Tutorial T04

Digital Filters, Filter Banks and Their Design for Audio Applications

With Python Examples

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Linear Filters, Convolution

Examples of linear filters: low pass filters, band pass filters

Linearity:

- Same result if we **add** two signals before or after the filter
- Same result if we apply **gain or attenuation** before or after the filter

A digital audio signal can be seen as a **sum of its samples** $x(n)$.

Each sample of a signal can be seen as an impulse of the sample value, producing the filters "Impulse Response" $h(n)$ multiplied by the sample value.

Hence the sum of all samples at the filter input results in the "**convolution**" of the signal $x(n)$ with the filter impulse response as output $y(n)$,

$$y(n) = \sum_{m}^{L-1} x(n-m) \cdot h(m) = x(n) * h(n)$$

"$*$" is a short notation for convolution.

We see: from **earlier signal samples later impulse response samples** are added.

Observe: In effect this is a **sliding weighted sum of past samples**.

In Python this is implemented in the function `scipy.signal.lfilter`.

The z-Transform

- If these samples where coefficients of polynomials, then this convolution sum would be the result of the multiplication of the polynomials.
- This leads to the "**z-Transform**", which converts samples $x(n)$ into polynomials $X(z)$

$$X(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$$

- The **convolution** in the time domain then turns into a **multiplication** in the z-domain,
- This is an important and convenient simplification for **filter design and analysis**,

$$x(n) * h(n) \rightarrow X(z) \cdot H(z)$$

- Imagine the z-domain as a **generalized frequency domain**.
- The usual frequency domain with normalized frequency ω (with π corresponding to the Nyquist frequency) is obtained by setting

$$z = e^{j\omega}$$

Example Application: Lowpass Filtering for Sampling Rate Change

- If we want to reduce the sampling rate of an audio signal, we first need to lowpass filter it
- to avoid **"Aliasing"** artefacts
- Aliasing results from audio components at frequencies above the new Nyquist Frequency (half the new sampling frequency)
- **Example:** We reduce the sampling rate to **half** the original sampling rate
- then the lowpass filter should only pass the **lower half** of the original frequency range.

Example for Aliasing

- To show the **effect of aliasing**, we take a chirp or sweep signal at 16 kHz original sampling rate
- It has a sinusoid which "sweeps" from 100 Hz up to its Nyquist frequency of 8 kHz
- Then we **downsample** it by a factor of 2 down to 8 kHz sampling rate, **without** lowpass filtering
- And listen to the original and downsampled version.

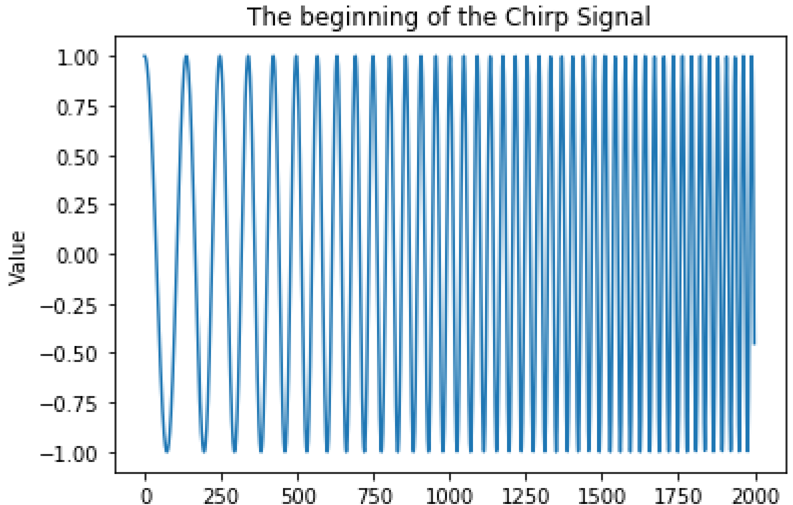
Chirp Audio Signal for Testing

In [1]:

```
from scipy.signal import chirp
import scipy.io.wavfile as wav
import numpy as np
#%matplotlib notebook
import matplotlib.pyplot as plt
from IPython.display import Audio

samplerate=16000  #sampling frequency in Hz
t1=2.0 #End time
f0=100 #start frequency in Hz
f1=8000 #end frequency in Hz
t=np.linspace(0,t1,int(t1*samplerate)) #sample times
chirpsig=chirp(t, f0, t1, f1)
wav.write("chirp.wav",samplerate,np.int16(chirpsig*2**14))
```

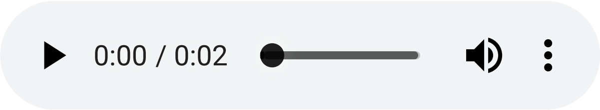
In [2]:
```python
plt.plot(chirpsig[0:2000])
plt.title('The beginning of the Chirp Signal')
plt.xlabel('Sample Index')
plt.ylabel('Value')
plt.show()
```



The Chirp Sound

In [3]:
```
Audio("chirp.wav")
```

Out[3]:



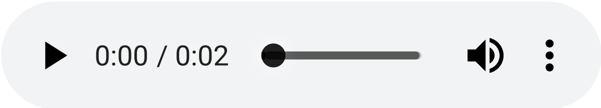
Downsampling

- Next we downsample it by a factor of 2 without lowpass filtering.
- We do that be keeping only every second sample
- The argument [::2] means: from beginning to end with index steps of 2.

In [4]:

```
chirpsigdownsampled=chirpsig[::2]
wav.write("chirpdownsampled.wav",samplerate/2,np.int16(chirpsigdownsampled*2**14))
Audio("chirpdownsampled.wav")
```

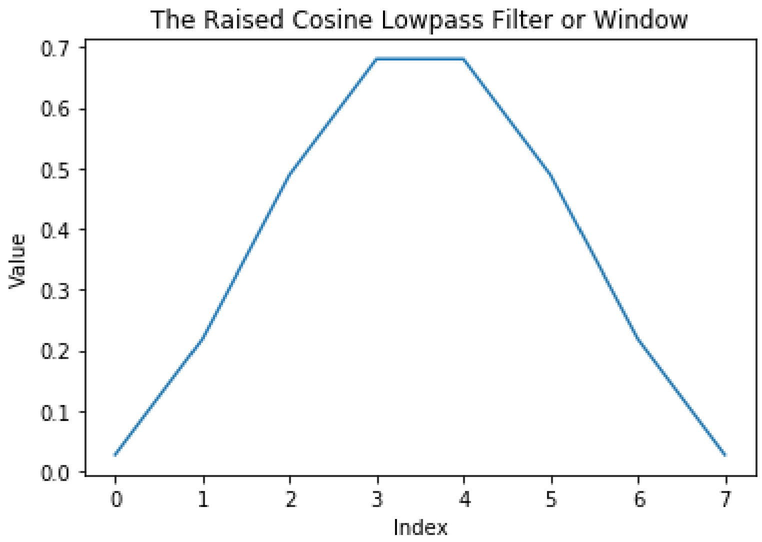
Out[4]:



Aliasing, Filtering

- Observe: We can hear "artificial" frequencies, the **aliasing**
- This shows that we need a lowpass filter to **suppress the higher frequencies** in the original which cause the aliasing
- A very simple lowpass filter is the so-called "raised cosine" function,
- in our example with N=8 samples or coefficients.

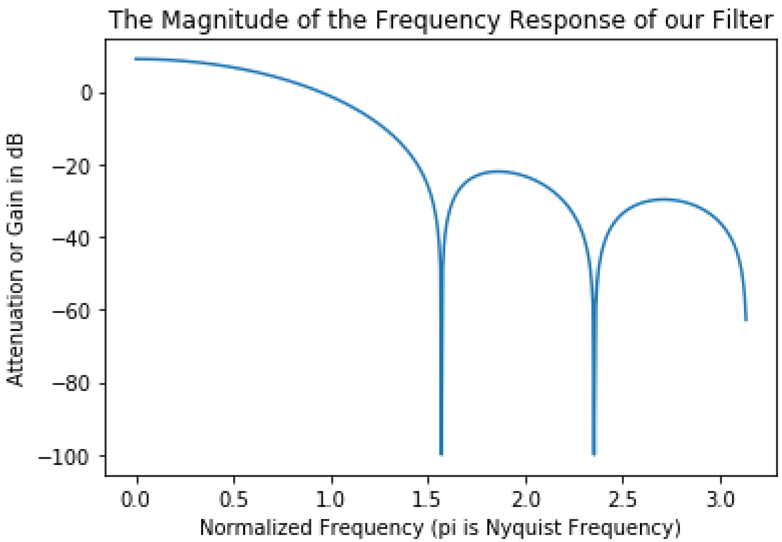
In [5]:
```
N=8 #length of filter
rc=(1-np.cos(2*np.pi/N*np.arange(0.5,N)))/np.sqrt(N) #raised cosine
plt.plot(rc)
plt.title('The Raised Cosine Lowpass Filter or Window')
plt.xlabel('Index')
plt.ylabel('Value')
plt.show()
```



Frequency Response

- Observe that this filter is **symmetric around its center**, which results in the **linear phase** property.
- This means all frequencies have the **same signal delay**.
- To see how much **attenuation** this filter provides, we plot its frequency response
- We use `scipy.signal.freqz` for it

In [6]:
```
import scipy.signal as sp
w,H=sp.freqz(rc)
plt.plot(w, 20*np.log10(abs(H)+1e-5))
plt.title('The Magnitude of the Frequency Response of our Filter')
plt.xlabel('Normalized Frequency (pi is Nyquist Frequency)')
plt.ylabel('Attenuation or Gain in dB')
plt.show()
```



- Observe: our desired stop band is above normalized frequency 1.5
- We only obtain roughly 30 dB attenuation, which is not much for our application.
- We can test it by applying it to our downsampling example,
- with the function `scipy.signal.lfilter` for the **lowpass filtering before** downsampling.

The Lowpass Filtered and Downsampled Chirp (with Raised Cosine)

In [7]:

```
chirplowpass=sp.lfilter(rc,1,chirpsig)
chirplowpasssampled=chirplowpass[::2]
wav.write("chirplowpasssampled.wav",samplerate/2,np.int16(chirplowpasssampled*2**13))
Audio("chirplowpasssampled.wav")
```

Out[7]:

0:00 / 0:02

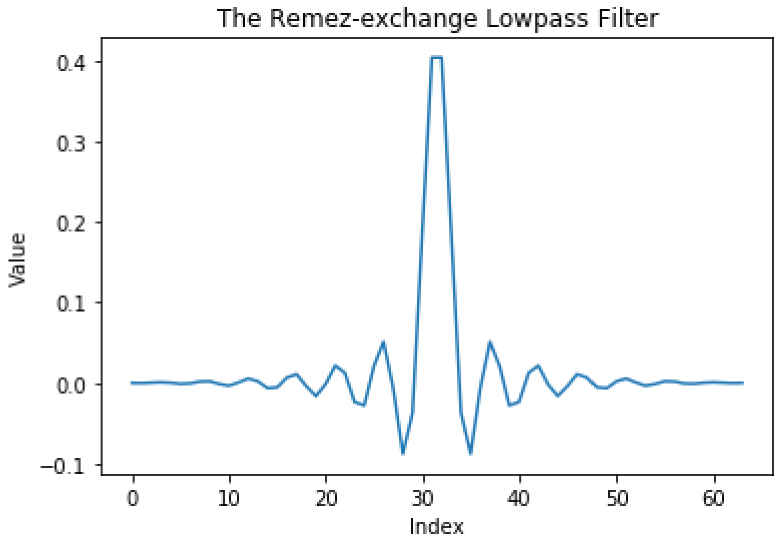
Improved Filter

- Observe: The aliasing was still clearly audible
- To better supress the aliasing, we need **more stopband attenuation**
- We try the "Remez-exchange" algorithm `scipy.signal.remez`.
- Now also with **more coefficients** to obtain more attenuation: N=64

The Remez Lowpass Filter

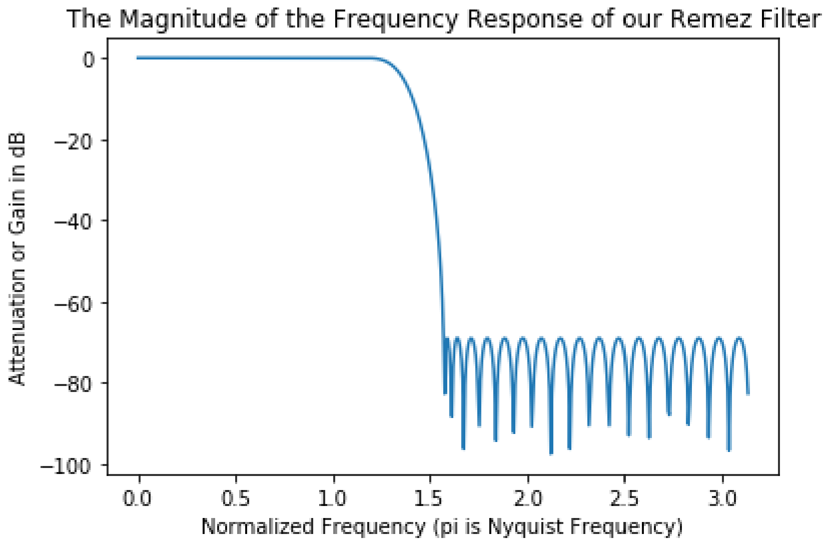
In [8]:

```
# Usage: remez(numtaps, bands, desired, weight=None, Hz=1)
#Passband: 0, 3000
#Stopband: 4000, 8000
#Desired band output: 1, 0
#Sampling frequency: Hz=16000
lpremez=sp.remez(64,[0, 3000, 4000, 8000],[1,0],Hz=16000)
plt.plot(lpremez)
plt.title('The Remez-exchange Lowpass Filter')
plt.xlabel('Index')
plt.ylabel('Value')
plt.show()
```



In [9]:

```
w,H=sp.freqz(lpremez)
plt.plot(w, 20*np.log10(abs(H)+1e-5))
plt.title('The Magnitude of the Frequency Response of our Remez Filter')
plt.xlabel('Normalized Frequency (pi is Nyquist Frequency)')
plt.ylabel('Attenuation or Gain in dB')
plt.show()
```



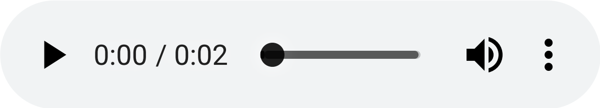
- Observe: We now have much more stopband attenuation, about 70 dB!
- Also the passband is much more flat, which is desirable to not change those frequency components.

- We can now test it with our downsampling example.

In [10]:

```
chirplowpass=sp.lfilter(lpremez,1,chirpsig)
chirplowpasssampled=chirplowpass[::2]
wav.write("chirplowpassremezsampled.wav",samplerate/2,np.int16(chirplowpasssampled*2**13))
Audio("chirplowpassremezsampled.wav")
```

Out[10]:



- Observe: The **aliasing is now indeed completely gone!**

Upsampling

- For upsampling we get the reverse order
- **First upsampling by inserting a 0 after each sample, then lowpass filtering**
- That we need lowpass filtering also after upsampling shows the following Python example

Upsampling

In [11]:
```
chirpupsampled=np.zeros(2*len(chirplowpasssampled))
chirpupsampled[::2]=chirplowpasssampled
wav.write("chirpupsampled.wav",samplerate,np.int16(chirpupsampled*2**13))
Audio("chirpupsampled.wav")
```

Out[11]:

0:00 / 0:02

- Observe: There is **again aliasing**, heard as a "reverse" chirp
- Hence we need again our lowpass filter to supress the aliasing
- Observe: both times the **filtering** happens **at the higher sampling rate**

In [12]:
```
chirpupsampledlp=sp.lfilter(lpremez,1,chirpupsampled)
wav.write("chirpupsampledlp.wav",samplerate,np.int16(chirpupsampledlp*2**13))
Audio("chirpupsampledlp.wav")
```

Out[12]:

0:00 / 0:02

Minimum Phase Filters

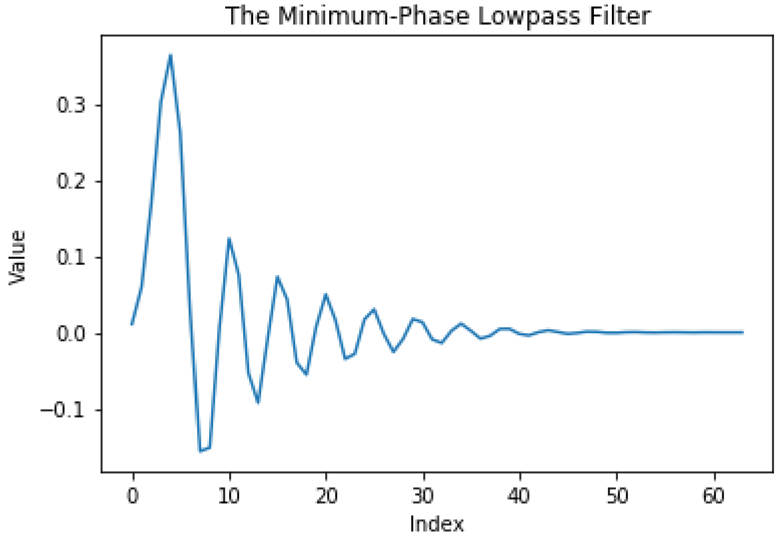
- Our better filter also became **longer**
- If the this impulse response becomes on the order of a few Milliseconds, there will be a danger of **"pre-echos"**
- This happens if the audio signal consists of a short attack, like from castanets
- The filter "smears" such a pulse or attack
- Particularly a tail before the attack can be easily picked up by the ear
- a tail after the attack is more likely masked by the temporal masking effects of the ear
- Hence a **non-symmetric** impulse response would be useful,
- with a shorter tail before the main lobe and a longer tail after it

Minimum Phase Filters

- This leads to **"minimum-phase"** filters.
- In Python, we have the function `scipy.signal.minimum_phase`.
- It approximates the **square root** of the magnitude frequency response of a symmetric impulse response by a minimum-phase version.
- Hence we need to input the **square of the magnitude**,
- which we obtain by **convolving the remez filter with itself**.

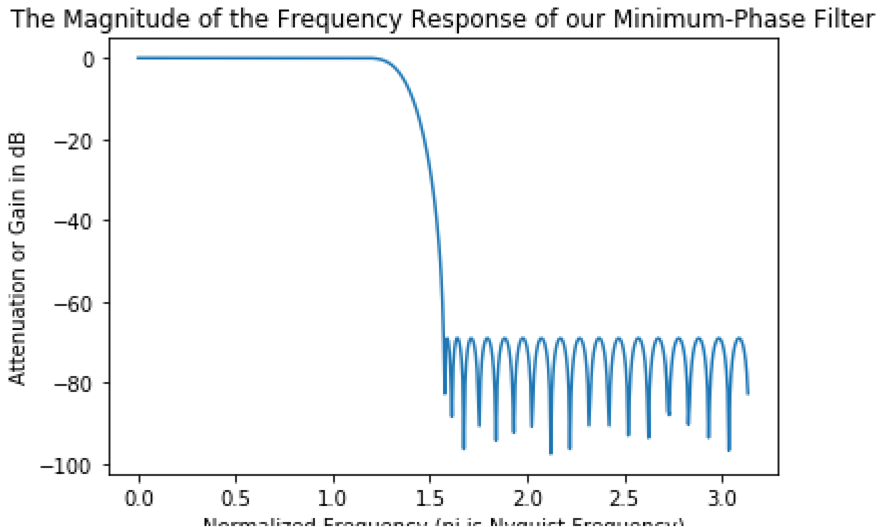
In [13]:

```
lp_minphase=sp.minimum_phase(sp.convolve(lpremez,lpremez))
plt.plot(lp_minphase)
plt.title('The Minimum-Phase Lowpass Filter')
plt.xlabel('Index')
plt.ylabel('Value')
plt.show()
```



- Observe: The main lobe is indeed now in the beginning,
- we have mainly a **tail after** it
- The filter has a **signal delay** which can be estimated as the duration from the beginning to the main peak
- Hence this filter also has **lower delay**
- We can now compare the magnitude of the frequency response.

In [14]:
```
w,H=sp.freqz(lp_minphase)
plt.plot(w, 20*np.log10(abs(H)+1e-5))
plt.title('The Magnitude of the Frequency Response of our Minimum-Phase Filter'
)
plt.xlabel('Normalized Frequency (pi is Nyquist Frequency)')
plt.ylabel('Attenuation or Gain in dB')
plt.show()
```

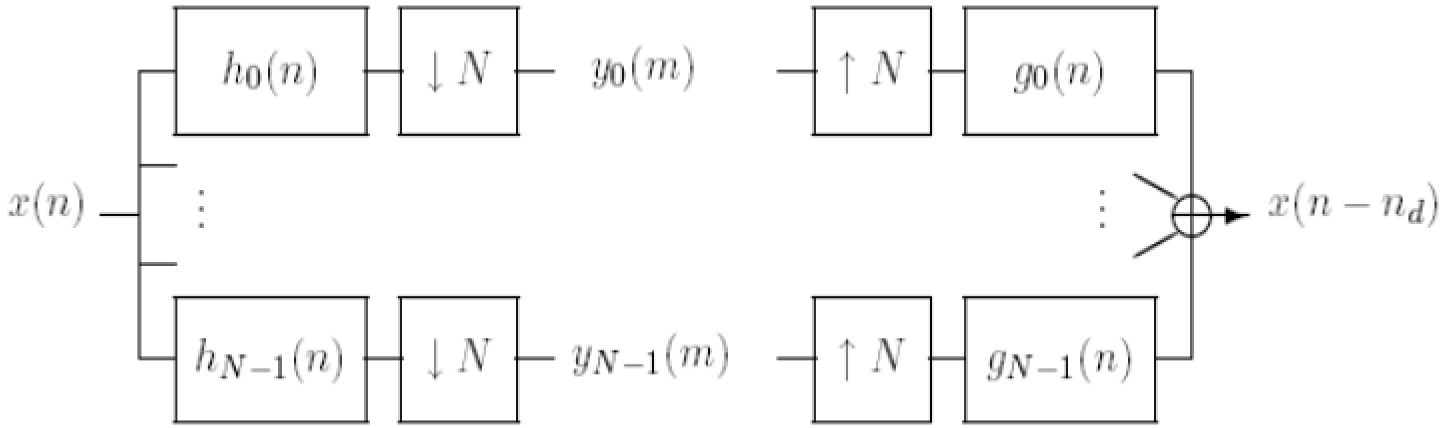


Filter Banks

- We now have a signal at a lower sampling rate, but half of the original **frequecy ranges gone**.
- To avoid this, we would need **another filter** with the same bandwidth, but keeping the remaining frequencies.
- In general, it we **downsample by a factor of N, we need N bandpass filters** to cover the original spectrum.
- This leads to "**critical sampled filter banks**".
- Together the filters cover the **entire original frequency range** (no lost frequencies).
- For the synthesis we now have the **addition of all of our subbands**
- This gives us the possibility of **cancelling all the alias** that the filters did not supress!

Filter Bank Block Diagram

- Left hand side: analsysis filter bank and downsampling
- Right hand side: upsampling and Synthesis filter
- This gives us an invertible "time/frequency" representation with the subband samples $y_k(m)$
- k: subband index (frequency), m: donwsampled time index



$x(n)$
$h_0(n)$
$\downarrow N$
$y_0(m)$
$\uparrow N$
$g_0(n)$
$h_{N-1}(n)$
$\downarrow N$
$y_{N-1}(m)$
$\uparrow N$
$g_{N-1}(n)$
$x(n-n_d)$

Filter Bank Applications

- Filter banks are widely used for **audio coding**
- In the **encoder**, the audio signal is feed into the **analysis filter bank**,
- the subbands are then quantized and encoded according to a **"psycho-acoustic" model**,
- this models the **ears sensitivities** to noise and artefacts at different frequencies
- In this way we **minimize the bit-rate** while keeping artifacts and noise mostly inaudible.

Filter Bank Applications

- In effect we shape the unavoidable quantization noise such that it it **below the "masking threshold"** of the ear, hence it is inaudible.
- This is called **"perceptual coding"**.
- The **decoder** uses the **synthesis filter bank** to reconstruct the audio signal, with hopefully inaudible distortions
- The signal should **sound** the same, but would **look** quite different in its waveform, or even its spectrum

The MDCT Filter Bank

- The **"Modified Discrete Cosine Transform"** (MDCT) filter bank is used for instance in the MPEG audio coding standards
- It consists of **N bandpass filters**, commonly $N = 1024$ or $N = 128$.
- Each has a bandwidth of $\mathbf{1/N}$ **th of the original frequency range**.
- Its filters are obtained by **multiplying cosine functions by a "window function" of length $\mathbf{2N}$**.
- The simplest window function is the **"sine window"**.
- It has **"Perfect Reconstruction"**
- Meaning: If there is no change to the subband samples, the reconstructed audio signal is identical to the input, except for its **"sytem delay"** of $2N - 1$ samples.

The MDCT Filter Bank

- Here we see the **multiplication of the cosine functions by window functions** $h(n)$ and $g(n)$ resp.,
    - The impulse responses for the bandpasses of the "**analysis**" filter bank, which is doing the downsampling, are
    $$h_k(n) = -h(n) \cdot \sqrt{\frac{2}{N}} \cos\left(\frac{\pi}{N}\left(k+\frac{1}{2}\right)\left(n \pm \frac{N}{2} + \frac{1}{2}\right)\right)$$
    for the subbands $k = 0, \ldots, N-1$ and the time index $n = 0, \ldots, 2N-1$
    - The impulse responses for the bandpasses of the "**synthesis**" filter bank, which is doing the upsampling, are
    $$g_k(n) = g(n) \cdot \sqrt{\frac{2}{N}} \cos\left(\frac{\pi}{N}\left(k+\frac{1}{2}\right)\left(n \mp \frac{N}{2} + \frac{1}{2}\right)\right)$$

The Sine Window

- For the MDCT filter bank, this happens by **limiting the filter length** to $2N$
- and by **imposing suitable conditions on the window functions** $h(n)$ and $g(n)$
- A particularly simple and often used window which fulfills these condition is the "sine window",

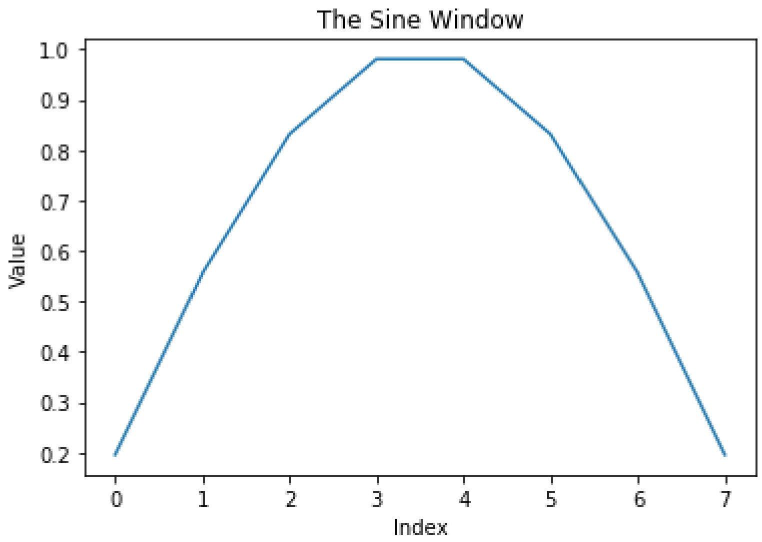
$$h(n) = g(n) = \sin\left(\frac{\pi}{2N}(n+0.5)\right)$$

for $n = 0, \ldots, 2N-1$.

- This is not a great filter, it has a **quality similar to our raised cosine window**
- but here it **cancels the aliasing!**

In [15]:

```
N=4
h=np.sin(np.pi/(2*N)*np.arange(0.5,2*N))
plt.plot(h)
plt.title('The Sine Window')
plt.xlabel('Index')
plt.ylabel('Value')
plt.show()
```



Alternative Windows

- An alternative Window which is used in MPEG coders is the "Kaiser Bessel Derived" (KBD) Window
- It also fulfills the alias cancellation conditions
- and has an optimized frequency response

Audio Coding Applications

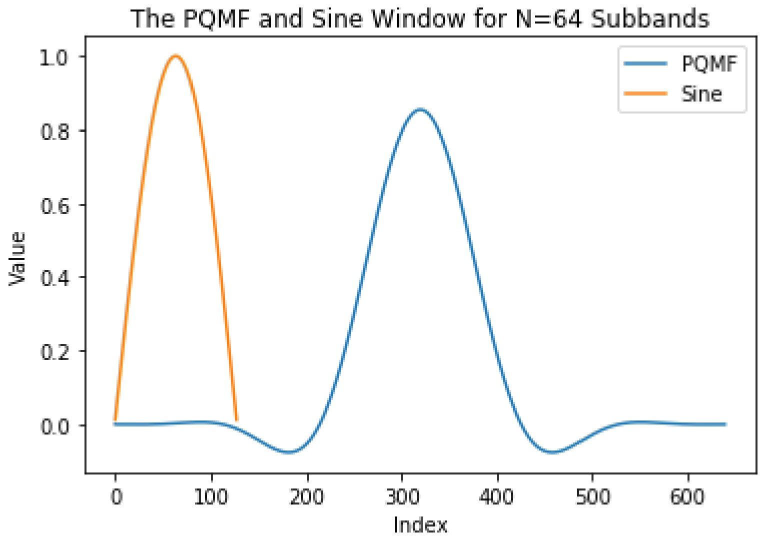
- The MDCT is used to **quantize and encode the time/frequency representation** $y_k(m)$
- The ear has different sensitivities for different frequencies and times, with **psycho-acoustic frequency and temporal maskings**
- This also depends on the signal itself
- The quantization step sizes are controlled by a psycho-acoustical model
- In effect the time/frequency representation is used for **time and frequency adaptive (quantization) noise shaping**
- In this way, ideally the quantization distortions stay inaudible, while minimizing bit-rate

Pseudo Quadrature Mirror Filter Bank (PQMF)

- This type of filter bank is used when very **narrow filter with high attenuation** are desired
- Drawback: **no perfect reconstruction**
- Aliasing cancels only between neighbouring subbands
- Beyond these neighbouring subbands, the stopband attenutation needs to be **high enough to sufficiently supress aliasing**
- Its window functions are obtained by **numerical optimization**, which tries to minimize the reconstructuion error while maximizing the stopband attenuation.

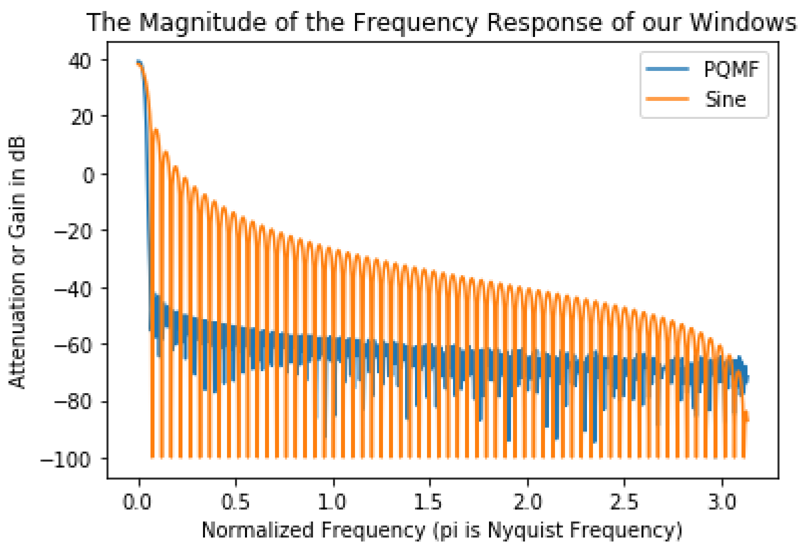
In [16]:

```
qmfwin=np.loadtxt('qmf.dat');
plt.plot(qmfwin)
plt.title('The PQMF and Sine Window for N=64 Subbands')
plt.xlabel('Index')
plt.ylabel('Value')
N=64
hsin=np.sin(np.pi/(2*N)*np.arange(0.5,2*N))
plt.plot(hsin)
plt.legend(('PQMF', 'Sine'))
plt.show()
```



In [17]:

```
w,Hqmf=sp.freqz(qmfwin)
w,Hsin=sp.freqz(hsin)
plt.plot(w, 20*np.log10(abs(Hqmf)+1e-5))
plt.plot(w, 20*np.log10(abs(Hsin)+1e-5))
plt.legend(('PQMF', 'Sine'))
plt.title('The Magnitude of the Frequency Response of our Windows')
plt.xlabel('Normalized Frequency (pi is Nyquist Frequency)')
plt.ylabel('Attenuation or Gain in dB')
plt.show()
```



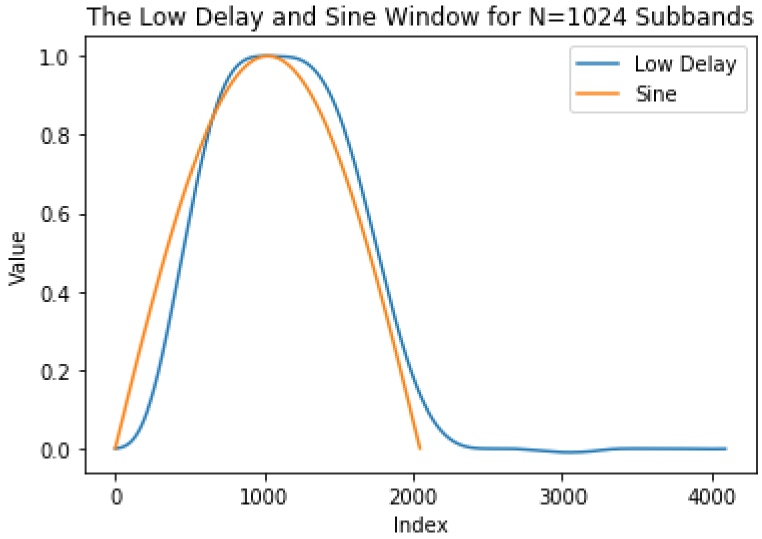
- Observe: The sine window does not really have enough attenuation to sufficiently suppress aliasing
- Hence we really need the **alias cancellation** property of the MDCT filter bank
- This means we **cannot change subbands too much** (like setting neighbouring subbands to zero)
- If we want to have more **substatial changes** over frequency we need to use the **PQMF** for its far better stopband attenuation.
- In MPEG coders this is for instance for spacial surround encoding or parametric high frequency regeneration.

Low Delay Filter Banks

- Similar to minimum-phase filters, we can design low delay filter banks
- They have a **lower system delay** than filter banks with symmetric window functions, with similar stopband attenuation
- or have **better stopband attenuation** at the same system delay through longer window functions
- In the following is an example for N=1024 subbands, where the low delay filter bank has **better stopband attenuation** due to the longer window function.

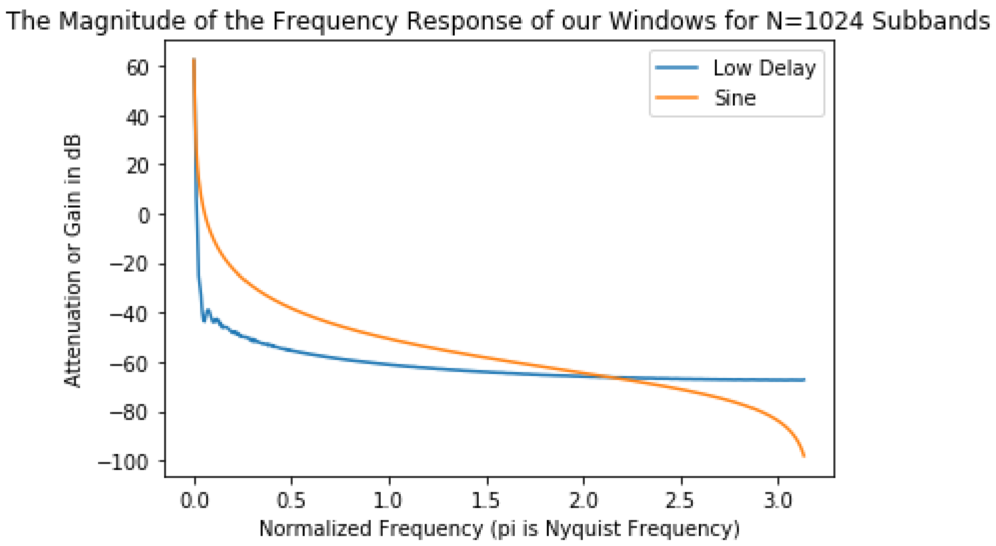
In [18]:

```
ldfbwin=np.loadtxt('h4096t2047d1024bbitc.txt');
plt.plot(-ldfbwin)
plt.title('The Low Delay and Sine Window for N=1024 Subbands')
plt.xlabel('Index')
plt.ylabel('Value')
N=1024
hsin=np.sin(np.pi/(2*N)*np.arange(0.5,2*N))
plt.plot(hsin)
plt.legend(('Low Delay', 'Sine'))
plt.show()
```



In [19]:

```
w,Hldfb=sp.freqz(ldfbwin)
w,Hsin=sp.freqz(hsin)
plt.plot(w, 20*np.log10(abs(Hldfb)+1e-5))
plt.plot(w, 20*np.log10(abs(Hsin)+1e-5))
plt.legend(('Low Delay', 'Sine'))
plt.title('The Magnitude of the Frequency Response of our Windows for N=1024 Su
bbands')
plt.xlabel('Normalized Frequency (pi is Nyquist Frequency)')
plt.ylabel('Attenuation or Gain in dB')
plt.show()
```



Low Delay Filter Banks

- Observe that we indeed obtain a much better stopband attenuation.
- We can also use it to obtain a **lower system delay for real time communications applications** like teleconferencing.
- It is part of the MPEG4 Enhanced Low Delay AAC audio coder (**ELD-AAC**)
- which is part of the i-OS and Android operating systems.

Implementation using a Neural Network Framework

- We can use a Neural Network framwork, like "Keras", because a critically sampled filter bank is a **special case of a one layer convolutional neural network**, with N "nodes", "strides" of size N, and no non-linearity.
- Advantage: it uses the parallel processing for fast implementation, even without a fast implementation like using an FFT.
- This makes a fast implementation simpler.

Github Repository for Filter Bank Implementation

- Implementation using a Fast Fourier Transform:
- `https://github.com/TUIlmenauAMS/FilterBanks_FastPythonImplementation`
- Implementation using the Keras Neural Nework Python library.
- This has the advantage that it works for more generic filter banks,
- and it uses a GPU for fast implementation
- `https://github.com/TUIlmenauAMS/FilterBanks_PythonKerasNeuralNetworkImplemention`

De-Nosing example

- We can use the ability of filter banks for **noise shaping** for a de-nosing example
- If a subband at a certain time **mostly contains noise** (is below a threshold), its signal is **set to zero**.
- If the signal is **above a threshold, it remains unchanged**
- This assumes that we **estimated** the strength of the noise before.
- In our example we take the **chirp signal with added noise** (hence we know the noise in this case)
- We choose uniformly distributed noise, which could result from quantization.

MDCT Spectrogram of our Clean Chirp Signal

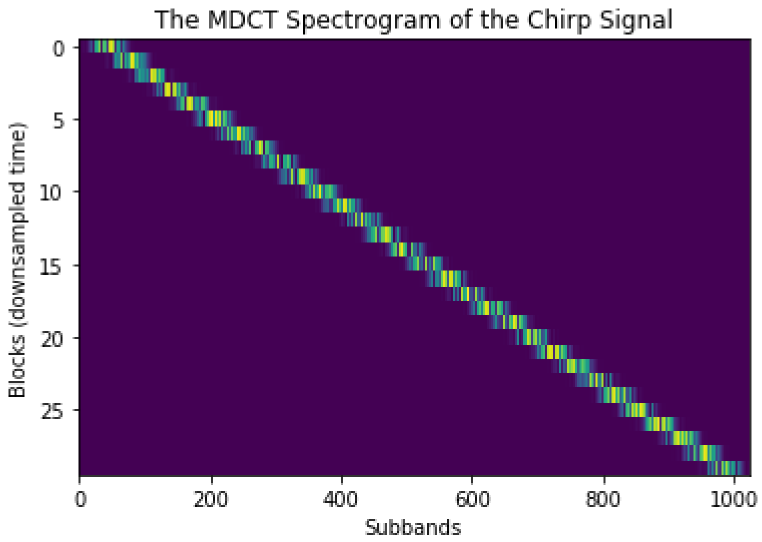
In [35]:

```
from keras_MDCTanalysis import *
N=1024 #Number of filters, stride
filtlen=2048 #Length of filter impulse response
modelana = generate_model_ana(N,filtlen)     # Compile an neural net analysis f
ilter bank
Y=keras_MDCT_ana(chirpsig,modelana)
plt.imshow(abs(Y),aspect='auto')
plt.xlabel('Subbands')
plt.ylabel('Blocks (downsampled time)')
plt.title('The MDCT Spectrogram of the Chirp Signal')
```

```
Initializing MDCT analysis weights
```

Out[35]:

```
Text(0.5,1,u'The MDCT Spectrogram of the Chirp Signal')
```



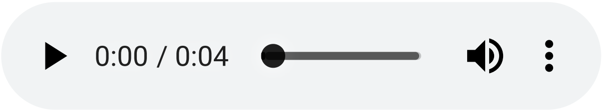
Noisy Chirp Test Example

- We now add white noise to our chirp signal for testing

In [21]:

```
chirpsignoise=chirpsig+ (np.random.rand(len(chirpsig))-0.5)*1.0
wav.write("chirpsignoise.wav",samplerate/2,np.int16(chirpsignoise*2**14))
Audio("chirpsignoise.wav")
```

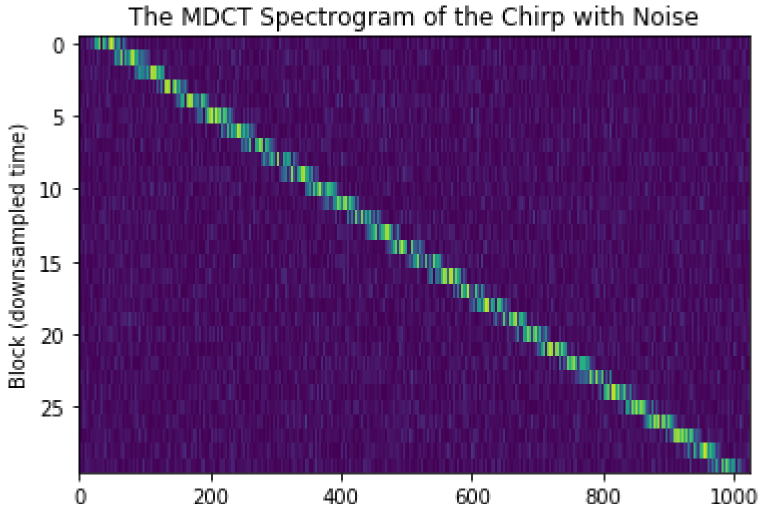
Out[21]:



MDCT Spectrogram of the Noisy Chirp

In [22]:
```
Y=keras_MDCT_ana(chirpsignoise,modelana)
plt.imshow(abs(Y),aspect='auto')
plt.xlabel('Subband')
plt.ylabel('Block (downsampled time)')
plt.title('The MDCT Spectrogram of the Chirp with Noise')
```

Out[22]:
```
Text(0.5,1,u'The MDCT Spectrogram of the Chirp with Noise')
```



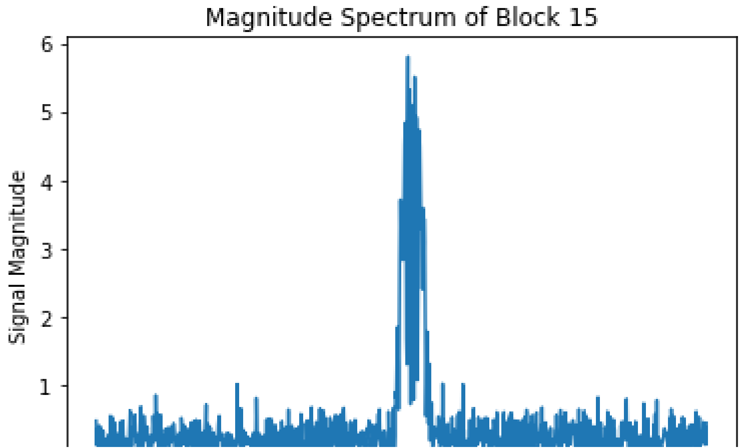
- Observe: the noise is visible as a **"snow" like pattern**.

Noise Estimation

- To **estimate the level of the noise** we plot the magnitude spectrum of block number 15
- Observe: the noise is below magnitude 1, the chirp signal is above.
- We can use this threshold to distinguish between noise and signal.

In [23]:
```
plt.plot(abs(Y[15,:]))
plt.xlabel('Subband')
plt.ylabel('Signal Magnitude')
plt.title('Magnitude Spectrum of Block 15')
```

Out[23]:
```
Text(0.5,1,u'Magnitude Spectrum of Block 15')
```



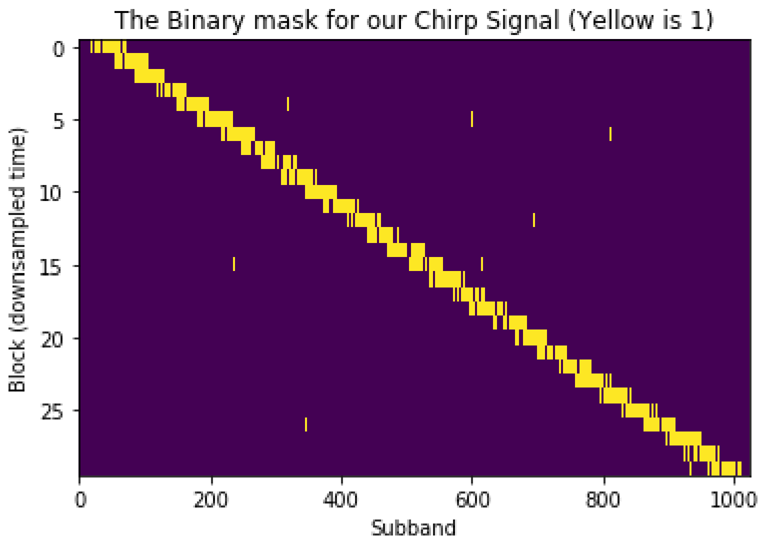
Signal Separation

- Now we can build a **"binary mask"** for our MDCT representation
- With it, we **multiply** time/frequency bin which we classify as noise **with 0, hence discarded**
- Time/frequency bins which we classify as **signal are kept by multiplying it with 1**
- Our binary mask has the same size as our MDCT representation, and has the **1's and 0's at the corresponing positons**
- Such masks are also used to separate signals in **audio source separation**
- We separate the signals by **elementwise multiplication**
- In our case we **separate the signal from the noise**

The Binary Mask

In [24]:
```python
# Create the binary mask:
binarymask=abs(Y)>1.0
plt.imshow(binarymask,aspect='auto')
plt.xlabel('Subband')
plt.ylabel('Block (downsampled time)')
plt.title('The Binary mask for our Chirp Signal (Yellow is 1)')
```

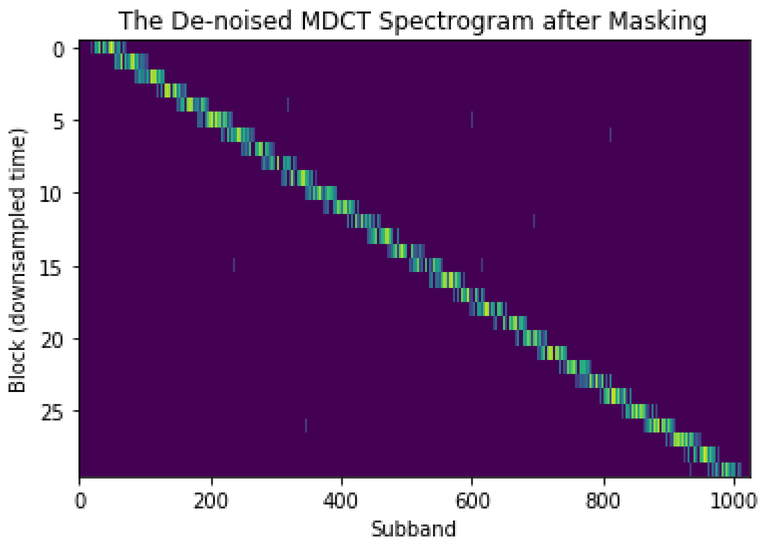
Out[24]:
```
Text(0.5,1,u'The Binary mask for our Chirp Signal (Yellow is 1)')
```



The MDCT Spectrogram after Applying the Mask

In [25]:
```python
# Elementwise multiplication with the binary mask:
Ydenoise=Y*binarymask
plt.imshow(abs(Ydenoise),aspect='auto')
plt.xlabel('Subband')
plt.ylabel('Block (downsampled time)')
plt.title('The De-noised MDCT Spectrogram after Masking')
```

Out[25]:
```
Text(0.5,1,u'The De-noised MDCT Spectrogram after Masking')
```



The MDCT Synthesis Filter Bank

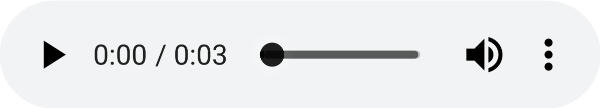
- To return to the time domain, we now apply the MDCT synthesis filter bank to our de-noised MDCT representation

In [26]:

```
from keras_MDCTsynthesis import *
modelsyn = generate_model_syn(N,filtlen)
#MDCT Synthesis:
xrek= keras_MDCT_syn(Ydenoise,modelsyn)
wav.write("xrekMDCTdenoised.wav",samplerate/2,np.int16(xrek*2**14))
Audio("xrekMDCTdenoised.wav")
```

```
Initializing MDCT synthesis weights
subbands.shape= (1, 30, 1, 1024)
xrek.shape= (1, 31744, 1, 1)
```

Out[26]:



- Observe: the noise is almost completely gone, except for some musical noise
- It results from non-cancelled aliasing from switching off neighbouring subbands
- And from some time/frequency bins outside the chirp turned falsely on.

Predictive Filters

- Filters can also be used to **predict the next sample** of an audio signal
- In an **encoder**, the predicted sample is **subtracted** from the true sample, resulting in the **prediction error**, which is encoded and transmitted to the decoder
- in the **decoder** does the same prediction, which is **added back** to the **received prediction error**

Predictive Filters

- **Smooth signals**, like sinusoids, are **easy to predict**
- **Noise** like signals, like quantization noise, are **difficult to predict**
- This means: the **predictor output** contains mostly the **predictable** part
- Hence the predictor output can be used for **de-nosing**

Adaptive Predictive Filter

- Adaptive filters are **(continuously) adapted** to the audio signal to optimize prediction performance
- A well know example is the **"Least Mean Squares"** (LMS) filter
    - It **minimizes the mean squared prediction error** over time
- The "Normalized" LMS (NLMS) filter normalizes its update steps to the signal power, and hence becomes less dependend on the signal amplitude.
- In Python it is implemented in `adaptfilt.lms` and `adaptfilt.nlms`.

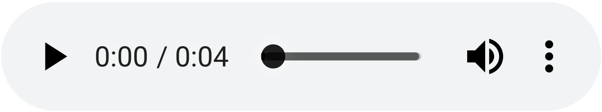
Python Example for (N)LMS De-Nosing

- First we generate a noisy chirp signal for testing

In [30]:

```
chirpsignoise=chirpsig+ (np.random.rand(len(chirpsig))-0.5)*1.0
wav.write("chirpsignoise.wav",samplerate/2,np.int16(chirpsignoise*2**14))
Audio("chirpsignoise.wav")
```

Out[30]:



- Next we apply `adaptfilt.nlms` for de-noising,

In [40]:

```
import adaptfilt
#predictor for the chirp signal, signal to predict is 1 sample ahead:
predicted, error, lastweights = adaptfilt.nlms(chirpsignoise[:-1], chirpsignoise[1:] , 20, step=0.1)
wav.write("chirppredicted.wav",samplerate/2,np.int16(predicted*2**14))
Audio("chirppredicted.wav")
```

Out[40]:

0:00 / 0:03

- Observe: we hear **less noise and noise shaping**,
- but the de-noising is **less effective than with the filter bank**
- but due to the predictor structure, it has **no system delay**
- This make it suitable for **real time speech communications**

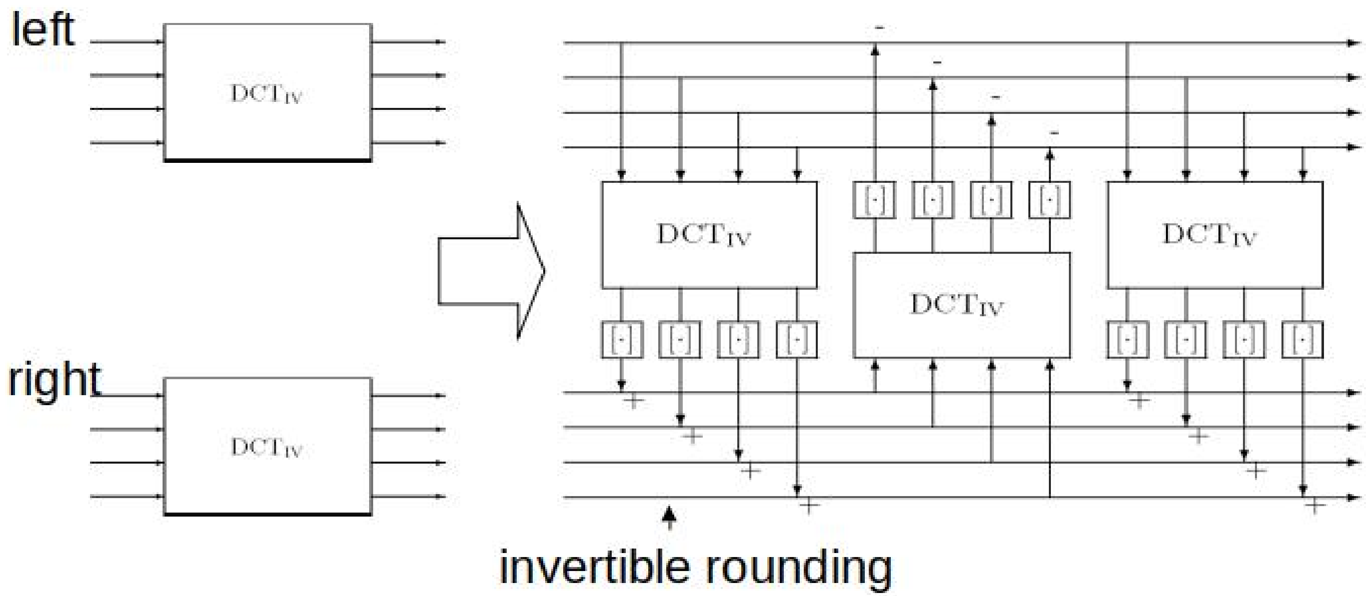
Predictive Lossless Coding

- Predictors can be easily applied to **lossless coding**,
- if the input of the encoder is integer valued audio samples, the predicted value can be **rounded to be integer valued** to produce an integer valued prediction error
- This leads to a **reduced bit-rate**
- The decoder simply **adds the same rounded predicted value back** to obtain the reconstructed **original integer audio samples**
- This principle is used for instance in **MPEG-4 ALS lossless coding**

Lossless Coding with Integer-to-Integer MDCT (IntMDCT)

- In general the MDCT filter bank produces **float valued** samples in its subbands,
- even if the input was integer valued, like for instance the usual 16, 24, or 32 bit audio samples
- For lossless coding, **integer valued subband values are needed**, which can be reconstructed to the original audio samples
- If we just round the samples from an MDCT to the nearest integers, it is not reconstructing the exact same integers
- The **IntMDCT** solves this problem: its analysis filter bank produces **integer subband values**
- Its synthesis filter bank **reconstructs the exact original integer values**

The Block Diagram of (Part of) the Encoder IntMDCT for Stereo



The Block Diagram of the IntMDCT

- The **decoder** IntMDCT has the same structure, except it is **mirrored** left to right, and **subtractions and additions are switched**.
- Observe: the IntMDCT uses the **same principle** as lossless predictive coding:
- on the encoding side, **rounded values are subtracted (or added)**,
- on the decoding side, the **same rounded values are added (or subtracted)** for exact reconstruction.

The Integer-to-Integer MDCT (IntMDCT)

- It can be combined with lossy perceptual MDCT based audio coders, like the MPEG-AAC coder
- In this way we can create a scalable extension layer for lossless coding,
- There the decoder can choose if it wants to decode the low bitrate lossy version or to include the lossless layer.
- It is used in **MPEG-4 SLS lossless audio coding**, also known as **HD-AAC**.

Conclusions

- We saw: digital filters can be seen as a running weightes average of previous samples
- Filter banks allow us to donwsample without loosing bandwidth and with perfect reconstruction
- We can use them for coding, noise shaping, and de-noising
- Minimum-Phase filter can reduce the signal delay while keeping the same or similar magnitude response and stopband attenuation
- Similar, Low Delay filter banks can reduce the system delay, for realy time coding applications, like teleconferencing

Conclusions

- Predictive filters can be used for low delay coding and also noise shaping and de-noising
- They can also be easily used for lossless coding
- The IntMDCT produces integer valued subband signals
- It can be used for lossless coding and lossless enhancement layers for perceptual audio coders.
- Slides later available at: https://www.tu-ilmenau.de/en/applied-media-systems-group/publications/ (https://www.tu-ilmenau.de/en/applied-media-systems-group/publications/)