Visualization of magnetoconvection

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The opacity of liquid metals represents a seemingly insurmountable barrier for visualizing flows of electrically conducting fluids under the influence of a magnetic field. We demonstrate that this fundamental limitation of experimental magnetohydrodynamics (MHD) can be overcome by using (transparent) electrolytes in place of (opaque) liquid metals if a superconducting magnet with field strength of the order of 10 Tesla is used instead of traditional electromagnets. We study the Rayleigh–Bénard convection as a prototype problem under a horizontal magnetic field using a shadowgraph method and tracer particles. It is shown that recent observations of Burr and Müller [J. Fluid Mech. 453, 345 (2002)] of an increased heat transfer in magnetoconvection can be attributed to the two-dimensionality and a smaller length scale of motion due to the magnetic field. The present approach provides a rational framework for the application of modern optical flow measurement techniques like laser Doppler velocimetry and particle image velocimetry to MHD problems.


Flows of liquid metals under the influence of a magnetic field occur under a wide range of circumstances ranging from the flow of liquid iron in the Earth’s core to the movement of molten steel, aluminum or titanium in materials processing. Since liquid metals are opaque to visible light, their interior cannot be visualized by relatively low cost and accessible classical optical flow visualization methods. That limits our understanding of MHD flow structures. Radioscopy is employed with much success to visualize convection in liquid metals, but this method requires special equipment and is not available for the broad MHD experimental practice. The purpose of the present Brief Communication is to demonstrate, using the example of Rayleigh–Bénard convection, that this technical limitation of experimental MHD can be overcome thanks to the increasing availability of high magnetic fields and that MHD flows in the laboratory are not necessarily limited to fluids with low Prandtl numbers.

The strength of electromagnetic forces in relation to inertia and viscosity is described by the dimensionless magnetic interaction parameter \( N = \sigma LB^2/\mu \) and the Hartmann number \( \text{Ha} = B^2L^2/\sigma/\mu \), respectively, where \( L \), \( u \), and \( B \) are the length scale, characteristic velocity, and magnetic-field intensity, while \( \sigma \), \( \rho \), and \( \mu \) denote the electrical conductivity, density, and dynamic viscosity of the fluid. Both parameters are proportional to \( \sigma B^2 \) implying that the same flow conditions can be obtained either using mercury (\( \sigma \approx 10^6 \, \Omega^{-1} \text{m}^{-1} \)) and traditional electromagnets (\( B \approx 0.1 \, \text{T} \)) or by using electrolytes (\( \sigma \approx 10^2 \, \Omega^{-1} \text{m}^{-1} \)) and high magnetic fields (\( B \approx 10 \, \text{T} \)). Although the idea of using electrolytes for MHD experiments is at least 30 years old, it was not until the advent of superconducting magnets with large warm bore that this idea could be systematically explored.

We use weakly supercritical Rayleigh–Bénard convection in a small rectangular box as a prototype problem to test the feasibility of the outlined approach. The influence of a steady homogeneous magnetic field on Rayleigh–Bénard convection in an electrically conducting fluid has been well studied both experimentally and theoretically. The majority of MHD-Rayleigh–Bénard theories was formulated in the two-dimensional (2D) approximation. In the frame of this assumption a magnetic field applied horizontally along the convective rolls does not have any effect on convective motion of a liquid conductor in the absence of the lateral boundaries. Normally the real convective flow is bounded from the side. The basic theory of MHD tells us that the walls perpendicular to the magnetic field create the Hartmann boundary layer. This layer does not break the two-dimensionality but creates a drag effect on the flow. In recent works the effect of the Hartmann boundary layers was introduced implicitly in the frame of the so-called quasi two-dimensional MHD theory.

According to this theory the electromagnetic force is proportional to the magnetic field induction and aspect ratio of the flow. Hence, if the convective rolls are essentially elongated (being compared with the diameter) then the MHD effect might be neglected despite the significant value of the Hartmann number. Experimental studies of MHD convection with horizontal magnetic field are presented in Refs. 1, 9–11.

Two ordinary acid and alkaline electrolytes were used as working liquids. The first one was a solution of sulfuric acid \( \text{H}_2\text{SO}_4 \) in water, the second one was a solution of potassium hydroxide KOH in water. Both electrolytes are not volatile and can be used in laboratory without any additional venti-
The properties of those electrolytes, taken from the handbooks, are summarized in Table I. The experiments were carried out in a rectangular cell of 30 mm in length and 15 mm in width. The thickness of the liquid layer was 6 mm in width. The thickness of the liquid layer was 0.05 mm. A principle sketch of the experimental device is shown in Fig. 1. The lower and top surfaces of the cell were made of 0.5 mm thick transparent sapphire plates. The heat conductivity of sapphire is 41 W/(m·K). The bottom plate had a reflecting aluminum coating. This optical arrangement of the cell was designed with the aim to obtain visual images of convection by means of the standard shadowgraph technique. A temperature sensor consisting of three differential copper-constantan thermocouples connected in series was used for the measurements of the temperature of the heated bottom. The total sensitivity of the sensor was 120 μV/°C. The reference ends of the thermocouples were placed in the feeding pipe of the top cold frame and were washed by water. A thermostat maintained a constant temperature of ±0.03 °C. The bottom of the cell contained two electrical heaters: The main and compensating heater. The electric power of the secondary heater was automatically controlled to eliminate a heat loss through the base of the cell. At last, the process of thermal diffusion (Soret effect) in binary mixture \( \text{H}_2\text{SO}_4=\text{H}_2\text{O} \), or electrolysis by induced potential differences may play a role.

![FIG. 1. Schematic of the experimental apparatus.](image)

The main dimensionless parameters of the experiments are the following. The Rayleigh number \( \text{Ra} = \frac{ag\Delta T\lambda^3}{ηκ} \) denotes the ratio of buoyant to viscous forces. The integral heat transfer across the layer is characterized by the Nusselt number \( \text{Nu} = \frac{qH}{λ\Delta T} \). The Prandtl number is defined as \( \text{Pr}=\frac{η}{κ} \). Here \( g \) is the acceleration of gravity, \( ΔT \) is the temperature difference between the lower and the upper fluid–wall interface, \( q \) is the heat flux density on the heated bottom, \( B \) is the induction of the magnetic field. The other quantities are defined in Table I.

In the experimental series the values of the critical Rayleigh number were found in the range of 1650<\text{Ra}<1700. These values are smaller than \( \text{Ra}_c=1707.76 \) to be predicted by the linear theory of convective instability in a laterally unbounded horizontal layer with both rigid top and bottom surfaces. This discrepancy may be caused by the several factors: (i) A slightly inhomogeneous heating of the cell because of a small extension of the cell in horizontal direction with respect of the vertical dimension (moderate aspect ratio of 7.5); (ii) the finite thermal conductivity of the top and bottom plates made of sapphire. At last, (iii) the process of thermal diffusion (Soret effect) in binary mixture \( \text{H}_2\text{SO}_4=\text{H}_2\text{O} \), or electrolysis by induced potential differences may play a role.

We managed to make the shadowgraph images with a distinct flow structure for Rayleigh numbers exceeding 5000. The dependence of the Nusselt number on the Rayleigh number for experiments with sulfuric acid is shown in Fig. 2(a). The first experimental series corresponds to ordinary convection of \( \text{H}_2\text{SO}_4 \) solution without magnetic field, the second one corresponds to the maximal value of the Hartmann number \( \text{Ha}=10 \). Both series are performed at increasing Rayleigh numbers. One can see from the figure that an essential difference in the behavior of ordinary and MHD convection is visible for Rayleigh numbers exceeding 10000. It is remarkable that, in spite of an additional electromagnetic braking, MHD convection ensures more intensive heat transfer compared with ordinary convection. This feature can be clarified by the visual images of those flows shown in Figs. 2(b)–2(e). For Rayleigh numbers less than 15000, the convective flow is represented by stable transversal rolls. The number of rolls is 7 for the nonmagnetic case [Fig. 2(b)] and 8 for the MHD convection [Fig. 2(d)]. Consequently, with applied horizontal stabilization we were able to make the shadowgraph images with a distinct flow structure for Rayleigh numbers exceeding 5000.

### Table I. Properties of the working fluids (aqueous solutions of KOH and H\textsubscript{2}SO\textsubscript{4}).

<table>
<thead>
<tr>
<th>Property</th>
<th>KOH</th>
<th>H\textsubscript{2}SO\textsubscript{4}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>20 °C</td>
<td>15 °C</td>
</tr>
<tr>
<td>Concentration</td>
<td>( c ) % weight</td>
<td>21</td>
</tr>
<tr>
<td>Density</td>
<td>( ρ ) kg/m\textsuperscript{3}</td>
<td>1193</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>( α \times 10\textsuperscript{4} ) K\textsuperscript{-1}</td>
<td>2.56</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>( μ \times 10\textsuperscript{3} ) Pa/s</td>
<td>1.63</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>( ν \times 10\textsuperscript{6} ) m\textsuperscript{2}/s</td>
<td>1.37</td>
</tr>
<tr>
<td>Heat conductivity</td>
<td>( λ ) W/(m·K)</td>
<td>0.598</td>
</tr>
<tr>
<td>Specific heat capacity</td>
<td>( c_p ) J/(kg·K)</td>
<td>3237</td>
</tr>
<tr>
<td>Thermal diffusivity</td>
<td>( κ \times 10\textsuperscript{7} ) m\textsuperscript{2}/s</td>
<td>1.549</td>
</tr>
<tr>
<td>Electrical conductivity</td>
<td>( σ ) Ω\textsuperscript{-1}·m\textsuperscript{-1}</td>
<td>53</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>( Pr )</td>
<td>8.84</td>
</tr>
</tbody>
</table>
magnetic field, the natural convection became short-wave and provided more effective heat transfer. The same evolution of the wave number is predicted by the linear stability analyses and numerical simulation for the Rayleigh–Bénard convection performed in Ref. 1. It should be mentioned that the studied flow was essentially restricted in the direction transversal to the magnetic field. This feature caused a discrete development of the flow wavelength upon growth of the Rayleigh number. One can see in Fig. 2 that at some critical values of the Rayleigh number the Nusselt number jumps to a lower value. Each level of the Nusselt number corresponds to a certain wave number $\lambda$. For example, in the range $24000 < Ra < 26000$, the wave numbers of the MHD and ordinary convective flows coincide $\lambda = 7.7$. As a result, the values of the Nusselt number for both flows also coincide. The applied magnetic field conserves the short-wave condition of convective flow at large values of the Rayleigh number. In the final stage of the experimental run ($Ra > 45000$, $Ha = 0$) the two-dimensional convective rolls have a tendency to transform to three-dimensional rectangular convective cells [cf. Fig. 2(c)].

The experiments described above were performed at fixed values of magnetic field. In Fig. 3 we present the experiment performed at fixed heat flux. In spite of a constant heating the corresponding Rayleigh number was not quite constant in the experimental run. It slightly changed in the range of $20500 < Ra < 20800$ because of the variable Nusselt number. The experiment was started at the maximal value of magnetic field corresponding to $Ha = 9.8$. The initial flow pattern was obtained by slow increase of the Rayleigh number from zero to $Ra = 20500$. The picture reveals the fact that while the wave number remained constant ($\lambda = 7.7$), the electro-magnetic force became incapable of maintaining the short-wavelength flow and the number of rolls changed from eight to seven. It resulted in an essential decrease of the Nusselt number [$point 2 in Figs. 3(a)$ and $3(b)$]. At the critical value of the Hartmann number $Ha = 7.9$, the electromagnetic force became incapable of maintaining the short-wavelength flow and the number of rolls changed from eight to seven. It resulted in an essential decrease of the Nusselt number [$point 2 in Figs. 3(a)$ and $3(c)$]. Further reduction of the magnetic field did not transform the shape of the flow. A small decrease of the Nusselt number for $Ha < 4$ obviously was caused by a weak instability of convective rolls. It is illustrated by Fig. 3(d), where the rolls look slightly curved. The experimental run between points 3 and 4 was performed with increasing magnetic field. The curve section corresponding to $Ha > 5$ should be referred to first of all due to the most interesting behavior. The slight decrease of the Nusselt number is a result of an MHD drag effect. This effect is rather weak because of specific flow configuration. Namely a
sufficient elongation of the rolls along the magnetic field and insufficient area of the contact with the wall perpendicular to the field made the influence of the MHD braking negligible. In the final stage of the series even at the maximal value of the Hartmann number the flow did not change its shape [refer to point 4 in Figs. 3(a) and 3(e)]. In this situation the magnetic field was not able to restore the initial flow pattern with eight convective rolls.

In Fig. 4 the experiments with KOH are presented by two series. The first experimental run corresponds to ordinary convection (without magnetic field), the second one is obtained for Ha=6.5. One can see that the initial stages of the both curves coincide. But at a moderate Rayleigh number (3500<Ra<11 000) the MHD convection provides more effective heat transfer compared with the nonmagnetic case. The actual reason for this phenomenon is accounted for by the flow configuration, which might be seen in the insets of Fig. 4. It turned out that in the absence of a magnetic field the convective flow became three-dimensional already at Rayleigh numbers exceeding 3500, whereas the MHD convection remained two-dimensional even at Ra=10 500±500. One should notice that convection in KOH has a higher tendency to three-dimensional (3D) flow in the nonmagnetic case. While at the Ra<10 000 we did not observe any essential difference between the behavior of the ordinary and MHD convection in the H2SO4 solution.

We did not visualize the KOH convection by the shadowgraph method. Because of the strong chemical reactivity of the KOH solution the sapphire plates were replaced by a usual glass window and a nonreflective stainless steel bottom plate. To obtain a picture we added to the electrolyte small tracer particles. Results show the flows visualized using tracer particles. ——Ha=0, ◇—Ha=6.5.

In summary, we have demonstrated that strong electrolytes in combination with a superconducting magnet ensure a unique possibility to visualize experiments with MHD convection. The horizontal homogeneous magnetic field characterized by a Hartmann number of at least Ha=10 is strong enough to maintain the initial quasi-two-dimensional mode of convection up to values of the Rayleigh number 40 000. The influence of the magnetic field also results in reduction of the wavelength of the convective flow. For example, at Ha=9.5 and Ra=24 000, the transversal size of convective rolls is smaller by 12.5% than in the absence of a magnetic field. At the same value of Rayleigh number the convective flow with the shorter wavelength is characterized by a higher value of the Nusselt number. Thus the stabilizing effect of magnetic field in combination with the wavelength shortening leads to an increase of the Nusselt number by to 17%.

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