Control of vehicle platoons

Hans Zwart (University of Twente, TU/e)

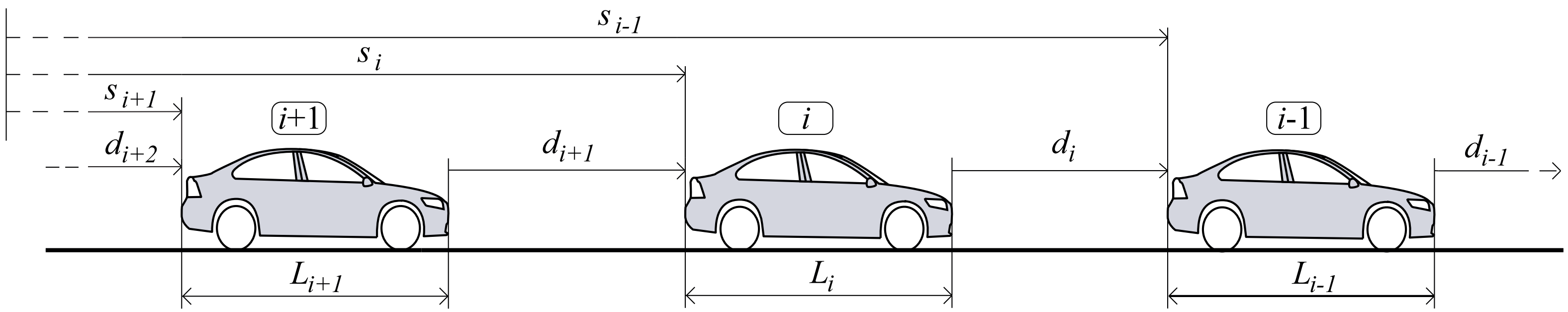
joint work with:

Amir Firooznia (TU-Delft), Jeroen Ploeg (TNO), Nathan van de Wouw (TU/e)

Ruth Curtain and Orest Iftime (RUG)

1 Introduction

Consider a string of vehicles (a platoon)



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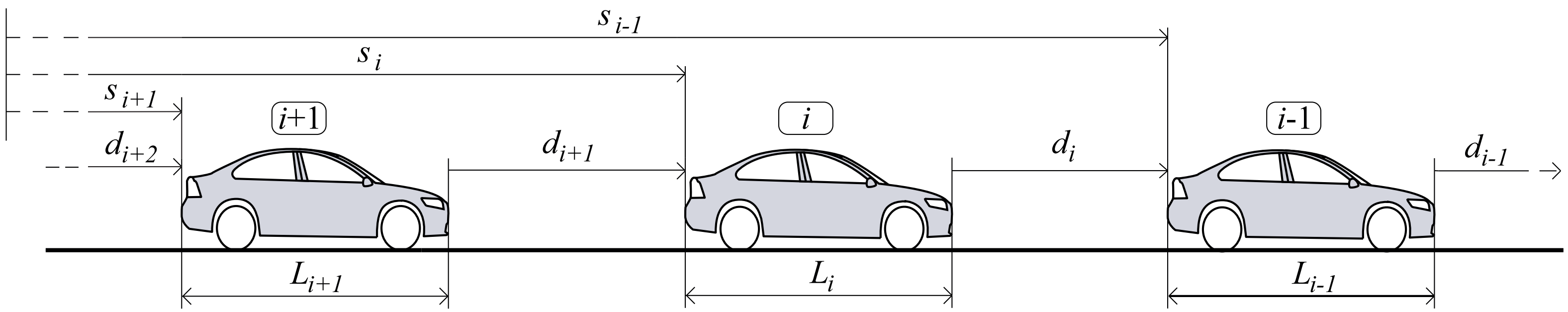

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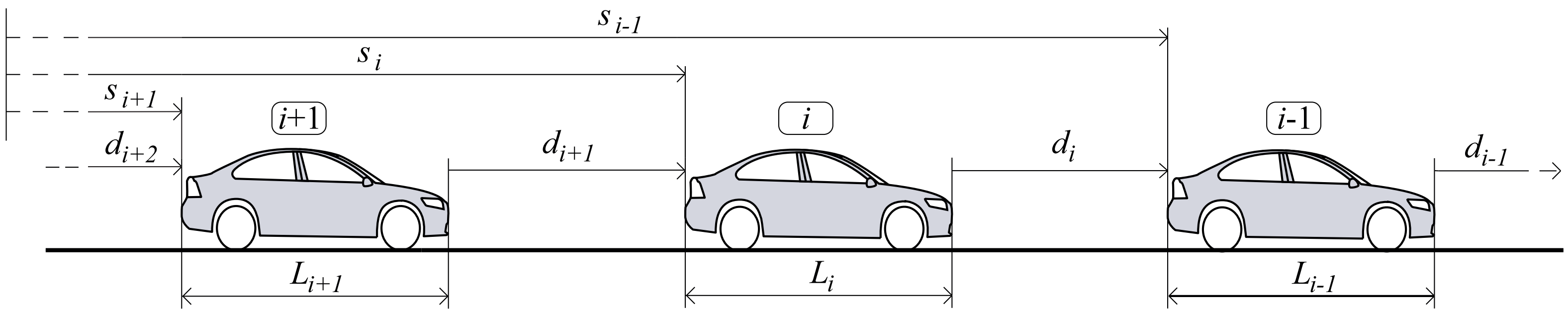

The aim is to control the distance d_i between the cars. Every car controls its jerk, i.e, the change of its force.

Normally the (desired) distance has to satisfy $d = d_0 + h * v$, with v the velocity and h the headway time (0.3 – 1.2 s).

Assumption

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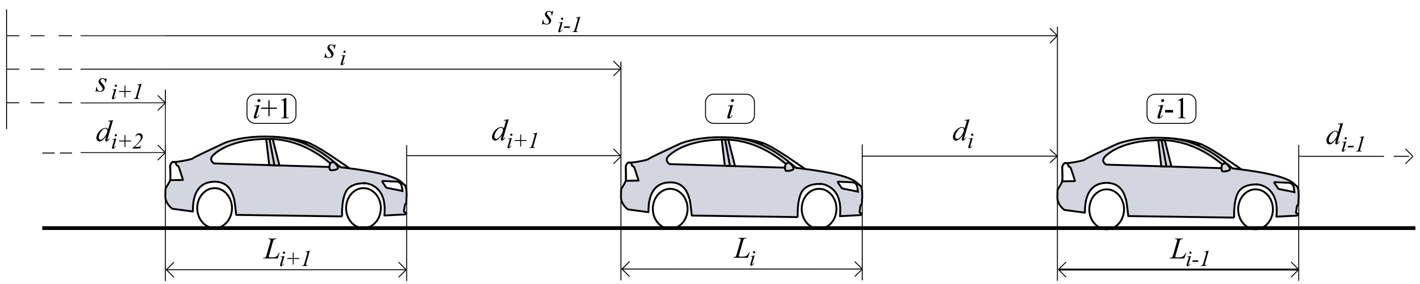
- A local controller will make different cars behave very similarly.
- Large platoons must be similar to infinite platoons. (Folk theorem)

2 Model



Standard mondeling gives the following model for the i'th car

$$\frac{d}{dt}\begin{pmatrix} d_i(t) \\ v_i(t) \\ a_i(t) \end{pmatrix} = \begin{pmatrix} v_{i-1}(t) - v_i(t) \\ a_i(t) \\ -\tau^{-1}a_i(t) + \tau^{-1}u_i(t) \end{pmatrix}, \qquad i \in \mathbb{Z}.$$

with d_i the distance, v_i the velocity, a_i the acceleration, and τ a constant.

We want to track a constant distance C_{eq} and constant velocity v_{eq}. So we choose our state to be

$$x_i(t) = \begin{pmatrix} x_{i,1}(t) \\ x_{i,2}(t) \\ x_{i,3}(t) \end{pmatrix} = \begin{pmatrix} d_i(t) - C_{eq} \\ v_i(t) - v_{eq} \\ a_i(t) \end{pmatrix}$$

With this state the model becomes

$$\frac{d}{dt} x_i(t) = \begin{pmatrix} x_{i-1,2}(t) - x_{i,2}(t) \\ x_{i,3}(t) \\ -\tau^{-1} x_{i,3}(t) + \tau^{-1} u_i(t) \end{pmatrix}, \qquad i \in \mathbb{Z}.$$

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To design a control, i.e., find u_i, we apply the z-transform to this infinite set of differential equations.

2.1 z-transform

The bilateral z-transformation transforms the sequence $f := (f_k)_{k=-\infty}^{\infty}$ to the function $\check{f}(z)$, as

$$\check{f}(z) = (\mathcal{Z}(f))(z) = \sum_{k=-\infty}^{\infty} f_k z^{-k}, \quad z \in \partial\mathbb{D},$$

where $\partial\mathbb{D}$ is the unit circle.

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It is well-known that the z-transform is an isometry between $\ell^2(\mathbb{Z};\mathbb{C}^n)$ (square summable sequences) and $L^2(\partial\mathbb{D};\mathbb{C}^n)$, i.e.,

$$\sum_{k=-\infty}^{\infty} \|f_k\|^2 = \frac{1}{2\pi}\int_0^{2\pi} \|\check{f}(e^{i\theta})\|^2 d\theta.$$

Furthermore, the z-transform has the following nice property.

Lemma

If $g := (g_k)_{k=-\infty}^{\infty}$ is the shifted version of f, i.e., $g_k = f_{k-1}$, $k \in \mathbb{Z}$, then

$$\check{g}(z) = z^{-1}\check{f}(z), \quad z \in \partial\mathbb{D}.$$

2.2 Transformed system

Applying the z-transform to our system it becomes

$$\frac{d\check{x}}{dt}(z,t) = \begin{pmatrix} 0 & z^{-1}-1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau^{-1} \end{pmatrix} \check{x}(z,t) + \begin{pmatrix} 0 \\ 0 \\ \tau^{-1} \end{pmatrix} \check{u}(z,t)$$

$$= \check{A}(z)\check{x}(z,t) + \check{B}(z)\check{u}(z,t).$$

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The multiplication operators $\check{A}$ and $\check{B}$ are bounded linear operators from X to X. For this transformed system we study the control problems.

3 Stability and Stabilization

Definition

Consider the differential equation for $z \in \partial\mathbb{D}$ and $t \geq 0$

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Can we characterize these properties point-wise?

Assume that $z \in \partial\mathbb{D} \mapsto \check{A}(z) \in \mathbb{C}^{n\times n}$ is continuous.

Theorem

The differential equation $\frac{d}{dt}\check{x}(t) = \check{A}\check{x}(t)$ is exponentially stable if and only if $\check{A}(z)$ is Hurwitz for all $z \in \partial\mathbb{D}$.

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Furthermore, it is strongly/asymptotically stable if and only if

- **For almost every $z \in \partial\mathbb{D}$ the (matrix) exponential $\left(e^{\check{A}(z)t}\right)_{t\geq 0}$ is exponentially stable, and**
- $\operatorname{ess\,sup}_{t\geq 0, z\in\partial\mathbb{D}} \left\| e^{\check{A}(z)t} \right\| < \infty$.

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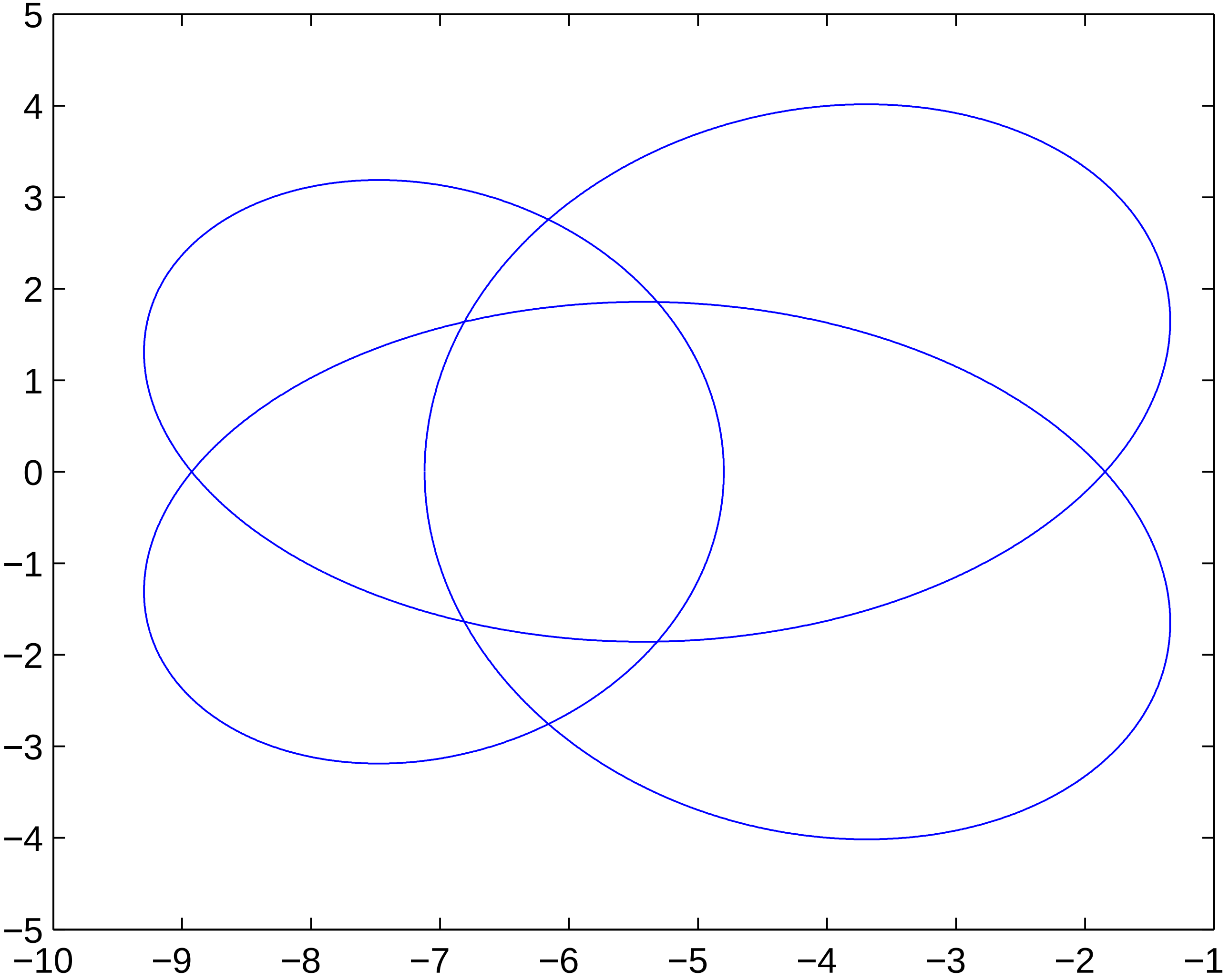
The second item is hard to check, but there are sufficient conditions.

The spectrum of $\breve{A}$ typically looks something like this.



3.1 Exponential stabilizability

Motivated by our platoon model we consider the differential equation for $z \in \partial\mathbb{D}$ and $t \geq 0$

$$\frac{d\check{x}}{dt}(z,t) = \check{A}(z)\check{x}(z,t) + \check{B}(z)\check{u}(z,t), \quad \check{x}(z,0) = \check{x}_0(z).$$

Here $\check{u}$ is the control signal. This we denote as the system $\Sigma(\check{A}, \check{B}, -)$.

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We assume that $\check{A}(\cdot)$ and $\check{B}(\cdot)$ are continuous functions on the unit circle $\partial\mathbb{D}$.

Definition

The system $\Sigma(\check{A}, \check{B}, -)$ is exponentially stabilizable if there exists an $\check{F}$ such that $\check{A} + \check{B}\check{F}$ is exponentially stable.

Hence the control $\check{u}(z,t) = \check{F}(z)\check{x}(z,t)$ makes the system exponentially stable.

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Theorem

The system $\Sigma(\check{A}, \check{B}, -)$ is exponentially stabilizable if and only if for all $z \in \partial\mathbb{D}$ the finite-dimensional system $\Sigma(\check{A}(z), \check{B}(z), -)$ is exponentially stabilizable.

Example

For our system we have that

$$\check{A}(z) = \begin{pmatrix} 0 & z^{-1}-1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau^{-1} \end{pmatrix}, \quad \check{B}(z) = \begin{pmatrix} 0 \\ 0 \\ \tau^{-1} \end{pmatrix}.$$

For $z = 1$ not exponentially stabilizable (always eigenvalue zero), and so $\Sigma(\check{A}, \check{B}, -)$ is not exponentially stabilizable.

4 Linear Quadratic Optimal Control

Consider the system $\Sigma(\check{A}, \check{B}, \check{C})$

$$\begin{aligned} \frac{d\check{x}}{dt}(t) &= \check{A}\check{x}(t) + \check{B}\check{u}(t), \quad \check{x}(0) = \check{x}_0 \\ \check{y}(t) &= \check{C}\check{x}(t), \end{aligned}$$

with $\check{B} \in \mathcal{L}(U, X)$ and $\check{C} \in \mathcal{L}(X, Y)$, where U and Y are Hilbert spaces.

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$$J(\check{x}_0, \check{u}) = \int_0^\infty \|\check{y}(t)\|^2 + \|\check{u}(t)\|^2 dt.$$

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The following general theorem holds:

Theorem

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- **The system is optimizable, i.e., for every $\check{x}_0 \in X$ there exists an input $\check{u}$ such that $J(\check{x}_0, \check{u}) < \infty$.**

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- **For every $\check{x}_0 \in X$ there exists an (unique) input $\check{u}^{\mathrm{opt}}$ such that $J(\check{x}_0, \check{u}^{\mathrm{opt}}) \leq J(\check{x}_0, \check{u})$ for all $\check{u}$.**

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- **There exists a non-negative solution to the ARE:**
  $\check{P} \in \mathcal{L}(X)$, $\check{P} \geq 0$ and
  $$\check{A}^*\check{P} + \check{P}\check{A} - \check{P}\check{B}\check{B}^*\check{P} + \check{C}^*\check{C} = 0.$$

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If one of the above holds, then $\check{u}^{\mathrm{opt}} = -\check{B}^*\check{P}\check{x}$.

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Theorem

For the system $\Sigma(\check{A}, \check{B}, \check{C})$ with cost-function $J(\check{x}_0, \check{u})$ the following are equivalent:

1. **The system is optimizable;**
2. **For almost all $z \in \partial\mathbb{D}$, the Algebraic Riccati Equation (ARE)**

$$\check{A}^*(z)\check{P}(z) + \check{P}(z)\check{A}(z) - \check{P}(z)\check{B}(z)\check{B}^*(z)\check{P}(z) + \check{C}^*(z)\check{C}(z) = 0$$

   possesses a non-negative solution $\check{P}(z)$ and $\mathrm{ess\,sup}_{z\in\partial\mathbb{D}}\|\check{P}(z)\| < \infty$.

The condition $\operatorname{ess\,sup}_{z\in\partial\mathbb{D}}\|\check{P}(z)\| < \infty$ does not necessarily hold even when $\Sigma(\check{A}(z), \check{B}(z), \check{C}(z))$ is optimizable for all $z \in \partial\mathbb{D}$.

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Example

Consider the following system

$$\check{A}(z) = 1 - z, \quad \check{B}(z) = (-1 + z)^2, \quad \check{C}(z) = -1 + z.$$

Since for $z \in \partial\mathbb{D}$, $z \neq 1$, the real part of $\check{B}(z)$ is non-zero, for such z the (finite-dimensional) system is exponentially stabilizable and hence optimizable.

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For $z = 1$, $\Sigma(\check{A}(z), \check{B}(z), \check{C}(z)) = \Sigma(0, 0, 0)$. This implies that $J(\check{x}_0(0), 0) = 0$, and hence the system is optimizable for all z.

Solving the ARE gives with $z = e^{i\theta}, \theta \in [0, 2\pi)$

$$\check{P}(z) = \frac{1 - \cos(\theta) + \sqrt{(1-\cos(\theta))^2 + 8(1-\cos(\theta))^3}}{(2 - 2\cos(\theta))^2} = \frac{1 + \sqrt{1 + 8(1-\cos(\theta))}}{4(1-\cos(\theta))}$$

Since now $\check{P}(z)$ is unbounded on the unit circle, the optimal control problem is not solvable for this system.

Remark

- If $\check{C} = I$, then optimizability is equivalent to exponential stabilizability.
- Normally, the optimizability is checked by checking exponential stabilizability.
- Our system is not exponentially stabilizable. Hence if we want to solve the optimal control problem we have to choose the $\check{C}$ carefully. Moreover, point-wise optimizability is necessary.

Lemma

Consider the system $\Sigma(\check{A}, \check{B}, \check{C})$ with cost functional $J(\check{x}_0, \check{u})$. Furthermore, assume that $\check{A}$, $\check{B}$, and $\check{C}$ are continuously depending on $z \in \partial\mathbb{D}$. If the system is optimizable, then for every $z \in \partial\mathbb{D}$, the finite-dimensional system $\Sigma(\check{A}(z), \check{B}(z), \check{C}(z))$ is optimizable.

4.1 Our platoon model

For our system we have that

$$\check{A}(z) = \begin{pmatrix} 0 & z^{-1} - 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau^{-1} \end{pmatrix}, \quad \check{B}(z) = \begin{pmatrix} 0 \\ 0 \\ \tau^{-1} \end{pmatrix}.$$

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By (analytical) computations, we could prove that the system is optimizable for

$$\check{C}(z) = \begin{pmatrix} 1 - z & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Furthermore, the optimal control, i.e.,

$$\breve{u}^{\text{opt}}(z,t) = -\breve{B}(z)^{*}\breve{P}(z)\breve{x}^{\text{opt}}(z,t)$$

is strongly/asymptotically stabilizing the system.

4.2 Another output

For our system with

$$\check{C}(z) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

we could not prove analytically that it is optimizable. However, numerical calculations showed it.

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Furthermore, the optimal control is strongly/asymptotically stabilizing the system. Approximation in “vehicle-index” gives similar results (4 look-ahead/4 look-behind).

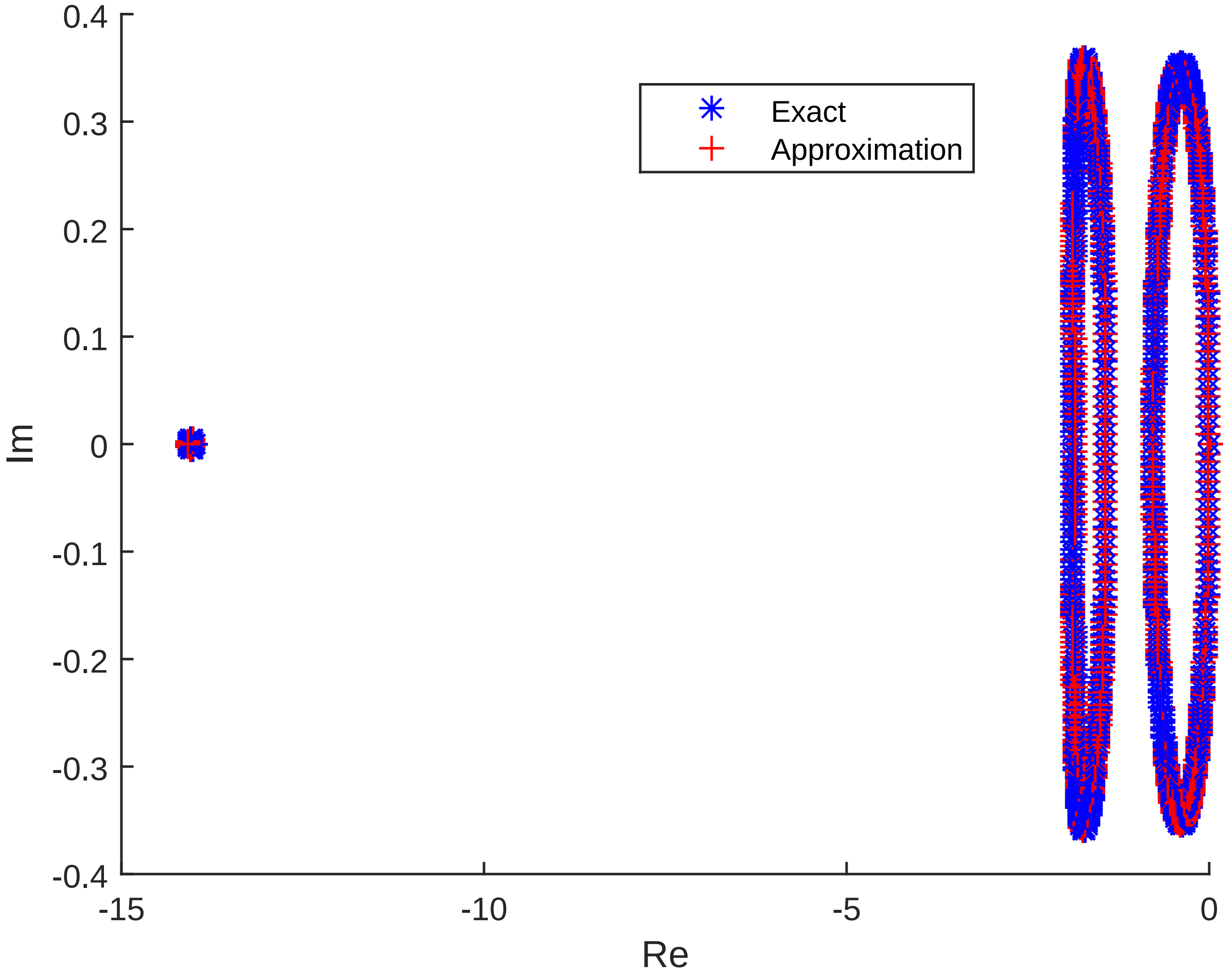

Figure 1: Spectrum of the closed loop system

5 Open problems

- **Find easy checkable (sufficient) conditions for optimizability.**
- **Find easy checkable (sufficient) conditions for which the optimal control strongly stabilizes the system.**
- **Find easy checkable (sufficient) conditions for (strongly) stabilizability, i.e., when does there exists a $\check{F}$ such that differential associated to $\check{A} + \check{B}\check{F}$ is strongly stable.**
- **What is stability?**

5.1 Stable or not?

Consider the following example

$$\frac{dx_i}{dt}(t) = -x_i(t) + x_{i-1}(t), \qquad i \in \mathbb{Z} \tag{1}$$

After z-transformation this becomes

$$\frac{d\check{x}}{dt}(z,t) = (-1 + z^{-1})\check{x}(z,t), \qquad z \in \partial\mathbb{D}. \tag{2}$$

It is not hard to see that this is strongly/asymptotically stable.

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Define now $v(t) = \sum_{i\in\mathbb{Z}} x_i(t)$, then by the differential equation (1) we find

$$\dot{v}(t) = 0.$$

So not stable.

Question?