Generation of liquid metal structures of high aspect ratio by application of an ac magnetic field

Oleg Andreev,1 Alban Pothérat,2,a and André Thess1
1Institute of Thermodynamics and Fluid Mechanics, Ilmenau University of Technology, P.O. Box 100565, 98684 Ilmenau, Germany
2Applied Mathematics Research Centre, Coventry University, CV15FB Coventry, United Kingdom

(Received 10 December 2009; accepted 24 March 2010; published online 16 June 2010)

We study how the shape of parts obtained through the LASER cladding process can be controlled by application of an ac magnetic field by means of two simple physical models: a numerical and an experimental one. More specifically, we show that straight metallic joints of high aspect ratio can be obtained by using inductors of triangular cross-section that concentrate electromagnetic forces at the bottom of the joint. The effect is first demonstrated on a numerical model for an infinitely long joint such as: we illustrate how the joint shape can be controlled by varying the inclination of the inductor and for a magnetic Bond number $Bo_m=60$ (which measures the ratio of electromagnetic to capillary forces), we obtain a joint of aspect ratio up to 7.2. We further find that inductor angles in the range $15°–25°$ lead to joint side faces that are close to vertical. These findings are then verified experimentally by placing a liquid metal drop in a purpose built inductor of triangular cross-section. We find a good agreement between the theoretical prediction of our two-dimensional model and the real three-dimensional drop. For the highest magnetic Bond number our generator could deliver, $Bo_m=20.19$, we achieved a drop aspect ratio of 2.73. © 2010 American Institute of Physics. [doi:10.1063/1.3409075]

I. INTRODUCTION

Melting powder using LASER is a widespread technique that makes it possible to precisely control material deposition on a substrate.1,2 It finds applications in cladding, welding, and the manufacturing of small parts of complex shapes that do not offer enough access space to micromachining tools. Such parts are obtained by blowing some powder onto a substrate through a nozzle and simultaneously melting it with a LASER beam. The powder injector and the LASER source are fitted on a single tool that moves along the substrate surface and leaves a solidified joint in its wake. The part is obtained by superimposing several such layers in several tool passes, switching the LASER on and off to form cavities where desired. For the best mechanical properties and process efficiency, the number of successive layers must be kept as low as possible, and so it is desirable to obtain joints with the highest height/width aspect ratio and that are as straight as possible. Unfortunately, the drop formed by the melted powder tends to collapse under the effect of gravity and surface tension forces tend to give it a rounded shape. For electrically conducting powders, these forces can be compensated by applying a high frequency ac magnetic field at the droplet surface.3,4 Typically, magnetic fields of the order of 0.1 T with a frequency of 100 kHz are applied to the liquid metal drop which is formed while welding with nickel (density $\rho=7900$ kg m$^{-3}$, electric conductivity $\sigma=1.2 \times 10^6$ S m$^{-1}$, surface tension at an interface with air $\gamma=1.27$ N m$^{-1}$). The dominant part of the resulting Lorentz force is normal to the surface5 so when applied on the side, it tends to counteract gravity and surface tension. This is achieved in practice by fitting an inductor on the moving tool that stretches along either side of the joint (the whole process is represented on Fig. 1). Since most common inductors have a rectangular or a circular cross-section though, the magnetic field they generate is highest around mid-height through the joint, and not at the bottom, where the pressure resulting from combined gravity and surface tension is highest. This gives the joint a triangular cross-section rather than the desired rectangular one. Inductors of circular cross-section induce an even more localized maximum magnetic field that leads to drop instabilities and can leave unwanted wavy shapes in the solidified joint as on Fig. 1(c).

In this article, we propose to use an inductor of triangular cross-section pointing at the bottom of the drop so as to
concentrate the strongest magnetic field there. The axis of the triangle is then inclined until the Lorentz force distribution leads to the straightest possible side faces of the drop. In the real process, a wealth of physical phenomena are involved such as fluid motion due to the movement of the tools and thermocapillary convection induced by localized LASER heating. Convective motion is measured nondimensionally by a Reynolds number $Re = Ma/Pr$ built out of the Marangoni $Ma=(dy/dT)\Delta T/(\rho \nu k)$ and Prandtl $Pr=v/\kappa$ numbers ($\Delta T$ is a typical temperature difference over a typical distance $L$ along the surface of the drop, $\nu$ and $\kappa$ are the kinematic and thermal diffusivities). The former represents the ratio of stresses induced by surface tension gradient to viscous stresses while the second is the ratio of kinematic to thermal diffusivities. In real processes, values of $Ma \sim 10^{-10^2}$ and $Re \sim 10^{-10^3}$ indicate that some moderate to intense fluid motion should be present, which a complete model of the process would need to take into account. Our approach, by contrast, focuses on evaluating the influence of the inductor geometry on the shape of the drop so instead of attempting a full modeling of the real industrial process, we study a simpler system made of a static liquid metal drop placed in an inductor, where fluid motion and thermal effects are ignored. We shall here illustrate the physical phenomenon in two ways. First, we work out numerically the static shape of an infinitely long drop in such an inductor of triangular cross-section (Sec. II). This theoretical approach allows us to vary inductor angle and magnetic field over a large enough range to identify the physical effects and work out an optimal inductor angle (Sec. III). Second, we build a simple experiment where a drop of liquid metal, that is liquid at room temperature, is placed in the gap of a purpose built inductor (Sec. IV). The drop shapes obtained by varying inductor current and geometry are compared to our numerical results. This allows us to assess the validity range of the theoretical model and confirm that it can be used as a predictive tool to optimize inductors of more complex design.

II. THEORY

A. Governing equations

We start with a two-dimensional (2D) theoretical approach where we model the joint as a drop of liquid metal (density $\rho$, electric conductivity $\sigma$, magnetic permeability $\mu_0=4\pi \times 10^{-7}$ and surface tension $\gamma$) resting on a plane, electrically insulating substrate at $z=0$, surrounded by an electrically insulating fluid of density $\rho_a$ and placed between the two branches of an inductor (electric conductivity $\sigma_0$). Both are infinitely elongated in the $e_x$ direction, so all physical quantities are assumed invariant along $e_x$, too (see Fig. 2). Furthermore, the period of the electric current is of the order of a fraction of a millisecond, a much shorter timescale than that of the fluid motion. The high frequency oscillations of the Lorentz force are, therefore, not expected to have any visible effect on the flow, which justifies the existence of an equilibrium state. On these grounds, we shall use only the average over a period of all electromagnetic quantities, assume that mechanical quantities are time-independent, and look for steady states of the drop only.

Our first task is to determine the governing equations for the problem under these assumptions. These are valid regardless of the shape of the inductor cross-section, which we shall, therefore, not specify at this point. A total ac electrical current $I=\Re\{I_e^{\text{out}}\}$ flows in the right branch of the inductor, and is assumed uniformly distributed in its cross-section, while the opposite current flows in the left branch ($I$ denotes the complex current amplitude and $\Re$, the real part). The shape of the static drop then results from the balance between gravity, surface tension forces, and the Lorentz force, that are all included in the $(x,z)$ plane. In the absence of fluid motion, we shall also assume symmetry with respect to the $x=0$ plane. The first step in determining the drop shape is to find an expression of the Lorentz force. In the frame of our 2D approximation, the magnetic field $B$ and electric current density $J=J_x e_x$ can be expressed in terms of the electric potential $\phi$ and magnetic vector potential $A_y e_y$ as:

$$B = \nabla \times A_y e_y$$

$$J_x = -\sigma(\partial_z \phi + \partial_y A_y)$$

By virtue of our 2D approximation, $\partial_x \phi = 0$ holds in the drop so the electric current and the Lorentz force within the liquid metal are entirely determined by the potential vector $A_y e_y$. Since the imposed current is ac, any electromagnetic quantity $G$ can be expressed using complex notation $G=\Re\{G e^{j\omega t}\}$, and so the current density and the Lorentz force $F_L$ are written:

$$\bar{J}_x = -j\sigma \omega \bar{A}_y$$

$$\bar{F}_L = \bar{J}_x e_x \times \bar{B} = -\sigma \nabla A_y \partial_y \bar{A}_y.$$  

As explained at the beginning of this section, the oscillations of the Lorentz force are too fast to have any influence on the flow. Instead, the liquid metal should rather react to the average over a period of the Lorentz force, expressed as:

$$\langle F_L \rangle = \frac{1}{2} \Re \{ \bar{J}_x e_y \times \bar{B}^* \}$$
The pressure in the liquid just beneath the free surface can be obtained using Biot–Savart law:

\[ \mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{||\mathbf{x} - \mathbf{x}'||^3} d^3\mathbf{x}', \]

where \((\cdot)^*\) denotes the complex conjugate. For convenience, averaging brackets will be omitted in the rest of this work. From Eqs. (1), (3), and (6), it turns out that all electromagnetic quantities depend solely on \(A_y\). The latter is in turn obtained using Biot–Savart law:

\[ \mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{||\mathbf{x} - \mathbf{x}'||^3} d^3\mathbf{x}', \]

where \(x=(x,y,z)\) and \(\Sigma^{(D)}\) (respectively, \(\Sigma^{(J)}\)) represent the area covered by the whole drop (respectively, the whole inductor) of surface \(S^{(D)}\) (respectively, \(S^{(J)}\)), and integrating (1)-\(e\). The electric current density in the inductor is assumed homogeneously distributed in the inductor cross-section of surface area \(S^{(J)}\):

\[ \mathbf{j}^{(J)} = \frac{\mathbf{I}}{S^{(J)}}. \]

The coupled system made of Eqs. (1), (3), (5), and (7) determines all electromagnetic quantities for a given drop shape. We shall now establish how electromagnetic forces in turn determine the equilibrium shape of the liquid metal drop. The latter is represented in the \(\{x>0, z>0\}\) quarter-plane, by the curve \(x=\chi(z)\) that tracks the position of the free surface (see Fig. 2). For a static drop, the pressure field \(p\) inside the drop results from the hydrostatic balance between pressure forces, gravity, and the time-average of the Lorentz force

\[ -\nabla p + \mathbf{F}_L - \Delta p g e_z = 0, \]

where \(\Delta p = p - p_a\). Symmetry imposes that \(\mathbf{F}_L(0,z) \cdot e_z = 0\), so that integrating Eq. (9) along a path starting from any point of the surface drop at height \(z\) following an horizontal line to \(x=0\) and then a vertical line to the top of the drop located at \((0,h)\), yields:

\[ p(0,h) - \Delta p(h-z) - p(\chi(z),z) = \int_{0}^{\chi(z)} \mathbf{F}_L(x,z) \cdot e_z dx = 0 \]

The pressure in the liquid just beneath the free surface can be related to \(\chi\), as the pressure drop across the liquid metal free surface is proportional to the surface curvature \(\kappa(\chi(z))\) and surface tension \(\gamma\):

\[ p_a - p(\chi(z),z) = \gamma \kappa(z) = \frac{\gamma \chi''(z)}{[1 + \chi'(z)^2]^{3/2}}, \]

so the equation governing the shape of the drop takes the form

\[ \gamma [\kappa(z) - \kappa(h)] + \Delta p(h-z) + \int_{0}^{\chi(z)} \mathbf{F}_L(x,z) \cdot e_z dx = 0. \]

(12)

Then, normalizing lengths, \(A_y, J_y,\) and \(F_L\), respectively, by \(h_0, \mu_0 l^2/(4\pi), 1/\pi \delta^2,\) and \(\mu_0 l^2/(8\pi^2 h_0 \delta^2)\), where \(h_0\) is the drop height without magnetic field and \(\delta=\left[2/(\mu_0 \sigma \omega)\right]^{1/2}\) is the skin depth, i.e., the depth to which the magnetic field penetrates the metal, (12) takes the nondimensional form:

\[ \kappa(z) - \kappa(h) + B_0(h-z) + B_0 m \int_{0}^{\chi(z)} \mathbf{F}_L(x,z) \cdot e_z dx = 0 \]

(13)

On the top of the parameters that describe the inductor geometry, which is not specified at this stage, the model is governed by two dimensionless parameters: the Bond number \(B_0=\Delta \rho g h_0^2 / \gamma\), which expresses the ratio of gravity to surface tension forces and the modified magnetic Bond number \(B_0 m = \mu_0 l^2/(8\pi^2 h_0 \delta)\) measures the ratio of Lorentz forces to surface tension forces ignoring the skin effect, while \(S=h_0/\delta\) is the screen parameter. \(B_0\) and \(B_0 m\) are two control parameters for the problem and can be, respectively, thought of as the nondimensional volume of the drop and inductor current, which are two of the three adjustable parameters in the experiment (see Secs. II C and IV). The nondimensional form underlines that the effect of the Lorentz force on the shape of the free surface can be controlled either through the intensity or the frequency of the inductor current. In the numerical experiment, the Bond and magnetic Bond numbers were varied in the ranges of \(B_0=0.1–1.4\) and \(B_0 m=0–100\).

Finally, assuming the drop does not wet the substrate implies the boundary condition

\[ \frac{dz}{d\chi}(z=0) = 0, \]

which, by integration of Eq. (13) using Eq. (11), leads to the expression for the shape of the free surface, for a given distribution of the Lorentz force distribution \(\mathbf{F}_L(x,z)\)

\[ \chi(z) = \int_{0}^{z} \left( \frac{1}{f(z')^2} - 1 \right)^{1/2}, \]

\[ f(z) = \frac{B_0}{2} z^2 + \left[ \kappa(h) - B_0 h \right] z \]

\[ - B_0 m \int_{0}^{\chi(z)} \mathbf{F}_L(x,z') \cdot e_z dx dz' + 1. \]

(16)

B. Numerical system

We shall now describe the numerical methods used to solve the coupled system of equations found in Sec. II A. For all values of \(B_0\), we first determine the shape of the drop without any applied magnetic field (\(B_0 m=0\)) for which the target height of the drop is \(h=1\). The numerical solution is found using a shooting method where the curvature of the free surface at the top of the drop \(\kappa_0=\kappa(h)\) is the guessed parameter. The latter is initialized to a high arbitrary value and \(\chi(z)\) in Eq. (15) is obtained by a trapezoidal integration scheme of step \(\Delta z=-10^{-4}\) along the drop height. The bottom of the drop is found at the smallest value \(z_{\min}\) of \(z\) for which \(\chi(z)\leq0\). \(\kappa_0\) is then adjusted according to the sign of \(z_{\min}\) and...
Define physical parameters $B_0, B_{0m}, \beta$ (i.e. $h_0, I, \beta$)

Calculate drop shape numerically for $B_{0m} = 0$

If $B_{0m} > 0$, initialisation:
- $A_y = 0, J_y = 0, \kappa_0 = \kappa_0^{init}$
- Evaluate $A_f$

Evaluate $\mathbf{B}$ using (19)

Evaluate $A_y, J_y$ using (20, 3)

Evaluate $f(z)$ using (16)

Evaluate $\chi(z)$ for $z \geq z_{min}$ using (15)

Evaluate $A$ and $h$

$|1 - A/A_f| < 10^{-2}$

No
- if $A < A_f$, decrease $\kappa_0$
- if $A > A_f$, increase $\kappa_0$

Yes

Converged

FIG. 3. Sketch of the numerical algorithm used for the determination of the shape of the drop. The numerical procedure to obtain the shape of the drop for $B_{0m}>0$ is similar to that for $B_{0m}=0$ with two important differences: electromagnetic quantities are set to 0 and the loop iterates on the drop height $h$ until it reaches the specified target height of 1 to a relative precision of $10^{-4}$.

the process is iterated until $|z_{\min}| < 10^{-4}$. Once the solution converged, the area $A_f$ of the drop is calculated and used as a target value for all numerical solutions at the same Bond number and for $B_{0m}>0$. Those are obtained by first initializing $A_y, J_y$ in the drop and $\chi(z)$ to 0, to 1, and the electric current density in the inductor according to Eq. (8). The same shooting method as for $B_{0m}=0$ is then applied, but this time $A_y, J_y, \mathbf{B}, \mathbf{F}_L$ are evaluated at each iteration, using the available values of $\chi(z)$. After each iteration, $h$ and the drop area $A$ are evaluated and $\kappa_0$ adjusted, according to the sign of $A-A_f$. The calculation is deemed converged when $\varepsilon = |1 - A/A_f| < 10^{-3}$. This algorithm is schematically summarized on Fig. 3.

For the evaluation of electromagnetic quantities, the physical domain (which includes both the drop and the inductor) is discretized in the $(x, y)$ plane using a fixed mesh made of $n^2$ square elements where all quantities are assumed constant within each element. The contribution $\mathbf{B}^{(n)}(r)$, where $r=(x, z)$, to the integral $\mathbf{B}(x)$ in Eq. (7) from the infinitely long element $S_n \times R$ can be written as:

$$\mathbf{B}^{(n)}(r) = \frac{\mu_0}{4\pi} \int_{S_n \times R} \mathbf{e}_x \times (\mathbf{x} - \mathbf{x}') |(\mathbf{x} - \mathbf{x}')|^3 d'x'd'y'. \quad (17)$$

This integral is evaluated by decomposition into a sum of integrals over four triangular domains. Those triangles are, respectively, defined by one side of square $S_n$ and the point located at $r$ (these triangles are marked $r_01, r_12, r_23, r_30$ in Fig. 2, and corresponding integrals are, respectively, weighted $-1, 1, 1, -1$). After a tedious but straightforward calculation, and using the identity

$$\arctan(\theta) = \frac{j}{2} \ln \frac{1 - j\theta}{1 + j\theta} + 2k\pi, \quad k \in \mathbb{Z}, \quad \theta \in \mathbb{R}, \quad (18)$$

this leads to the expression of the total integral as a function of the coordinates of the four corners $(\mathbf{r}_i)_{i=0,1,2,3}$ of $S_n$, and the corners $(\mathbf{r}_i)_{i=4,5,6,7}$ of the symmetrical of $S_n$ with respect to $(O, \mathbf{e}_z)$:

$$\mathbf{B}^{(n)}(x) = \frac{\mu_0}{4\pi} j R \left( 2 \ln \prod_{i=0}^{7} K_i(r)^{(i+1)\delta_1 K_j(r)} - \ln \prod_{i=0}^{7} K_i(r)^{(i-1)\delta_1 K_j(r)} \right) \quad (19)$$

where $K_i(r) = (x_i-x) + j(z_i-z)$. The vector potential $A_y$ is obtained from (20) by virtue of (1):

$$A_y^{(n)}(r) = \frac{\mu_0}{4\pi} \left( \frac{1}{2} \left[ \ln \prod_{i=0}^{7} K_i(r)^{(i-1)\delta_1 K_j(r)} - \ln \prod_{i=0}^{7} K_i(r)^{(i+1)\delta_1 K_j(r)} \right] \right) = \frac{\pi}{2} \sum_{i=0}^{7} (-1)^i (z_i-z) (x_i-x) (z_i-z) (x_i-x)^2 ||x_i-x||^2, \quad (20)$$

Having obtained the numerical values of $\mathbf{J}$ from (3), the Lorentz force is evaluated numerically using (5) rather than (6), to circumvent the difficulty of calculating the gradients of $A_y$ numerically. Finally an under-relaxation factor of typically 0.05 is applied to the evolution of $A_y$. In practice, $Bo$ (and therefore, $A(Bo)$) is set to a smaller value than the prescribed one, in order to ensure that the drop does not come in contact with the inductor (this problem does not occur in experiments anyway, as the inductor is coated with an electrically insulating material). The value of $Bo$ is then progressively increased to the prescribed value as the drop shapes up.

The shooting procedure for the evaluation of $X(z)$ was tested on the pure hydrodynamic case of a liquid drop resting on a flat surface (see for instance Ref. 6). The maximum relative error was $10^{-5}$. Our procedure to evaluate the electromagnetic quantities was tested on the axisymmetric problem of a circular inductor, with a fixed cylindrical load of radius $R_0$ centered on the inductor axis. For an infinitely long inductor, the magnetic field induced in the load is axial, of complex amplitude $\mathbf{B}(r)/B(R_0) = \cosh(1 + j) r S_0)/\cosh((1 + j) S_0)$, where $S_0 = R_0 \mu_0 \sigma \omega / 2$ is the screen parameter (see...
for example Ref. 7). To assess the precision and the rate of convergence under grid refinement of our numerical method, we have calculated numerical solutions for this problem \( \{\overline{B}_i\}_{i \in \{1, \ldots, n\}} \) at mesh points \( \{r_j\}_{j \in \{1, \ldots, n\}} \) and reported the variations in the discrete \( L^2 \) norm of the relative discrepancy to the analytical solution \( \epsilon_2(n) = \frac{1}{\sqrt{\int_{0}^{n} |\overline{B}_i - \overline{B}(r_j)|^2/|\overline{B}(r_j)|^2}} \) against the number of cells \( n \) along the radius on Fig. 4, for \( S \in [0.58, 3.66] \). It turns out that \( \epsilon_2 \sim n^{-2} \) up to a critical value of \( n(S) \in [24, 36] \) beyond which the precision hardly improves. This establishes that our code converges well toward the correct solution and leads us to choose \( n=32 \) for all subsequent calculations.

C. Problem geometry

In the remaining of this work, all calculations will be performed for the inductor cross-section featured in the experimental part. It is triangular, with sides of length \( w_I = 10 \text{ mm} \) and \( h_I = 20 \text{ mm} \), and a right angle between these two sides, as sketched on Fig. 2. The gap between the two inductor branches is fixed at \( g_I = 6 \text{ mm} \). The angle of inclination of the side facing the drop with respect to the vertical is denoted \( \beta \). On the top of two control parameters that we already identified \( Bo \) and \( Bo_m \), this parameterization of the inductor shape implies that the full problem is entirely determined by the addition of four additional nondimensional numbers \( \beta, h_I/w_I, h_I/g_I, \) and \( h_I/h_0 \). In order to analyze the influence of the inductor geometry, \( Bo, Bo_m, \) and \( \beta \) will be varied systematically, while \( h_I/w_I \) and \( h_I/g_I \) will be left constant. Since \( Bo \) will be varied by changing the drop height \( h_0 \), the ratio \( h_I/h_0 \) will vary too. In all cases, it will, however, remain larger than unity, a regime for which the top end of the inductor (and, therefore, the ratio \( h_I/h_0 \)) has little influence on the shape of the drop. It is interesting to notice that this parameter becomes important for high drops (\( h_I/h_0 < 1 \)) and small inductor angles where it can be expected to determine the drop stability. For larger angles, the top of the inductor is further away from the drop so \( h_I/h_0 \) has less influence.

III. RESULTS OF THE NUMERICAL SIMULATIONS

A. Aspect ratio of the drop, case without magnetic field (\( Bo_m=0 \))

In the welding process, the ideal cross-section of the joint would be a rectangle of high aspect ratio. The shape of the droplet we are studying here is much more complex than this ideal rib shape. In order to measure how much it departs from it, we, therefore, parameterize it by defining the rectangle of same area \( A \) as the cross-section of the drop, of height \( b \) and width \( 2w \), that least deviates from the drop shape in the least-squares sense. From this “equivalent ideal rectangle,” we define the aspect ratio of the drop as \( \alpha = b/(2w) \), as well as the rms deviation to flatness from the top of the drop \( \delta_s \), from its side \( \delta_w \), and the total deviation \( D \) as (see Fig. 5)

\[
D^2 = \min_{2w=b=A} \left[ d_s^2(w,b) + d_w^2(w,b) \right],
\]

\[
d_s^2(w,b) = \frac{2}{Ab} \int_0^b \left[ \chi(z) - b \right]^2 dz,
\]

\[
d_w^2(w,b) = \frac{2}{Aw} \int_0^b \left[ \chi^{-1}(z) - w \right]^2 dz.
\]

The first calculations are done in the absence of magnetic field. Figure 6 shows the corresponding drop shapes, and equivalent rectangles for several values of \( Bo \), while the variations with \( Bo \) of the aspect ratio \( \alpha \), of the top and side deviations \( d_s \) and \( d_w \) are plotted on Fig. 7 (top). The aspect ratio of the drop \( \alpha \) decreases when the Bond number increases. Small drops, with a small Bond number are domi-
nated by surface tension effects that give them a near-spherical shape, with $\alpha = 1$, and $d_s = d_t$. When $Bo$ (or the drop volume) is increased, gravity progressively takes over surface tension effects so that larger drops become flatter with their top deviation quickly decreasing when $Bo \geq 1.5$, while the side deviation increases to reflect the rounded shape of the side edge. Increasing the drop volume, therefore, takes it away from the ideal rectangular shape sought for the welding joint.

### B. Effect of the magnetic field ($Bo_m \neq 0$)

We now calculate the shape of the drop for several magnetic Bond numbers $Bo_m$, for a fixed inclination angle of the inductor $\beta$ (see Fig. 2). We first set the Bond number to $Bo=0.8$ and increase the magnetic Bond number. The corresponding drop shapes are shown in Fig. 8 (left) and clearly illustrate the “squeezing” effect of the magnetic field that stretches the drop vertically. The side of the drop also becomes flatter under the influence of the applied magnetic field. The top and bottom regions, on the other hand, remain locally spherical.

Figure 8 (right) shows drop shapes and corresponding rib structures when the Bond number varies within the range 0.3–1.4, while the magnetic Bond number is held constant at $Bo_m=32$. For very large drops ($Bo \approx 1$), electromagnetic effects are slightly excessive, as the magnetic pressure becomes too high around mid-height of the drop, resulting in a pincushion deformation that slightly impairs the drop flatness. This saturation of the electromagnetic effects is confirmed by the graph on Fig. 7 (bottom), where the deviations to the ideal rib shape and the aspect ratio for the drop from Fig. 8 (right) are plotted against the Bond number: while the side deviation $d_s$ decreases noticeably for $Bo$ in the range 0.3–1.2, it varies little for larger drops ($Bo > 1.2$). Still, comparison to Fig. 7 (top) clearly shows that the application of the ac magnetic field efficiently prevents the drop from collapsing under the effect of gravity and keeps its side flat as the aspect ratio $\alpha$ increases with $Bo$ instead of decreasing when $Bo_m=0$.

In order to further quantify the influence of the ac field, we now turn our attention to Fig. 9, that shows the variations in the aspect ratio $\alpha$ with $Bo_m$ for several fixed values of the Bond number $Bo$. In all cases, the aspect ratio increases monotonously with $Bo_m$, at an all the greater rate as $Bo$ is high. The maximum aspect ratio we could achieve was $\alpha = 7.2$, for $Bo=1$ and $Bo_m=60$. Beyond these values, the numerical
scheme became unstable, probably because the drop itself became unstable, as our experiments suggest (see Sec. IV).

Plots of the top and side deviations to the ideal rib shape versus $B_{0_m}$ for several fixed values of $Bo$ in Fig. 9 show that

the presence of the magnetic field also “flattens” the side of the drop very efficiently, as the side deviation $d_s$ indeed quickly decreases with $B_{0_m}$. As for the aspect ratio, it decreases at a faster rate for higher values of $Bo$ (larger drops). The deviation to the ideal rib shape on the top of the drop $d_t$ is, on the other hand only weakly affected by the presence of the magnetic field. $d_t$ even slightly increases for $B_{0_m} < 20$, then slightly decreases for $B_{0_m} > 20$ to reach an asymptotic value $d_t = 0.042$ at high $B_{0_m}$. This can be understood as the shape of our inductor tends to concentrate the field lines around the bottom of the drop, so the field is much weaker around the top. The top region, therefore, behaves as a small drop of its own, of radius $w$ and whose shape is essentially determined by the balance between gravity and surface tension, and, therefore, by the Bond number scaled on $w$, $Bo w^2/h_0^2$, and not by the magnetic Bond number $B_{0_m}$. Similarly to the case without magnetic field, when this number becomes small (i.e., for small $w$), surface tension becomes locally dominant and the top of the drop tends to assume a hemispherical shape. As far as the welding process is concerned, the top of the drop could be flattened by addition of a secondary inductor near the top of the drop, that would increase the magnetic field locally.

C. Influence of the inductor angle $\beta$

Having now established that the application of an ac magnetic field on both sides of the drop could significantly increase the aspect ratio of the drops while keeping its side flat, we shall now look for the inductor geometry that achieves this effect most efficiently.

The influence of the angle of inclination of the inductor $\beta$ on the shape of the drop is shown in Fig. 10 for constant

![Figure 9](image1.png)  
**FIG. 9.** Aspect ratio (top) and deviation in the liquid metal drop from the rib structure vs magnetic Bond number $B_{0_m}$, for several values of $Bo$ and $\beta=15^\circ$.

![Figure 10](image2.png)  
**FIG. 10.** Shape of the liquid metal drop and corresponding rib structures for different angles of inclination of the inductor $\beta$, fixed magnetic Bond number $B_{0_m}=36$ and Bond number $Bo=0.8$.
magnetic Bond number $B_{0m}=36$ and Bond number $B_0=0.8$. The inductor closest to the conventional “straight” inductor design ($\beta=0$) leads to a pearlike drop shape that is clearly not optimal [see curve 1 in Fig. 10(a)]. A widely opened inductor ($\beta>30^\circ$) does not yield the sought rib shape either, as it laces the drop tight in the bottom region, leaving the top part weakly influenced by electromagnetic effects and, therefore, quite round. This results in and inverted pear shape that can be unstable as observed in our experiments (see Sec. IV). When $\beta$ spans the middle range $15^\circ$ to $25^\circ$, the drop shape hardly varies and is closest to a rectangular one. This range is, therefore, the optimal one for $\beta$ to obtain both a high aspect ratio and a flat side of the drop for $B_0=0.8$ and $B_{0m}=36$. The variations in the aspect ratio of the drop $\alpha$ versus $\beta$ for several values of $B_{0m}$ and a constant Bond number $B_0=0.8$ are shown in Fig. 11. The maximum aspect ratio $\alpha=6.2$ is achieved for $B_{0m}=60$ and corresponds to $\beta=15^\circ$. For smaller magnetic Bond numbers, the value of $\alpha$ that achieves the maximum aspect ratio shifts toward slightly higher values in the region of $35^\circ$ to $40^\circ$.  

IV. EXPERIMENT  

A. Experimental setup

The experiments were performed in a rectangular vessel depicted on Fig. 12, of 200 mm length, 150 mm width, and 50 mm depth, filled with a 5% aqueous solution of hydrochloric acid of density $\rho_h=1000$ kg m$^{-3}$ that ensured both cooling and protection against oxidation of the liquid metal drop. The working fluid for the drop was an eutectic alloy of Ga–In–Sn of density $\rho_{m}=6360$ kg m$^{-3}$, electrical conductivity $\sigma=3.4\times10^{4}$ S m$^{-1}$ and surface tension at the interface with the solution of $\gamma=0.61$ N m$^{-2}$. It is liquid at room temperature and its physical properties are consistent with those of liquid metals used in the real process (such as nickel, mentioned in Sec. I). The surface tension was measured experimentally as follow: we placed on the substrate a drop that was large enough to be very flat in the $(x,z)$ plane and circular in the $(x,y)$ plane. The drop radius in the latter plane was a lot larger than that at the edge of the drop in the former plane, so that surface tension effects resulted almost exclusively from the curvature observed in the $(x,z)$ plane, as in a purely 2D case. In the particular case where the drop is so large that the curvature vanishes at the top, the drop reaches its maximum achievable height $h_{0}^{\text{max}}$. $h_{0}^{\text{max}}$ can be expressed analytically by eliminating $B_0$ between Eq. (16) taken at $z=0$ and 1, respectively: it turns out to be related to the surface tension between the liquid metal and the electrolyte by $h_{0}^{\text{max}}=2\sqrt{\gamma/\Delta pg}$. Measuring $h_{0}^{\text{max}}$ experimentally, therefore, yields a precise indirect measure of the surface tension. In the present case, $h_{0}^{\text{max}}=6.79$ mm corresponds to a maximum Bond number of $B_0=4$.

The side walls of the vessel were transparent to allow for direct optical observation (see Fig. 12). A 0.5 mm thick, 15 $\times$ 30 mm$^2$ sapphire plate was inserted in the bottom of the vessel at the location of the drop to provide a nonwetting substrate for it, that satisfied boundary condition (14). A thermostat-controlled cooling system was fitted underneath this plate, that provided extra cooling for the drop. The ac magnetic field was generated by a 120 mm long linear inductor of hollow triangular cross-section, with walls made of copper (electric conductivity $\sigma_{c}=5.88\times10^7$ S m$^{-1}$), of thickness $t=1.5$ mm. The inner side (i.e., facing the drop side) of the triangle was at an angle $\beta$ with the vertical. This design, with a cross-section identical to that studied numerically in the previous sections ensured that the magnetic field lines were concentrated near the bottom of the drop, were pressure forces are highest, and further allowed us to cool the inductor by letting a water flow pass through it. The inductor was connected to the poles of 150 kHz ac electric current generator that delivered up to 911A for a maximum magnetic Bond number of $B_{0m}=20.19$. At higher frequencies, the drop heated very quickly and induced a strongly inhomogeneous optical index in the surrounding electrolyte that made observations impossible. The angle of inclination of the inductor was varied in the range $15^\circ$–$45^\circ$. The drop height was between 2.27 and 6.79 mm and corresponding Bond numbers...
lied within the interval (0.35–4). The height and area of the drop were extracted from pictures taken from the side of the drop alongside a 5 mm ruler used as a reference. Once the contour of drop cross-section identified, these quantities were, respectively, obtained directly and by integration.

At the beginning of each set of measurements, the drop was placed between the two branches of the inductor using a glass pipette. It is noteworthy that no additional mechanical confinement was required. As soon as the inductor was switched on, the drop underwent a strong squeezing force in the \(x, z\) plane so it naturally stretched in the \(y\) and \(z\) directions. Note that in none of the cases studied quantitatively did the liquid metal come in contact with the insulating coating of the inductor, even before the latter was switched on. This did happen, however, at \(B_{om}=4\) with the inductor switched off (Fig. 16, left), but not when \(B_{om}=20.19\) (Fig. 16, right). Once the drop in place, experiments at \(B_{om}=0\) were performed before those at \(B_{om}>0\) in order to obtain the values of \(Bo\) through the measure of \(h_0\), taken directly from pictures similar to those of Figs. 14 and 15. The setup symmetry implied that in all cases, the drop cross-section in the \((x,z)\) plane was highest and widest in the middle of the drop \((y=0)\), with no other maxima along \(e_x\), \(h_0\) and the drop shapes depicted on Figs. 14 and 15 are thus representative of the drop cross-section in this particular plane.

In order to evaluate the extend to which temperature variations could influence our results by affecting the physical properties of the metal and the electrolyte, we have observed the shape of a drop at rest with no magnetic field applied when the temperature of the surrounding electrolyte was varied in the range 20 °C–65 °C but did not notice any significant variation.

**B. Comparison between experiment and numerical simulations**

Figure 14 shows the steady states of the drop for a magnetic Bond number in the range 0–20.19. These pictures were obtained keeping the volume of the liquid metal drop constant. For the corresponding drop without applied magnetic field, the Bond number was \(Bo=1.4\). Since the drop was three-dimensional (3D), however, it became shorter (along the inductor) when \(B_{om}\) was increased and, unlike in our 2D numerical simulations, the drop area seen from the side increased from \(A=0.24\) cm\(^2\) to \(A=0.34\) cm\(^2\). The profiles obtained by the 2D numerical simulations of Sec. III for the values of \(A\) corresponding to each drop are reported on the pictures. On overall, the agreement between experiment and 2D numerical simulations is very good, considering the simplicity of our theoretical approach. For \(B_{om}=0\), [Fig. 14(a)] the height of experimental drop is about 4.5% lower.

FIG. 13. Top (top) and side (bottom) views of two drops with the same Bond number \(Bo=2.18\) for \(B_{om}=0\) (left, drop is 3.7 mm high and 5.8 mm diameter) and \(B_{om}=20.19\) (right, drop is 7.5 mm high, 4.2 mm along \(e_x\) and 4.9 mm along \(e_y\)).

FIG. 14. (Color online) Pictures of the liquid metal drop obtained experimentally for a constant Bond number \(Bo=1.4\), constant inductor angle \(\alpha=15^\circ\), and for different values of the magnetic Bond number. Solid lines represent the results of the 2D numerical simulations. Lengths are normalized by \(h_0^{\max}\).
than that obtained numerically. This is a direct consequence of the 2D approximation in our numerical simulations: near the top, surface tension dominates gravity and the shape of the real 3D drop is indeed determined by the two components of the curvature in both $x$ and $y$ directions. In two dimensions, the pressure drop across the surface results from the surface curvature in only one direction, that must, therefore, be higher than in 3D. This results in the 3D drop having a flatter top, leading to a wider drop, of smaller height. This small height discrepancy between the experiments and 2D numerical simulations decreases when the magnetic Bond number is increased, as the influence of surface tension is reduced.

For large magnetic Bond numbers ($Bo_{m} > 10$), the drop becomes nearly circular in the $(x,y)$ plane, so electromagnetic forces are not only balanced by surface tension related to the curvature in the $(x,z)$ plane (as they are in the 2D numerical simulations), but also by surface tension in the $(x,y)$ plane. Below the top, in the upper half of the drop, where electromagnetic forces are weaker, the $(x,z)$ surface curvature of the real 3D drop is, therefore, lower than that of the 2D one. This effect is illustrated on the top and side views of the drop in Fig. 13. It results in a more triangular, pointier drop, in the experiments than in the numerical simulations. As a consequence, the experimental drop tends to be narrower than the simulated one in its top half but also wider in its bottom half as mass is redistributed there. Despite this effect, gravity-induced pressure becomes important in the bottom half so the relative importance of surface tension in the $(x,y)$ plane is reduced there and 2D numerics are in slightly better agreement with experiments in this region.

Figure 15 shows pictures of the drop obtained for a constant magnetic Bond number $Bo_{m}=20.19$, for several values of the Bond number. For this value of $Bo_{m}$, we could obtain a drop of aspect ratio of 2.73. Comparison with the drop shape predicted by the numerical model confirms that the highest discrepancy between experiment and 2D numerics happens when surface tension dominates and more importantly, when the surface curvature in the $(x,y)$ plane, neglected in the 2D simulation becomes important. In this regard, the worst discrepancies are found at the lowest values of the Bond number. Interestingly, for such a high value of $Bo_{m}$ as $Bo_{m}=20.19$, even when the drop is nearly circular in the $(x,y)$ plane ($Bo=1.54$ and 1.76), the agreement between experiments and 2D numerics remains reasonably good, so it can be concluded that in the situations most relevant to the welding process, where electromagnetic forces are high enough to achieve a large aspect ratio of the drop, the simple 2D model from Sec. III yields a realistic drop shape with, importantly, the correct aspect ratio.

Lastly, Fig. 16 shows a drop obtained for a large angle of inclination of the inductor. The inverted pear shape found in the numerical simulations is so strongly accentuated here, that the drop starts to oscillate in the $(x,z)$ plane (i.e., it alternately leans toward either branch of the inductor) and becomes unstable. The same phenomenon appears for drops of aspect ratios higher than the stable drops presented earlier as the top of the drop escapes the influence of the magnetic field. This effect, therefore, strongly depends on the inductor height.

V. CONCLUSIONS

We have developed a simple numerical model to calculate the static shape of a liquid metal drop under the influ-
ence of an ac magnetic field. Its assessment against experiments performed on a liquid metal drop placed in an inductor has proved that it could predict the equilibrium shape of the drop realistically enough for a parametric analysis. Such an analysis was performed in order to optimize the angle of inclination of an inductor of triangular cross-section placed on either side of the drop. The main outcome was that drops of high height-to-width aspect ratio could be obtained by using an inductor of triangular cross-section that concentrates the magnetic field near the bottom of the drop. In the range of parameters we investigated \( Bo \in [0.35, 4] \) and \( Bo_m \in [0.60] \), the drop shapes closest to rectangular were achieved for an inclination of the inductor elements at an angle between 15° and 30° to the vertical, depending on the value of the magnetic Bond number \( Bo_m \). Aspect ratios of up to 2.73 could be obtained for \( Bo_m = 20.19 \) in the experiment, while numerical simulations predicted aspect ratios of up to 7.2 for \( Bo_m = 60 \), which our experimental setup could not reach. An important aspect in view of the application is that these high values of attainable aspect ratio are very robust to reasonable variations in the inductor angle.

We have singled out the influence of the inductor design on the static shape of the drop. For direct application to the real process itself though, a more refined model taking into account fluid motion due to convection and tool motion should be implemented in further studies. Also, our approach can easily accommodate more complex cross-sections of the inductor to suit the exact joint shape required for a particular application so it could also be used to optimize the cross-section of the inductor branches themselves. Another interesting point for further investigation is the question of the drop stability, as instabilities might be responsible for the unwanted wavy pattern observed in the real process. Such instabilities are also known to develop when a ac magnetic field is applied to a free surface, \(^8\)\(^\sim\)\(^10\) and the condition of their appearance in the configuration studied here could be analyzed by modifying the numerical model developed in this paper.

ACKNOWLEDGMENTS

The authors are grateful to the Deutsche Forschungsgemeinschaft for their financial support in the framework of Schwerpunktprogramm “Erweiterung der Prozessgrenzen bei der Werkstoffbearbeitung mit Laserstrahlung” (SPP 1139/3, Grant No. PO1210/2-1). We would also like to thank Dr. Markus Dolles and Professor Johannes Wilden (TU Berlin) for their helpful input on the industrial process.