A Constraint Method in Nonlinear Multi-Objective Optimization

Gabriele Eichfelder

Institute of Applied Mathematics, University of Erlangen-Nuremberg, Martensstr. 3, D-91058 Erlangen, Germany Gabriele.Eichfelder@am.uni-erlangen.de

Summary. We present a new method for generating a concise and representative approximation of the (weakly) efficient set of a nonlinear multi-objective optimization problem. For the parameter dependent ε-constraint scalarization an algorithm is given which allows an adaptive controlling of the parameters –the upper bounds– based on sensitivity results such that equidistant approximation points are generated. The proposed method is applied to a variety of test-problems.

1 Introduction

In many areas like economics, engineering, environmental issues or medicine the complex optimization problems cannot be described adequately by only one objective function. As a consequence multi-objective optimization which investigates optimization problems like

$$\begin{array}{l} \min f(x) = (f_1(x), \ldots, f_m(x))^\top \\ \text{s. t. } x \in \Omega \subset \mathbb{R}^n, \end{array} \tag{1}$$

with a function $f: \mathbb{R}^n \to \mathbb{R}^m$ with $m \in \mathbb{N}$, $m \geq 2$, and $\Omega \subset \mathbb{R}^n$ a closed set, is getting more and more important. For an introduction to multi-objective optimization see the books by Chankong and Haimes, [3], Ehrgott, [6], Hwang and Masud, [14], Jahn, [16], Miettinen, [21], Sawaragi et al., [24], and Steuer, [27]. Further see the survey papers by Hillermaier and Jahn, [13], and by Ruzika and Wiecek, [23], with a focus on solution methods.

In general there is not only one best solution which minimizes all objective functions at the same time and the solution set, called efficient set, is very large. Especially in engineering tasks information about the whole efficient set is important. Besides having the whole solution set available the decision maker gets a useful insight in the problem structure. Consequently our aim is to generate a representative approximation of this set. The importance of this aim is also pointed out in many other works like e. g. in [5, 9, 20, 26].