Measuring edges on pivot-mounted objects during rotation

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Introduction

For quality assurance during a production process it is necessary to check the condition of the tools used in that process. For cutting edge tools like drills, end mills and so on the main influence on the resulting quality is the geometry of the cutting edges. Measurement of such edges can be performed using 2D image processing technology and a setup according to figure 1, in which the tool is pivot-mounted with its rotary axis inside the plane of focus of the camera.

Fig. 1. Scheme of the assembly (left) and sample camera image of an edge (right)
State of the Art

For this measurement task the state of the art uses 2 methods of measurement. Both of which rely on measurement of runout values $x(\varphi)$ for the tool according to the images acquired at different angles of rotation $\varphi$. The first method is to rotate the tool through the plane of focus and continuously measure the runout for which the local maximum value is calculated. Afterwards the tool is rotated backwards until the user decides that the maximum position is reached once again using a graphical indicator based on the current and maximum runout values. At the position of maximum runout a static image is acquired which is used for measurement. The second method also requires the tool to be rotated through the plane of focus while measuring runout values. When the sequence of runout values shows a local maximum, the image that contained the maximum runout is used for measurement. The first method suffers from high user influence due to the interpretation of a graphical indicator and manual return to focus position, the second method produces systematic errors. The new approach presented in this paper is an extension to the second method calculating a maximum speed at which the tool can be rotated and applying a mathematical correction to the results that compensates for the systematic errors.

The new approach

The runout measured in an image acquired at rotary angle $\varphi$ can be described as

$$x(\varphi) = x_0 + r \cdot \cos(\varphi - \varphi_0), \quad (1)$$

where $x_0$ is the offset of the rotary axis, $r$ is the actual distance of the edge point to the rotary axis (radius) and $\varphi_0$ is the angle of rotation at which the edge is in the plane of focus.

While rotating the edge through to the focus plane, the camera acquires a sequence of images and for each image an additional rotary encoder samples the angle of rotation at the rotary axis. By evaluation of the $k$-th image the runout value $x_k$ measured at the rotary angle of $\varphi_k$ is calculated. By iterating through these values the index $\hat{k}$ for which $x_{\hat{k}}$ is maximal can be determined.

Assuming that the rotary axis is relatively near to the origin of the coordinate system, $\tilde{r} := x_{\hat{k}}$ is a rough approximation of the radius and $\tilde{\varphi}_k$ approximates $\varphi_0$. Let $\delta$ be the depth of focus of the optical system, then
the interval of rotary angles for which the edge is in focus can be estimated by the following approach.

\[ r \cdot |\sin(\alpha)| \leq \delta \]  

(2)

\[ \alpha \in [-\sin^{-1}\left(\frac{\delta}{r}\right), \sin^{-1}\left(\frac{\delta}{r}\right)] \]  

(3)

The substitution \( \alpha = \varphi - \varphi_k \) and \( r = \bar{r} \) leads to the estimated interval of feasible rotary angles and the set of indices of values inside this interval.

\[ \varphi \in \Phi := [\varphi_k - \sin^{-1}\left(\frac{\delta}{\bar{r}}\right), \varphi_k + \sin^{-1}\left(\frac{\delta}{\bar{r}}\right)] \]  

(4)

\[ I := \{k|\varphi_k \in \Phi\} \]  

(5)

Using the Gaussian method of least squares the selected rotary angles \( \varphi_k \) and associated runout values \( x_k \) with \( k \in I \) can be used to approximate the parameters \( x_0, \varphi_0 \) and \( r \) of equation 1. The sum of square errors is given by:

\[ E(x_0, r, \varphi_0) := \sum_{k \in I} (x_k - (x_0 + r \cdot \cos(\varphi_k - \varphi_0)))^2 \]  

(6)

Using the trigonometric identity \( \cos(x - y) = \cos(x) \cdot \cos(y) + \sin(x) \cdot \sin(y) \) and some substitutions this term can be transformed to the following representation:

\[ E_0(x_0, r, \varphi_0) = n \cdot x_0^2 + 2 \cdot r \cdot x_0 \cdot sc \cdot \cos(\varphi_0) + 2 \cdot r \cdot x_0 \cdot ss \cdot \sin(\varphi_0) \]

\[ - 2 \cdot x_0 \cdot sx - 2 \cdot r \cdot sx \cdot \cos(\varphi_0) - 2 \cdot r \cdot sx \cdot \sin(\varphi_0) \]

\[ + r^2 \cdot sc^2 \cdot \cos(\varphi_0)^2 + r^2 \cdot ss^2 \cdot \sin(\varphi_0)^2 \]

\[ + 2 \cdot r^2 \cdot sc \cdot \sin(\varphi_0) \cdot \cos(\varphi_0) + sx^2 \]  

(7)

\[ sc := \sum_{k \in I} \cos(\varphi_k), \quad ss := \sum_{k \in I} \sin(\varphi_k), \]

\[ sc^2 := \sum_{k \in I} \cos(\varphi_k)^2, \quad ss^2 := \sum_{k \in I} \sin(\varphi_k)^2, \]

\[ sx := \sum_{k \in I} x_k, \quad sx^2 := \sum_{k \in I} x_k^2, \]

\[ sx \cdot sc := \sum_{k \in I} x_k \cdot \cos(\varphi_k), \quad sx \cdot ss := \sum_{k \in I} x_k \cdot \sin(\varphi_k) \]  

(8)

Solving the equation
\[ \frac{\partial E_0(x_0, r, \phi_0)}{\partial x_0} = 0 \]  
with respect to \(x_0\) leads to the partial solution \(\hat{x}_0\) and creates the simplified error term \(E_1\).

\[ E_1(r, \phi_0) := E_0(\hat{x}_0, r, \phi_0) \]  
Now let \(\hat{r}\) be the solution of

\[ \frac{\partial E_1(r, \phi_0)}{\partial r} = 0 \]

with respect to \(r\), then the last stage error term \(E_2\) can be calculated by substitution.

\[ E_2(\phi_0) := E_1(\hat{r}, \phi_0) \]

In order to calculate the optimal value for \(\phi_0\) the equation

\[ \frac{\partial E_2(\phi_0)}{\partial \phi_0} = 0 \]

must be solved with respect to \(\phi_0\). This leads to the following formula, which can be used to calculate the approximate rotary angle at which the edge would be in the focus plane.

\[ \hat{\phi}_0 = \tan^{-1} \left( -s s c \cdot s x c \cdot n + n \cdot s x s \cdot s c_2 + s s \cdot s c \cdot s x c - s x s \cdot s c \cdot s c \right. \]
\[ \left. - s s \cdot s x \cdot s c_2 + s c \cdot s x \cdot s s c, -s x s \cdot s s c \cdot n - s s_2 \cdot s c \cdot s x \right) \]
\[ + s s \cdot s s c \cdot s x - s s \cdot s s \cdot s x c + s x c \cdot s s_2 \cdot n + s s \cdot s c \cdot s x s \]  
(Notice the use of the two argument version of arcus tangens.)

Based on the difference between \(\phi_k\) and \(\hat{\phi}_0\) a correction factor

\[ \gamma_k := \frac{1}{\cos(\phi_k - \hat{\phi}_0)} \]

can be calculated for each image that compensates the measurement error in horizontal direction. This allows for the use of any image in the interval of feasible rotary angles to be used for the measurement of the edge. In order to reduce the influence of random error it is even possible to use the mean value of the corrected results of all images.
Since this procedure requires the approximation of a circle it is necessary, that the interval $I$ contains at least the 3 indices $\hat{k} - 1, \hat{k}, \hat{k} + 1$. Let $\omega$ be the (mean) rotational speed and $f$ the sampling frequency of the camera, then the absolute difference $\beta$ between the selected maximum image and the actual focus position theoretically satisfies equation (16). The smaller residual interval of in-focus angles ($\alpha - \beta$) must contain another sample, which leads to the maximum permissible rotational speed in inequation (17).

$$\beta = |\varphi_{\hat{k}} - \varphi_0| \leq \frac{\omega}{2 \cdot f}$$  

(16)

$$\alpha - \beta_{\text{max}} \geq \frac{\omega}{f}$$  

(17)

$$\omega \leq \omega_{\text{max}} = \frac{2}{3} \cdot f \cdot \sin^{-1}\left(\frac{\delta}{r}\right)$$  

(18)

**Experimental results**

Multiple image sequences have been captured at a very low speed of rotation of roughly $0.5^\circ/s$. This speed was not constant since the rotation was performed manually. The camera in the system acquires 20 images per second and has a depth of focus of $1\text{mm}$. The outrun of the edge was approximately $30.38\text{mm}$ at the vertical position of measurement.

Based on these image sequences new sequences were calculated which simulate accelerated rotation during acquisition. Let the $i$-th sequence be

$$S_i = \{\text{img}_{i,j}|0 \leq j < n_i\}$$  

(19)

then the subsampled sequence $S_{i,j,k}$ with SpeedUp-factor $j$ and Offset $k$ consists of a subset of these images.

$$S_{i,j,k} := \{\text{img}_{i,k+j+l}|l \in \mathbb{N} \land k + j \cdot l < n_i\}$$  

(20)

The maximum runout calculated by evaluating sequence $S_{i,j,k}$ is denoted as $x_{i,j,k}$, the associated angle of rotation is $\varphi_{i,j,k}$. Since the offset of sampling poses a random influence to the measurement result, only the mean values ($\bar{x}_{i,j}$ and $\bar{\varphi}_{i,j}$) and standard deviation ($\sigma_{x_{i,j}}$ and $\sigma_{\varphi_{i,j}}$) of these values for all possible offsets $k$ are considered.

Figure 2 illustrates $\bar{x}_{i,j}$ for different SpeedUp-factors $j$ comparing the results of selection of maximum runout and calculation of the average/mean value of all corrected measurements in images acquired in the focus interval $I$ of sequence $S_{i,j,k}$. Obviously $\bar{x}_{i,j}$ is stable when the rotation is accelerated.
Fig. 2. mean value $\bar{x}_{i,j}$ over different sampling offsets $k$ at SpeedUp-factor $j$ for selection and averaging method.

if the averaging procedure is used. The method of selection is much more dependent on the speed of rotation.

Additionally figure 3 shows that the averaging method is about 2-3 times less dependent on the offset of sampling.

Fig. 3. standard deviation $\sigma_{x,i,j}$ over different sampling offsets $k$ at SpeedUp-factor $j$ for selection and averaging method.

When considering the position of the edge the angle of rotation at which the maximum runout was located is also of high interest. Figure 4 compares the mean values $\phi_{i,j}$ of these angles for different SpeedUp-factors $j$. The influence of rotation speed on the calculated angle for selecting the maxi-
The comparison of the standard deviations $\sigma_{\varphi, i, j}$ for the selection and averaging methods at different SpeedUp-factors indicates, that the angle calculated by the averaging method is much less influenced by the sampling offset $k$, since there is about one magnitude of difference between the respective standard deviations as depicted in figure 5.

Fig. 4. mean value $\bar{\varphi}_{i, j}$ over different sampling offsets $k$ at SpeedUp-factor $j$ for selection and averaging method

Fig. 5. standard deviation $\sigma_{\varphi, i, j}$ over different sampling offsets $k$ at SpeedUp-factor $j$ for selection and averaging method
The figures above represent one of the captured image sequences. The other sequences showed similar advantages for the averaging method.

When comparing the results of different sequences $S_{i,1,0}$ using all images of the sequence, the standard deviation of the set of measurement results $\{\bar{x}_{i,j}|1 \leq i \leq 10\}$ was approximately the same for both evaluation methods ($\approx 0.2\mu m$). Since the value $\bar{x}_{i,j}$ is relatively independent of the SpeedUp-factor when using the averaging method, this kind of evaluation is clearly favorable when using a higher speed of rotation during image acquisition.

Concerning the reproducability of the estimated focus angle $\tilde{\varphi}_0$ and the selected best fit angle $\varphi_k^\wedge$, the standard deviation of measurements at full sample rate is clearly in favor of the averaging method. The observed standard deviation of approximately $0.00825^\circ$ was about 10 times lower for this method than for the selection of the maximum runout image.

Conclusion

The method of evaluation for image sequences presented in this paper has been shown to produce nearly identical results as the state of the art method when used at a very high sampling rate. Contrary to the state of the art method the new averaging method loses far less of it’s precision when used at a lower sampling rate. Additionally the new evaluation method can be used to calculate the angle of rotation at which the edge is located at much higher accuracy and reproducability.

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References