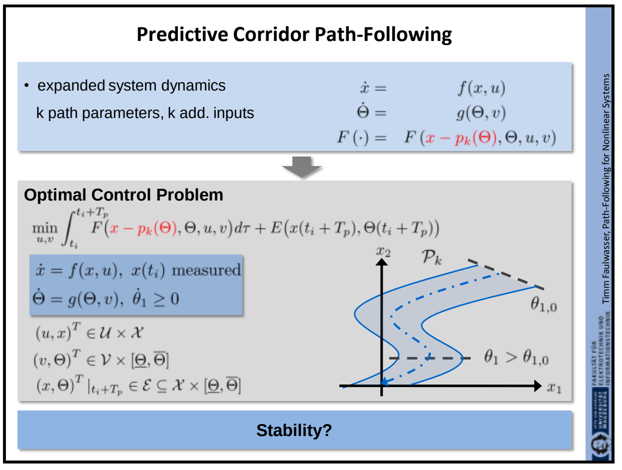
- **path** = regular, kD **surface**
  $\mathcal{P}_k \subset \mathbb{R}^n$
  $\mathcal{P}_k = \{p_k(\Theta) : \Theta \in \mathbb{R}^k \mapsto p_k(\Theta)\}$
- endpoint = origin $\lim_{\Theta\to 0} \|p_k(\Theta)\| = 0$
- given initial point $p_k(\Theta_0)$

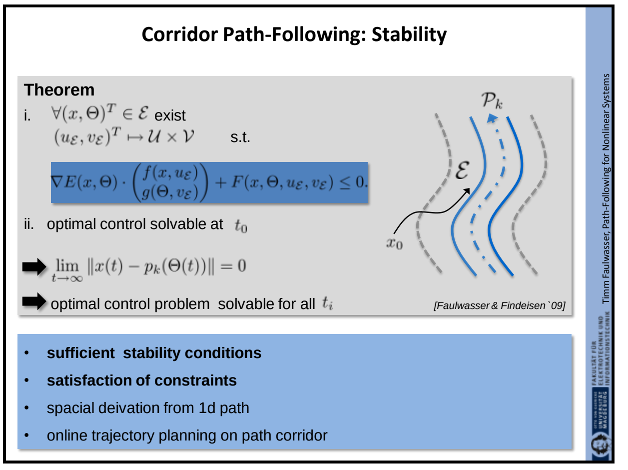
Control Task

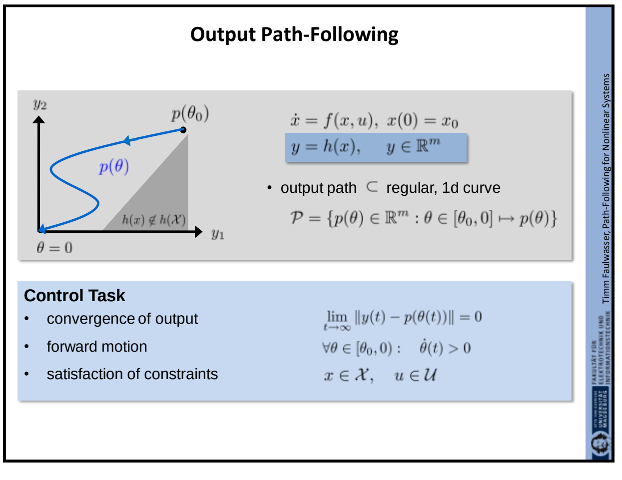
- convergence to corridor $\lim_{t\to\infty} \|x(t) - p_k(\Theta(t))\| = 0$
- forward motion $\forall\Theta \in [\overline{\Theta}, \underline{\Theta}): \quad \dot\theta_1(t) > 0$
- satisfaction of constraints $x \in \mathcal{X}, \quad u \in \mathcal{U}$

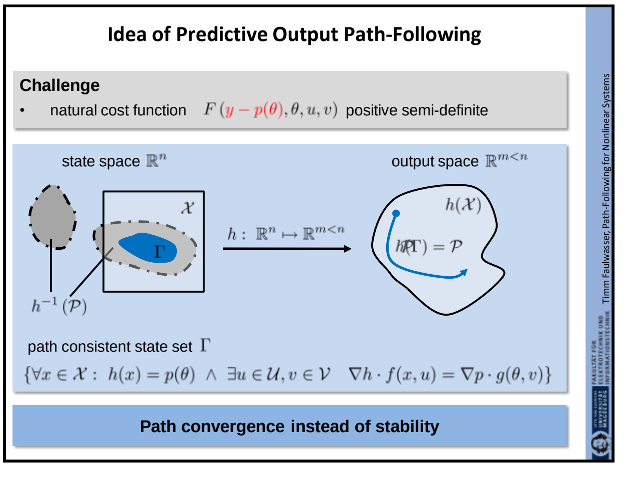
Timm Faulwasser, Path-Following for Nonlinear Systems

Path-Following for Nonlinear Systems Subject to Constraints
Timm Faulwasser
Lehrstuhl für Systemtheorie und Regelungstechnik (Prof. Rolf Findeisen),
Otto-von-Guericke Universität Magdeburg





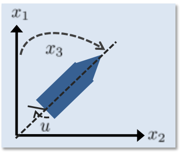




Path-Following for Nonlinear Systems Subject to Constraints

Timm Faulwasser
Lehrstuhl für Systemtheorie und Regelungstechnik (Prof. Rolf Findeisen),
Otto-von-Guericke Universität Magdeburg

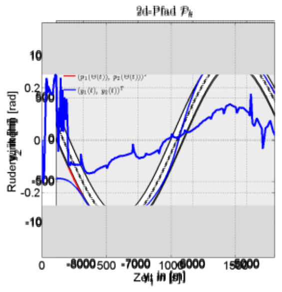
Example: Ship Control



$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix} = \begin{pmatrix} w\cos x_3 - Lwx_4 \cdot \sin x_3 \\ w \sin x_3 + Lwx_4 \cdot \cos x_3 \\ x_4 \\ \frac{1}{T}(-x_4 + Ku)\end{pmatrix}$$

$$\begin{pmatrix}y_1\\ y_2\end{pmatrix} = \begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

$$\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{pmatrix} = \begin{pmatrix}-\lambda_1\theta_1 + v_1\\ -\lambda_2\theta_2 + v_2\end{pmatrix} \quad \begin{matrix} v_1 \in [0, \overline{v}_1] \\ v_2 \in [\underline{v}_2, \overline{v}_2]\end{matrix}$$



2D Output Path Corridor

$$p(\theta_1, \theta_2) = \begin{pmatrix}\theta_1 \\ \alpha \sin(\beta\theta_1)\end{pmatrix} + \theta_2 \left\| \begin{matrix}-\alpha\beta\cos(\beta\theta_1) \\ 1\end{matrix} \right\|^{-1} \begin{pmatrix}-\alpha\beta\cos(\beta\theta_1) \\ 1\end{pmatrix}$$

Summary

new approach to constrained path-following problems

- nonlinear path-following problems subject ot constraints
- rigorous stability conditions
- extensions to path corridors and output paths
- outlook: optimal control on manifolds, robustness, …

Possible Applications

- CNC-machines, robotics
- autonomous vehicles & planes
- crystallisation processes
- …

Many applications ➡ path-following problems

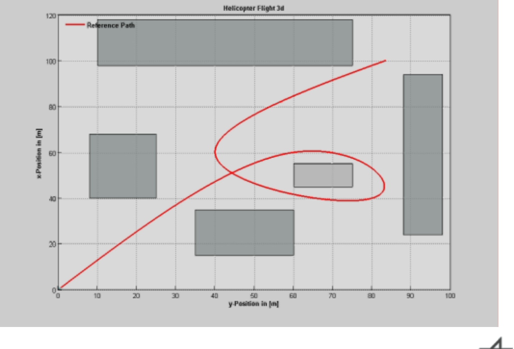
Timm Faulwasser, Path-Following for Nonlinear Systems

Literatur

- ***Aguiar et al. (2005).*** *Path-following for nonminimumphase systems removes performanc e limitations. IEEE Transactions on Automatic Control 50, 234-239.*
- ***Dacic & Kokotovic (2006).*** *Path-following for linear systems with unstable zero dynamics. Automatica 42, 1673-1683.*
- ***Aguiar et al. (2008).*** *Performance limitations in reference tracking and path-following for nonlinear systems. Automatica 44, 598-610.*
- ***Skjetne et al. (2005).*** *Robust output maneuvering for a class of nonlinear systems. Automatica 40, 373-383.*
- ***Do & Pan (2006).*** *Global robust adaptive path followng of under actuated ships. Automatica 42, 1713-1722.*
- ***Shin & McKay (1985).*** *Minimum-time control of robotic manipulators with geometric path constraints. IEEE Transactions on Automatic Control 30, 531-541.*
- ***Verscheure et al. (2009).*** *Time-optimal oath tracking for robots: A convex optimization approach. IEEE Transactions on Automatic Control 54, 2318-2327.*
- ***Faulwasser & Findeisen (2009).*** *Nonlinear model predictive path-following control. In Magni et al. (ed.) Assesment and future directions of nonlinear model predictive control, 335-343. Springer.*
- ***Faulwasser et al. (2009).*** *Model predictive path-following for constrained nonlinear systems. Proc. of 48th CDC, 8642-8647.*
- ***Faulwasser & Findeisen (2010).*** *Constrained output path-following for nonlinear systems using predicitve control. Submitted to NOLCOS 2010.*

Timm Faulwasser, Path-Following for Nonlinear Systems

Example: Path-Following for an Autonomous Helicopter



joint project with Institute for Flight Systems, DLR (S. Lorenz)

Timm Faulwasser, Path-Following for Nonlinear Systems