1. Introduction and Motivation

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"Information is not knowledge."
Albert Einstein

"Information is not knowledge. Knowledge is not wisdom..."
Frank Zappa
1.1. Introduction and Motivation

A System

“A system is a self-contained entity with interconnected elements, process and parts. A system can be the design of nature or a human invention.”

- A system has a clearly defined boundary. Outside this boundary is the environment surrounding the system.
- The interaction of the system with its environment is the most vital aspect.
- A system responds, changes its behavior, etc. as a result of influences (impulses) from the environment.

▶ We are interested in physical systems.
1.2. Some examples of systems

- Mechanical systems
- Electrical systems
- Mechatronic systems
- Communication network systems
- Chemical processing systems
- Water reservoir and distribution network systems
- Power generation and distribution systems
- Renewable-energy generation and distribution systems
- Transportation network systems
- Social Systems
- Ecological and environmental system
- Biological system
- Financial system
- Planning and budget management system
- etc
1.3. Purpose and importance of Systems Optimization

- Optimization refers to either minimization or maximization of a quantity.

Systems Optimization methods provide strategies for optimal product design, efficient planning, scheduling, and control strategies for better system performance.

- Optimization methods provide prior knowledge before the actual system is constructed, manufactured, or launched into action. Hence, it helps avoid wrong decision, wastage of materials, and resources, etc.

- Solutions of optimization problems serve as decision support strategies:
  - to obtain high-quality and high-performance product with less consumption of resources
  - to manipulate a system to best achieve a desired goal
  - to make a system profitable, or to identify whether a system will be profitable or not
  - to avoid possible future system-failure and/or risk of damage
  - to recover lost artifacts, and to optimally reconstruct desired features, etc.
1.4. Current Advanced Research and Applications

- Control and management of water distribution networks
- Renewable energy generation, storage, and transmission
- Operation and management of chemical plants
- Nanotechnology and applications
- Bionics and biomechanics
- Systems biology, biotechnology, and medicine
- Autonomous systems
- Distributed and multi-agent systems
- Artificial intelligence and machine learning
- Wireless and mobile communication
- Finance and economics
- Environmental sciences
- etc
(A) Water Distribution Networks

(B) Renewable Energy Resources

- wind speed

- solar intensity

Renewable Energy Generation, Storage, and Transmission

Important issues

- optimal power generation and storage
- optimal integration of renewable energy resources with conventional energy systems
- optimal management and control of transmission networks
- optimal cost benefit analysis, marketing and trading, etc
(B) Renewable energy resources...

Some References


1. Introduction and Motivation

Important issues:

- energy optimal and cost efficient plant operation and production
- optimal control of the production process
- guaranteeing high quality products
- timely adjustment for feed variations, changing market conditions, and new production strategies
- efficient pollution reduction and risk minimization, and environment protection
- observance of government policies, regularity conditions, etc.
Plenty of Room at the Bottom

Carbon Nanotube
Graphene

Efficient Solar Cells
Nanoelectronics and Circuit
Nanomedicine

Important issues
- the design of high performance materials
- optimal combination of geometries and material properties to construct new and improved materials
- high strength, high conductivity, thermal resistant materials
- optimal modification of material properties at molecular or atomic scales
- design of materials for highly efficient solar cells
- new type of medicine, efficient, and less intrusive medical treatment
"There’s Plenty of Room at the Bottom", Richard Feynman (Physicist, Nobel Prize 1965).

Some References

(E) Bionics

Learning from Nature to provide Engineering Solutions

Bionic Arm

Bionic Eye

Important issues

- optimal prosthetic shape design, structure, and weight
- efficient coordination and interaction of bionic organs with natural body muscles, skeletons, and the nervous system
- efficient brain-machine interface
- attaining natural like posture and functionality, high dexterity, and efficient performance
- optimal force transmission (for smoother motion and better quality of performance)
- efficient resource consumption for sustainable mobility with no or minimal fatigue
- optimal movement control, stable dynamics, and adaptive behavior, etc.
Some References


(F) Machine Learning and Data Mining

Important Issues

- Designing autonomous intelligent systems
- Understanding human genomics
- Designing new type of medicine
- Knowledge discovery from data (Data Mining)
- Online Shopping and Recommender Systems
- Business intelligence and financial forecast
- Speech and text processing
- Automatic Language translation
- Intelligent Computer vision

Traffic Sign Identification

Examples: Pedestrian Detection

Artificial Minds
Some References:


Autonomous Systems

Important issues:
- simultaneous localization, mapping, and optimal path planning
- faster motion and reliable mission accomplishment
- obstacle avoidance in complex environments
- energy efficient navigation and biped walking
- stable and faster biped locomotion
- fully autonomous navigation
- perception and learning
- high-performance sensor fusion
- efficient multi-robot communication and cooperation

UGV

Service Robots

Biped Humanoid Robot

UAV

AUV
**Some References**

- A. Goswami and V. Kallem: Rate of change of angular momentum and balance maintenance of biped robots, IEEE Int. Conf. on Robotics and Automation, New Orleans, April 2004.
Some References

1.4. Modeling Optimization Problems

Systems optimization requires a **valid** mathematical model.

The formulation of a **valid mathematical optimization model** needs:

- specifications
- a good knowledge and experience on the system to be optimized - the quality of the optimization model depends on the available information about the system.
- to consider only the **most important aspects** of the system under investigation
  - an optimization problem considering each and every aspect of system can be very complex and difficult to solve
  - the modelling should be a trade-off between: proper representation of the systems’ behaviors to be optimized and available computational resources.
- verification of practicality of optimal solutions
- model validation, frequent evaluation, and upgrading
1.5. Modeling Optimization Problems ...

Specifications

Formulate a Valid Optimization Model

Look for Additional Specifications, Evaluate, and Upgrade the Model

Identify Problem Type (Class)

Analyze the Problem, Identify an Appropriate Algorithm, and Solve the Problem

Verify the Viability of the Obtained Solutions for Practical Use
1.5. Modeling Optimization Problems ...

Some tips for modeling optimization problems

- identify what is to be optimized (either maximized or minimized) - this gives the **objective function** of the optimization problem
- identify the **decision variables**, with respect to which the optimization task is going to be performed
- determine a convenient mathematical model for the system to be optimized
- identify the **constraints** of the process and write their mathematical description
  - constraints usually arise as a result of limited availability of resources, capacity limitations, physical or natural limitations, etc.
  - constraints are commonly written as **equations or inequalities**
  - the mathematical description (model) of the process is also taken as part of the constraints
- write the complete mathematical description of the optimization problem

The optimization model can be:
(I) steady-state model
(II) time-dependent dynamic optimization problem
1.5. Modeling Optimization Problems...

\( (NLP) \quad \min_x f(x) \)

subject to:
\[
\begin{align*}
    h_i(x) &= 0, \quad i = 1, \ldots, m_1, \\
    g_j(x) &\leq 0, \quad j = 1, \ldots, m_2, \\
    x_{\text{min}} &\leq x \leq x_{\text{max}}.
\end{align*}
\]

- All variables and functions are time-independent, where
  - \( x \) - decision (optimization) variables
  - \( f(x) \) - objective function or performance criteria
  - \( h_i(x) = 0, \quad i = 1, \ldots, m_1 \) - equality constraints
  - \( g_j(x) \leq 0, \quad j = 1, \ldots, m_2 \) - inequality constraints
  - \( x_{\text{min}} \leq x \leq x_{\text{max}} \) - bound constraints on the decision variables
1.5. Modeling Optimization Problems.

**Dynamic Optimization**

\[(DynOpt) \quad \min_u J(x, u)\]

subject to:

\[
\dot{x} = f(x, u, t)
\]

\[g(x, u, t) = 0.\]

\[x_{\text{min}} \leq x(t) \leq x_{\text{max}}\]

\[u_{\text{min}} \leq u(t) \leq u_{\text{max}}.\]

- Some variables or functions are time-dependent, where
  - \(u(t)\) - control variables or decision (optimization) variables
  - \(x(t)\) - state variables (behaviors of the system to be controlled)
  - \(J(x, u)\) - objective function or performance criteria
  - \(\dot{x} = f(x, u, t)\) - equations describing systems dynamics or model equations
  - \(g(x, u, t) = 0\) - algebraic constraints on the dynamics of the system
  - \(x_{\text{min}} \leq x(t) \leq x_{\text{max}}\) - bound constraints on the trajectory of the state variables
  - \(u_{\text{min}} \leq u(t) \leq u_{\text{max}}\) - bound constraints on the control variables
1.5. Modeling Optimization Problems...

Objective Function \( f(x) \) or \( J(x(t), u(t)) \):

- a quantity or criteria that helps evaluate the performance of the system → performance criteria
- specifies the desired goals to be achieved
- specifies what is to be minimized or maximized
- In general, maximization problem and a minimization problem are equivalent

\[
\max_x f(x) = -\min_x (-f(x))
\]

- We usually attempt to minimize:
  - energy consumption, production cost, cost of transportation, environmental pollution, risk, wastage of raw materials, divergence from a set-point trajectory, arrival time, etc.

- We attempt to maximize:
  - profit, performance, product quality, amount of production, services coverage, quality of services, comfort, safety, etc.
1.5. Modeling Optimization Problems.

**Decision Variables** \((x, u(t))\)

- are optimization variables represent the parameters that we can manipulate to force the system attain a desired optimal performance.
- represent the available **degrees-of-freedom** for a better system performance
- represent available resources that should be used ingeniously

**Constraints on the Variables** \((x_{\text{min}} \leq x \leq x_{\text{max}}, u_{\text{min}} \leq u(t) \leq u_{\text{max}})\)

- describe the limits on **available system capacities or resources**; e.g., required minimum water level \(x_{\text{min}}\) and maximum water capacity \(x_{\text{max}}\) of a water tank, so that \(x_{\text{min}} \leq x \leq x_{\text{max}}\)
- **safety limits** (corridors); e.g., bounds on the trajectory of a mobile robot \(x_{\text{min}} \leq x(t) \leq x_{\text{max}}\)
- bounds on **required resources and system limitations**; e.g. minimum required voltage level \(u_{\text{min}}\) and maximum allowed voltage level \(u_{\text{max}}\) to drive a motor, so that \(u_{\text{min}} \leq u(t) \leq u_{\text{max}}\)
1.5. Modeling Optimization Problems...

**Constraints** could also arise from

- required dimensioning of a given design \((h(x) = 0, g(x) \leq 0)\)
- limitations of available resources
- the governing laws of a dynamic system \((\dot{x} = f(x, u, t))\)
- the governing laws coupled with physical constraints of a dynamic system, e.g., dynamics of an industrial robot with work-space limitations \((\dot{x} = f(x, u, t), g(x, u, t) = 0)\)
- mass-balance equations, volume-balance equations, conservation laws, etc.
- nodal equations in flow networks; e.g., in electrical circuits, in water networks, etc.
- policy constraints; e.g., governmental policies for pollution control, waste reduction, etc.
1.6. Examples

Example 1 - Optimal Design
Find the dimensions of a open-topped rectangular tank with smallest surface area and given volume $V = 500\, \text{cm}^3$.

Variables: $x_1$ width, $x_2$ breadth, $x_3$ height

Objective function: $f(x) = f(x_1, x_2, x_3) = x_1 x_2 + 2x_2 x_3 + 2x_1 x_3$ - surface area to be minimized.

Constraints: $x_1 x_2 x_3 = 500$ - the volume is given

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Optimization Model:

$$\min_{x} \{ f(x) = x_1 x_2 + 2x_2 x_3 + 2x_1 x_3 \}$$

subject to: $x_1 x_2 x_3 - 500 = 0,$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$
1.6. Example 2: Maximization of power absorption

Formulate an optimization problem to maximize the total power absorbed by the resistors $R_1$ in the circuit given below.

- Power absorbed by resistor $R_1$ is $p = i^2 R_1$.
- Kirchoff’s voltage Law: $V - iR_1 - iR_2 = 0 \Rightarrow V = i(R_1 + R_2)$
  \[ \Rightarrow i = \frac{V}{R_1 + R_2}. \]
- Objective function: $p(R_1, R_2) = \frac{V^2 R_1}{(R_1 + R_2)^2} = \frac{400R_1}{(R_1 + R_2)^2}$
1.6. Example 2: Maximization of power absorption

... 

- Constraints: $R_1 \geq 0$, $R_2 \geq 0$.

**Steady-State Optimization Model:**

$$
\min_{R_1, R_2} - \left\{ p(R) = \frac{400R_1}{(R_1 + R_2)^2} \right\}
$$

subject to

- $R_1 \geq 0$,
- $R_2 \geq 0$. 
1.6. Example 3: Control of water reservoirs

Consider two water tankers $T_1$ and $T_2$, each with capacities $1000\, m^3$, which are connected by a pipeline. At each time instant $t$, the water volume in the tankers is given by $V_1(t)$ and $V_2(t)$, respectively. Water flows into $T_1$ at a rate of $q(t)$, flows out of $T_1$ at a rate of $u(t)$, and out of $T_2$ at a rate of $d(t)$.
1.6. Example 3: Control of water reservoirs ...

The hourly average rate of water demand is equal to $530 m^3/h$. For a smooth and efficient operation, it is recommended to hold the flow rate $q(t)$ from the pump near to a constant value $K = 30 m^3/h$. Furthermore, at the beginning and end of the day the water tankers $T_1$ and $T_2$ should be 50% and 70% full, respectively. Write an optimization to keep the flow rate $q(t)$ almost constant by controlling the flows into the water tankers $T_1$ and $T_2$.

- Flow balance equations:
  \[ \dot{V}_1(t) = q(t) - u(t) \]
  \[ \dot{V}_2(t) = u(t) - d(t) \]

- Demand rate: $d(t) = 530 m^3/h$

- Initial states: $V_1(t_0) = 0.5 \times 1000 = 500 m^3$, $V_2(t_0) = 0.75 \times 1000 = 750 m^3$

- Terminal states:
  \[ V_1(t_f) = 0.5 \times 1000 = 500 m^3, \quad V_2(t_f) = 0.70 \times 1000 = 750 m^3 \]

- Control variables: $q(t), u(t)$

- Objective function:
  \[ J(q, u) = \int_{t_0}^{t_f} (q(t) - K)^2 \, dt \]

- State constraints: $0 \leq V_1(t) \leq 1000$, $0 \leq V_2(t) \leq 1000$

- Control constraints: $q(t) \geq 0$, $u(t) \geq 0$. 
1.6. Example 3: Control of water reservoirs ...

Dynamic Optimization Model:

\[
\min_{q(t), u(t)} \left\{ J(q, u) = \int_{t_0}^{t_f} (q(t) - 30)^2 \, dt \right\}
\]

subject to:
\[
\begin{align*}
\dot{V}_1(t) &= q(t) - u(t), \quad V_1(t_0) = 500, \quad V_1(t_f) = 500, \\
\dot{V}_2(t) &= u(t) - 530, \quad V_2(t_0) = 750, \quad V_2(t_f) = 750, \\
0 &\leq V_1(t) \leq 1000, \\
0 &\leq V_2(t) \leq 1000, \\
q(t) &\geq 0, \\
u(t) &\geq 0, \\
t_0 &\leq t \leq t_f.
\end{align*}
\]
1.6. Example 4: Minimum time navigation problem

Suppose a ship has to travel across a river from one shore at point $(x_0, y_0)$ to another shore at the point $(x_f, y_f)$. The speed $v$ of the ship is assumed to be a constant. However, its heading direction $\theta(t)$ is a function of time. The water current is in the $y$ direction and has a constant speed $w$. What should be the trajectory of the heading direction in order for the ship to arrive at the destination point $(x_f, y_f)$ in the shortest time possible.
1.6. Example 4: Minimum time navigation problem...

- State variables: $x(t), y(t)$
- Initial conditions $x(t_0) = x_0, y(t_0) = y_0$
- Terminal conditions $x(t_f) = x_f, y(t_f) = y_f$
- Control variable: $\theta(t)$
- Equations of motion:
  \[
  \begin{align*}
  \dot{x}(t) &= v \cos(\theta(t)) \\
  \dot{y}(t) &= v \sin(\theta(t)) + w
  \end{align*}
  \]
- Objective function $J(\theta) = t_f$

Dynamic Optimization model (minimum-time optimal control problem):

\[
\min_{\theta} \{ J(\theta) = t_f \}
\]

subject to:

\[
\begin{align*}
\dot{x}(t) &= v \cos(\theta(t)), x(t_0) = x_0, x(t_f) = x_f, \\
\dot{y}(t) &= v \sin(\theta(t)) + w, y(t_0) = y_0, y(t_f) = y_f, \\
0 &\leq t \leq t_f.
\end{align*}
\]
1.7. Solution framework

(I) Framework for Steady-state Optimization Problems

- **Solution Methods for Steady-State Optimization problems**
  - Heuristic Methods
    - Evolution algorithms
    - Genetic algorithms
    - Pattern search
    - Particle swarm optimization
    - etc.
  - Gradient-Based Methods
    - (Quasi-)Newton Method
    - Conjugate Gradient Method
    - Descent Methods
    - Penalty Method
    - Barrier and Interior Point Method
    - Sequential Quadratic Programming (SQP) Method

- **General Iterative Scheme for Gradient-based Methods**
  \[ x^{(k+1)} = x^{(k)} + \alpha_k d_k \]
  
  - \( d_k \) and \( \alpha_k \) are to be determined from the objective, constraint function, and their gradients.
1.7. Solution framework ...

(II) Framework for Dynamic Optimization Problems

![Diagram showing the solution framework for dynamic optimization problems. The framework includes steps such as Indirect Methods, Direct Methods, Simultaneous Method, State and Control Discretization, Nonlinear Optimization, and Solution Methods of Nonlinear Optimization.]

Systems Optimization Winter Semester 2016  1. Introduction and Motivation

TU Ilmenau