Electromagnetically induced chaotic mixing in a pipe mixer

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A numerical investigation is carried out to study the mixing by chaotic advection in an electromagnetically driven pipe mixer. It consists of a pipe with two inner electrodes which are energized alternately. An externally applied magnetic field along the flow direction interacts with the local electric currents inducing a spatially varying electromagnetic body force in the model fluid. The system is an extension of Aref’s blinking vortex model to three dimensions. The Lagrangian motion of passive tracer particles is numerically simulated to quantify the degree of stirring. The model predictions indicate that the chaotic mixing is strongly dependent on the modified Hartmann number, the electrode switching frequency and the electrode separation distance. A comparison with numerical simulation results obtained using a dimensional model with glass melt shows that the chaotic mixing behavior of the glass melt is very close to the non-dimensional model predictions. The computational results presented here will be useful for developing efficient glass melt homogenisation systems.

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1. Introduction

High energy laser (HEL) applications require glass windows with a very high optical quality (Bayya et al., 2006). Defects like bubbles and crystalline particles in the glass window give rise to scattering and refractive index inhomogeneity inducing optical distortion, which is highly undesirable for such applications. HEL glass windows, particularly of low melting and low viscosity glasses, are made by stirring the molten glass during cooling without the use of a mechanical stirrer within the glass, by rotating the mold or crucible in which the glass is cooling (Tran, 2009). Electromagnetically induced chaotic mixing in a pipe mixer can be a promising substitute for making high quality optical glasses as it eliminates the complex rotating system. Such a device can be well adapted for a continuous production. It is also superior to conventional static mixers which lead to pressure drops that are so high that the process becomes impractical for highly viscous glass melts.

Chaotic advection plays a significant role in a variety of mixing applications in engineering, especially in systems with low Reynolds numbers. In chaotic mixing, various layers of fluid are bent and folded repeatedly so as to bring initially distant parts close together and separate initially close parts exponentially farther. As a result, the interfacial area between the fluid-layers increases, whereas the thickness of each layer or striation decreases. These complementary actions improve the mixing process substantially. The theory of chaotic mixing has been well developed due to the pioneering work of Aref and co-workers as well as Ottino and co-workers (Phelan et al., 2008). A thorough review of the key concepts in chaotic mixing is given by Ottino (1989).

In Aref’s well-known blinking vortex model, the flow is induced by two corotating point vortices, separated by a fixed distance (2b), that blink on and off periodically with a constant period (T) in a two-dimensional, inviscid, incompressible fluid in a circular domain of a given radius (a) (Aref, 1984). For the first half of the period, the vortex with a strength T resides at (b,0) and for the remainder of the period, it remains at (−b,0) where b < a. The dynamics of the blinking vortex flow are governed by two dimensionless parameters: μ = 2π2a2T/a, a dimensionless period of the oscillation, and β = b/a, a dimensionless amplitude of the oscillation. For a given β, chaotic advection is localized when μ is small enough. As μ increases, the chaotic region grows and the extent of homogenisation increases. Mixing is greatly enhanced as the transverse intersection of the time-dependent streamlines increases.

Efficient mixing can be produced in pipe flows by spatial changes along the axis of the pipe. Two successive streamline portraits, say at z and z + Δz for spatially periodic flows, when superimposed, should show intersecting streamlines when projected onto x – y plane to create the chaotic advection (Ottino and Wiggins, 2004). The classical partitioned-pipe mixer (Khhkar et al., 1987) and the standard Kenics static mixer (Hobbs and Muzzio, 1997) are typical examples for the spatially periodic pipe mixers. In these mixers, a cross-sectional motion is induced either by the rotation of the pipe, in the case of the partitioned-pipe...
mixer, or by the static mixing element with a helical twist, in the case of the Kenics mixer, whereas the axial flow is caused by a pressure gradient. The main difference between the two mixers is that the direction of rotation of the cross-sectional flow is same in the adjacent elements of the partitioned-pipe mixer while it is opposite in the static mixer. The mixing strength in a partitioned-pipe mixer can be expressed in terms of the product of the characteristic strain rate of the cross-sectional flow and the average residence time based on the axial velocity. The analogous parameters in a Kenics static mixer are the pitch and aspect ratio of each helical element.

The present study has been motivated by the experimental works reported in the literature to investigate the influence of the Lorentz forces on homogenisation of electrically heated glass melts (Hülsenberg et al., 2004; Krieger et al., 2007). These studies demonstrate the capability of the Lorentz forces imposed by an externally applied magnetic field to effectively stir the glass melt in which an electric field is maintained by means of a pair of immersed electrodes. This permits the possibility of developing a spatially periodic blinking vortex flow model. The goal of the present work is to systematically investigate electromagnetically induced chaotic advection in such a spatially periodic blinking vortex flow model.

The paper is organized as follows. The mathematical model used for investigating the present problem is presented in Section 2 and implementation of the numerical solution is discussed in Section 3. Computational results and their discussion are given in Section 4, which is followed by final conclusions in Section 5.

2. Mathematical model

The problem to be considered here is defined in Fig. 1, which illustrates the configuration of the electromagnetically driven pipe mixer. It consists of an outer pipe, with diameter \( D \) and length \( L \), and two inner rod electrodes, with diameter \( d \) and length \( b \), which are separated by a radial distance \( h \). The inner electrodes are energized alternatively while the outer pipe is kept at zero electric potential invariably. The potential difference between the outer pipe and the inner electrode gives rise to an electric current in the plane perpendicular to the axis of the pipe. When a steady, uniform external magnetic field is applied along the flow direction, the interaction between the magnetic field and the current density field results in an electromagnetic body force field in the electrically conducting fluid. This Lorentz force field is capable of twisting the flow around the inner electrodes. The time-periodic energization of the inner electrodes can lead to transverse intersection of the successive streamline portraits when projected onto the \( x - y \) plane suggesting the occurrence of the chaotic advection.

The model fluid is assumed to be Newtonian. The time-dependent flow inside the pipe mixer can be computed by solving the unsteady Navier–Stokes equation using the Boussinesq approximation:

\[
\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla p + \nabla \cdot (\eta \nabla \mathbf{V} + \nabla \mathbf{V}^T) - \rho g b (T - T_0) + \mathbf{J} \times \mathbf{B},
\]

(1)

together with the incompressibility condition,

\[
\nabla \cdot \mathbf{V} = 0.
\]

(2)

where \( \mathbf{V} \) is the velocity vector; \( \mathbf{J} \) is the electric current density; \( \mathbf{B} \) is the externally applied magnetic flux density; \( p \) is the pressure; \( g \) is the acceleration due to gravity; \( \rho, \beta, \eta \) are the density, coefficient of thermal expansion and viscosity, respectively. The ratio of the electric body force to the electromagnetic body force can be expressed as \( (\epsilon \nabla \sigma \nabla \phi) / (\sigma^2 \beta) \), where \( \epsilon \) is the electric permittivity; \( \sigma \) is the electrical conductivity and \( \phi \) is the electric scalar potential. Eq. (1) assumes that the electric body force \( (\epsilon \nabla \sigma \nabla \phi) / \sigma \) is negligible compared to the electromagnetic body force \( (\sigma \nabla \phi \beta) \) as \( \| (\epsilon \nabla \sigma \nabla \phi) / (\sigma^2 \beta) \| \ll 1 \) for the present problem.

The temperature \( T \) in the model fluid can be obtained by solving the energy conservation equation:

\[
\rho c_p \frac{\partial T}{\partial t} + \rho c_v \mathbf{V} \cdot \nabla T = \nabla \cdot (k \nabla T) + \frac{\mathbf{J}^2}{\sigma},
\]

(3)

where \( c_p \) is the specific heat; \( k \) is the thermal conductivity; \( \sigma \) is the electrical conductivity and the term \( \mathbf{J}^2 / \sigma \) is the volumetric heat source on account of Joule heating. The viscous heating is neglected as the Brinkman number, \( Br = \eta \beta V_b / \kappa_0 \Delta T \), satisfies the condition \( Br < 1 \).

The electric current density \( \mathbf{J} \) can be expressed by the generalized Ohm’s law (Moreau, 1990):

\[
\mathbf{J} = \sigma \left( -\nabla \phi + \mathbf{V} \times \mathbf{B} - \frac{\partial \mathbf{A}}{\partial t} \right),
\]

(4)

where \( \phi \) is the electric scalar potential and \( \mathbf{A} \) is the magnetic vector potential. The first term \( -\sigma \nabla \phi \) results from the spatial distribution of the electric scalar potential. The second term \( \sigma \mathbf{V} \times \mathbf{B} \) is the induced current density due to the motion of the electrically conducting fluid in a magnetic field. For a very small magnetic Reynolds number \( (Re_m = \mu_b \sigma_0 V_0 D) \), where \( \mu_b \) is the magnetic permeability) the induced current density due to the motion of the fluid can be neglected. The third term \( -\sigma \mathbf{A} / \varepsilon t \) is the induced current density due to a time varying magnetic field and is zero for a steady (static) magnetic field. Thus, the expression for the electric current density becomes \( \mathbf{J} = -\sigma \nabla \phi \) for the present system. The continuity of the electric current states that \( \nabla \cdot \mathbf{J} = 0 \). Therefore, the electric current density in the model fluid can be obtained by solving the Laplace equation:

\[
\nabla \cdot (\sigma \nabla \phi) = 0,
\]

(5)
For time-periodic energization of the inner electrodes, the three-dimensional flow in the pipe mixer becomes time-periodic and its numerical simulation requires a substantial amount of computational time and resources. However, if spatially periodic, time-independent boundary conditions are used to describe the conditions of the inner electrodes, a steady three-dimensional flow field can be obtained easily. The Lagrangian particle motion in the steady three-dimensional flow can be numerically simulated to study the influence of various governing parameters on the mixing behavior of this flow. The results, thus obtained, can be useful to understand the mixing behavior of the time-periodic three-dimensional flows. Therefore, a steady three-dimensional model is used here to describe the flow in the electromagnetically driven pipe mixer. Following non-dimensional equations govern the electromagnetically driven pipe mixer operating under isothermal conditions:

\[ \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{V} + \frac{M}{Re} (\mathbf{J} \times \mathbf{B}), \]

(7)

\[ \nabla \cdot \mathbf{V} = 0, \]

(8)

\[ \nabla^2 \varphi = 0, \]

(9)

\[ \mathbf{J} = -\nabla \varphi. \]

(10)

All quantities used in Eqs. (7)–(10) are dimensionless. The non-dimensionalization was effectuated by using \( D, V_0, D/V_0, \varphi_0, \sigma_0, \sigma_0 D/V_0, \) and \( B_0 \) as reference scales for length, velocity, time, electric potential, current density and magnetic flux density, respectively. Here \( V_0 \) is the average axial velocity, \( \varphi_0 \) is the electrode potential and \( B_0 \) is the externally applied magnetic flux density in the direction of flow. The dimensionless numbers \( Re \) and \( M \) are, respectively, the Reynolds number and the square of the modified Hartmann number which will be defined below. \( Re \) is the representative of the ratio of an inertial force to viscous force and \( M \) is the representative of the ratio of the Lorentz force to viscous force. Using the above scales, \( Re \) (the Reynolds number) can be expressed as

\[ Re = \frac{DV_0}{v}, \]

(11)

and \( M \) (the square of the modified Hartmann number) can be expressed as

\[ M = \frac{\sigma_0 \varphi_0 B_0 D}{\rho v V_0}. \]

(12)

It should be noticed that the modified Hartmann number \( M = \sqrt{\sigma_0 \varphi_0 B_0 D/\rho v V_0} \) differs from the conventional Hartmann number \( H = \frac{B_0 D}{\sigma_0} \) in that the former describes the ratio between the imposed Lorentz forces and the viscous forces, whereas the latter describes the ratio between the induced Lorentz forces and the viscous forces. The conventional Hartmann number is negligibly small in the present application. Unlike in the Stokes flow, the inertial term in Eq. (7) cannot be neglected in the present problem for which \( Re \geq 1 \) and \( 0 \leq M \leq 1000 \).

In the present study, \( L = 10D \) and \( d = D/5 \) are used. The dimensionless radial separation distance between the left and right electrodes is

\[ s = \frac{b}{D}. \]

(13)

All velocity components were set to zero at the solid boundaries while \( (V_x=0, V_y=0, V_z=1) \) was specified at the inlet boundary and \( p=0 \) was specified at the outlet boundary. The inlet and outlet boundaries were treated as electrically insulated. The model includes an entrance zone and an exit zone, each having a unit length, in which both the pipe and inner electrode potentials were set to zero. This implies that the Lorentz force can exist only in the zone \( 1 \leq z \leq 9 \). In this zone \( (1 \leq z \leq 9) \), the electric potential on the pipe wall was set to zero while \( \varphi = \max(0, \sin(2\pi N_i (z-1))) \) was specified on the left electrode wall and \( \varphi = \max(0, -\sin(2\pi N_i (z-1))) \) was specified on the right electrode wall. Here \( N_i \) is the dimensionless switching frequency at which the left and right electrodes are energized to produce a spatially periodic electric field. In other words, \( 1/N_i \) is the dimensionless periodicity length of the spatially varying electric field.

3. Numerical solution

The model equations, Eqs. (7)–(10), were solved using the commercial computational fluid dynamic software FLUENT™. A GAMBIT-generated non-uniform structured grid with 910400 cells was used in this study. Although symmetry (Cortelezzi and Mezić, 2009) and mapping (Anderson and Meijer, 2000) can be used to obtain the Poincare sections relatively faster in steady three-dimensional incompressible flows with rotational symmetry, local stretch ratios and progressive snapshots of the spread of tracer particles were used in the present study to compare the extent of mixing under different operating conditions of the pipe mixer. In order to study the chaotic advection in the pipe mixer, the Lagrangian motion of 25,600 passive tracer particles were tracked employing the discrete phase model of FLUENT™. These particles, initially distributed uniformly between points \( (x=-0.005, y=0, z=1) \) and \( (x=0.005, y=0, z=1) \), were tracked using the Runge–Kutta scheme with a time step of \( \Delta t = 0.001 \).

The box counting method (Liu et al., 1994) was utilized to quantify the particle dispersion in the pipe mixer. In this study, positions of the Lagrangian particles at a given time are projected onto a given horizontal plane. (This is analogous to a snapshot of the particles taken by a camera at one end of the pipe with an illuminating lamp on the other end.) As the box size is related to the number of particles tracked, the box size and the number of particles are chosen such that 98% of the boxes would contain at least one out of the 25,600 randomly distributed particles. The degree of mixing can be quantified in terms of a stirring index, \( \varepsilon \), which is defined as

\[ \varepsilon = \frac{1}{K} \sum_{i=1}^{K} \omega_i, \]

(14)

where \( K \) is the total number of boxes used and \( \omega_i \) is a weighting factor for the \( i \)th box (Kim and Beskok, 2007). The weighting factor \( \omega_i \) can be defined in terms of the number of particles in the box \( (n_i) \) and number of particles per box for a homogeneous mixing state \( (n_{mix} = 25600/K) \) as

\[ \omega_i = \frac{n_i}{n_{max}} \quad \text{if} \quad n_i \leq n_{max}, \]

\[ \omega_i = 1 \quad \text{if} \quad n_i \geq n_{max}. \]

4. Results

4.1. Effect of the modified Hartmann number

In order to illustrate the effect of the modified Hartmann number on chaotic mixing in an electromagnetically driven pipe
mixture, a non-uniform distribution of a passive scalar was introduced into the pipe inlet and its behavior in an Eulerian framework at high Péclet number was monitored along the pipe length. A similar approach was reported by Rodrigo et al. (2003). The concentration of the passive scalar inside the mixer was obtained by solving numerically the scalar transport equation:

$$\mathbf{V} \cdot \nabla C = \frac{1}{Pe} V_0 \nabla^2 C,$$  \hspace{1cm} (15)

where $C$ is the species concentration and $Pe$ is the Péclet number. Here the Péclet number is defined as $Pe = V_0 D/\rho_0$, where $D_0$ is the mass diffusivity. Since the present study emphasizes mixing by advection, a high value of $Pe=1000$ was used for the numerical solution of Eq. (15) to keep the mass transport by diffusion much smaller than that by advection. The pipe wall was assumed to be non-diffusive. The inlet concentration was specified as a function of the $x$- and $y$-coordinates, $C_0=x^2+y^2$, which has a standard deviation $\sigma_C=0.0727$ for the given geometry with $s=0.5$. Standard deviations of the species concentration across the pipe cross-section along the pipe length for different values of $M$ with $Re=1$, $Ns=12$ and $s=0.5$ are compared in Fig. 2. For $M=0$, the $\sigma_C$ at the pipe exit (i.e. $z=10$) is $0.0328$. Fig. 2 shows that $\sigma_C \approx 10^{-4}$ at $z=3$ when $M=10^3$. Thus, Fig. 2 clearly illustrates the mixing effect as a function of $M$.

In order to visualize the flow behavior as a function of $M$, streamtraces of 20 points uniformly distributed between ($x=0$, $y=-0.4$, $z=0$) and ($x=0$, $y=0.4$, $z=0$) obtained for different values of $M$ with $Re=1$, $Ns=12$ and $s=0.5$ are compared in Fig. 3. For $M=0$, the flow is purely axial and all the streamtraces are vertical lines as seen in panel (a). The Lorentz force imposed by the externally applied magnetic field results in twisting of the streamtraces and the extent of twisting increases with increase in $M$ (panel (b)). As $M$ increases further, the streamtraces turn around the left and right electrodes alternatively while traveling axially (panels (c)–(d)). This can lead to transverse intersection of the successive streamline portraits when projected onto the $x$–$y$ plane suggesting the occurrence of the chaotic advection.

To evaluate the extent of chaotic mixing in the pipe mixer, the Lagrangian motion of 25,600 passive tracer particles was tracked by integrating the Eulerian velocity field ($\mathbf{V} = \mathbf{d} \mathbf{X} / dt$). These particles were initially distributed uniformly between points ($x=-0.005$, $y=0$, $z=1$) and ($x=0.005$, $y=0$, $z=1$) to form a material line. Time evolution of this material line for $M=1000$, $Re=1$, $Ns=12$ and $s=0.5$ is illustrated in Fig. 4. It clearly suggests a non-linear stretching of the passive material line. Snapshots of

![Fig. 2. Standard deviations of species concentration across the pipe cross-section along the pipe length for different values of $M$ with $Re=1$, $Ns=12$ and $s=0.5$. The inlet concentration is $C_0=x^2+y^2$ with standard deviation, $\sigma_C=0.0727$.](image)

![Fig. 3. Streamtraces of 20 points uniformly distributed between ($x=0$, $y=-0.4$, $z=0$) and ($x=0$, $y=0.4$, $z=0$) obtained for different values of $M$ with $Re=1$, $Ns=12$ and $s=0.5$. Panels shown are (a) $M=0$; (b) $M=100$; (c) $M=500$; (d) $M=1000$.](image)

![Fig. 4. Time evolution of the material line formed by 25600 tracer particles initially distributed uniformly between points ($x=-0.005$, $y=0$, $z=1$) and ($x=0.005$, $y=0$, $z=1$) for $M=1000$, $Re=1$, $Ns=12$ and $s=0.5$. Panels shown are at times (a) $t=1$; (b) $t=2$; (c) $t=3$; (d) $t=4$; (e) $t=5$.](image)
leads to a globally chaotic flow at \(M = 1000\) (Panel (d)). Table 1 presents slopes of the linear fits of \(\ln(l/l_0)\) for \(0 < t \leq 0.5\) for different values of \(M\). It indicates that the rate of mixing increases with increase in \(M\). Time evolution of local stretch ratio \((l/l_0)\) and stirring index \((\iota)\) for different values of \(M\) with \(Re = 1\), \(N_s = 12\) and \(s = 0.5\) are presented in Fig. 6(a) and (b), respectively. Table 1 and Fig. 6 clearly show that both the stretch ratio and the stirring index strongly depend on the modified Hartmann number for a given axial flow.

In a partitioned-pipe mixer, the product of the characteristic strain rate of the cross-sectional flow and the average residence time in the pipe is a measure of cross-sectional stretching (Khhkar et al., 1987). For the present problem with a constant aspect ratio \((L/D = 10)\), the dimensionless number \(M\) defined by Eq. (12) is a measure of the cross-sectional stretching. Therefore, the modified Hartmann number is expected to govern the cross-sectional stretching irrespective of the Reynolds number. In order to verify this aspect, local stretch ratio calculations were carried out for \(M = 500\) and \(M = 1000\) with \(Re = 2\), \(N_s = 12\) and \(s = 0.5\). These results are compared with those for \(Re = 1\) in Fig. 7(a), which shows a good agreement between them. This was further substantiated by comparing the slope of the linear fit of \(\ln(l/l_0)\) for \(0 < t \leq 0.5\). The results are presented in Fig. 7(b), which supports the inference made from Fig. 7(a).

### 4.2. Effect of electrode switching frequency

For the present study, the pipe is invariably kept at zero potential while the left electrode potential is given by \(\phi = \max(0, \sin(2\pi N_s(z-1))]\) and the right electrode potential is given by \(\phi = \max(0, -\sin(2\pi N_s(z-1))]\) for \(1 > z > 9\). Here \(N_s\) is the dimensionless switching frequency at which the left and right electrodes are energized spatially. For \(N_s = 0\), there exists no Lorentz force. In the present model, the parameter \(N_s\) is responsible for the blinking action and, therefore, controls the transverse intersection of the successive streamline portraits in a given system. As a result, the chaotic advection in the electromagnetically driven pipe mixer is expected to be dependent on \(N_s\).

The effect of \(N_s\) on the chaotic mixing inside the pipe mixer is illustrated in Fig. 8, which presents snapshots of spread of passive tracer particles at \(t = 4\) projected onto the \(x-y\) plane for different values of \(N_s = 1, 2, 4, 12\) with \(M = 1000\), \(Re = 1\) and \(s = 0.5\). Panel (a), which corresponds to \(N_s = 1\), shows that the material line undergoes rotational stretching due to the Lorentz forces. As \(N_s\) increases, the repeated stretching and folding result in better

---

**Table 1**

<table>
<thead>
<tr>
<th>Modified Hartmann number ((M))</th>
<th>Slope of the linear fit of (\ln(l/l_0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.94</td>
</tr>
<tr>
<td>100</td>
<td>1.28</td>
</tr>
<tr>
<td>500</td>
<td>3.89</td>
</tr>
<tr>
<td>800</td>
<td>6.30</td>
</tr>
<tr>
<td>1000</td>
<td>7.61</td>
</tr>
</tbody>
</table>

---

**Fig. 5.** Snapshots of spread of passive tracer particles at \(t = 4\) projected onto the \(x-y\) plane for different values of \(M\) with \(Re = 1\), \(N_s = 12\) and \(s = 0.5\). Panels shown are (a) \(M = 0\); (b) \(M = 100\); (c) \(M = 500\); (d) \(M = 1000\).

**Fig. 6.** Time evolution of local stretch ratio (a) and stirring index (b) for different values of \(M\) with \(Re = 1\), \(N_s = 12\) and \(s = 0.5\).
mixing (see Panels (b)-(d)). Time evolution of the local stretch ratio \( (l/l_0) \) and the stirring index \((\gamma)\) for different values of \(N_s\), ranging from 1 to 20, with \(M=1000, \; Re=1,\) and \(s=0.5\) are presented in Fig. 9(a) and (b), respectively. Fig. 9 clearly shows that there exists an optimum \(N_s\) away from which the chaotic mixing effect starts deteriorating. Fig. 10 shows that the optimum mixing occurs at \(N_s=12\) for \(M=1000, \; Re=1\) and \(s=0.5\).

The effect of the switching frequency \((N_s)\) can be reflected on \(M\) by multiplying it with a factor \([1-\exp(-N_s/2)]\) if \(N_s \leq N_{optimum}\). In order to illustrate the combined effect of \(M\) and \(N_s\), the stirring index is plotted as a function of \(M[1-\exp(-N_s/2)]\) for different values of \(N_s\), ranging from 1 to 12, with \(M=1000, \; Re=1\) and \(s=0.5\) in Fig. 11.

4.3. Effect of electrode separation distance

Another parameter which significantly affects the performance of the electromagnetically driven pipe mixer is the electrode separation distance, \(s\). In order to investigate the effect of the electrode separation distance, numerical simulations were carried out for four different values of \(s\) (= 0.3, 0.4, 0.5 and 0.6). Fig. 12 compares the snapshots of spread of passive tracer particles at \(t=2\) projected onto the \(x-y\) plane for different values of \(s\) with \(M=1000, \; Re=1\) and \(N_s=12\). Panel (a), which corresponds to \(s=0.3\), shows the presence of a regular region near the pipe wall. This can be attributed to the low electric current density near the pipe wall for \(s=0.3\) case. Panel (d), which corresponds to \(s=0.6\), shows a substantial reduction in the material stretching. This is on account of the reduction in the transverse intersection of the streamline portraits as the Lorentz force is very weak in the central core of the pipe mixer. Table 2 presents slopes of the linear fits of \(\ln(l/l_0)\) for \(0 < t \leq 0.5\) for different values of \(s\). It indicates a higher slope for \(s=0.3\) during the initial period. This is on account of the fact that the initial location of the material line lies in the region of a high electric current density and, therefore, a high Lorentz forces. Fig. 13 presents the time evolution of local stretch ratio and stirring index for different values of \(s\) with \(M=1000, \; Re=1\) and \(N_s=12\). It clearly shows that \(s=0.5\) gives rise to the best mixing among the various cases studied. These results show that both local stretching and global mixing (long term) behavior are required to compare the performance of the pipe mixer. Based on the model predictions, \(s=0.5\) is recommended as the optimum value of the electrode separation distance.

4.4. Comparison with glass melt homogenisation

In order to validate the applicability of the results obtained using the non-dimensional model to glass melt homogenisation, numerical simulations were carried out employing a dimensional model with glass melt as the model fluid. A 1 m long pipe with 0.1 m diameter was considered for this purpose. Two inner electrodes with a diameter of 0.02 m separated with a radial distance of 0.05 m were used for the numerical simulation. The temperature-dependent physical properties of the glass melt used for the simulation are given in Table 3 (Giessler and Thess, 2009; Gopalakrishnan et al., 2010). The Joule heat generation and the gravitational body forces in the molten glass were also taken into account. The dimensional form of the model equations were solved numerically using FLUENTTM.

A uniform inlet fluid temperature (1525 K) and an isothermal pipe wall (1525 K) boundary conditions were enforced such that the dimensional model is close to the isothermal, non-dimensional model. An axial velocity of 0.0025 m/s chosen based on a mechanically stirred industrial glass melt homogenizer with a
diameter of 100 mm and a length of 250 mm. The glass pull rate through this dynamic mixer is 550 g/min of a phosphate glass with a density of 3520 kg/m$^3$. An axial velocity which is approximately 10 times higher than that of the dynamic mixer was used in order to achieve a higher throughput. An axial magnetic flux density of 0.7232 T was used for the simulation. The maximum electrode potential $\theta_0$ was 10 V. These conditions correspond to $M=1000$ and $Re=1$. Other parameters were $N_s=12$.
and $s=0.5$. Fig. 14 compares the local stretch ratios obtained for the glass melt with the results of the non-dimensional model. It shows that the chaotic mixing behavior of the glass melt is very close to the non-dimensional model predictions. The maximum and minimum temperatures observed in the glass melt are 1526.4 and 1525 K, respectively. The numerical results are practically consistent with the assumption of isothermal condition of the non-dimensional model. The pressure drop calculated for the electromagnetically driven pipe mixer with $M=1000$, $Re=1$, $Nu=12$ and $s=0.5$ is 19.5 Pa.

### 4.5. Comparison with time-periodic electrode switching

As the spatially periodic electrode switching of the two inner electrodes is not physically realizable, a time-periodic electrode switching was considered in which the two inner electrodes were energized alternatively with a time period of $T$'s which

### Table 3

Physical properties of glass melt used for the numerical simulations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ($\rho$, kg/m$^3$)</td>
<td>3328.7</td>
</tr>
<tr>
<td>Coefficient of expansion ($\beta$, 1/K)</td>
<td>10$^{-4}$</td>
</tr>
<tr>
<td>Specific heat ($c_p$, J/kg K)</td>
<td>1285</td>
</tr>
<tr>
<td>Thermal conductivity ($k$, W/m K)</td>
<td>$3.4538 \times 10^{-8}T^3 + 2 \times 10^{-6}T^2$</td>
</tr>
<tr>
<td>Viscosity ($\eta$, Pa s)</td>
<td>$0.0992 e^{0.0642(T/1033.41)}$</td>
</tr>
<tr>
<td>Electrical conductivity ($\sigma$, S/m)</td>
<td>$1.74 \times 10^6 e^{(-20.300(T))}$</td>
</tr>
</tbody>
</table>

Fig. 14. Time evolution of local stretch ratio (a) and stirring index (b) for different values of $s$ with $M=1000$, $Re=1$ and $s=0.5$.

Fig. 15. Snapshots of spread of passive tracer particles projected onto the $x$–$y$ plane obtained for time-periodic electrode switching with $M=1000$, $Re=1$, $Nu=12$ and $s=0.5$. Panels shown are (a) $t=0.25$; (b) $t=0.5$; (c) $t=0.75$; (d) $t=1$; (e) $t=1.25$; (f) $t=1.5$. 
corresponds to a dimensionless switching frequency of \( N_s \) given by the expression \( N_s = 0.8L/V_0T \). (This expression was obtained based on the fact that the Lorentz force does not exist in the entrance zone and exit zone, each having a length of 0.1 \( L \).)

The dimensional model described in Section 4.4 was modified by replacing the spatially periodic electrode switching with time-periodic electrode switching. The time-periodic boundary condition for the left electrode was \( \phi = 10 \times \max[0, \sin(2\pi N_t t)] \) and that for the right electrode was \( \phi = 10 \times \max[0, -\sin(2\pi N_t t)] \). A time period \( T = 27 \) s, which is equivalent to \( N_t \approx 12 \), was used for the simulation. Other dimensionless parameters of the simulation are \( M = 1000, Re = 1 \) and \( s = 0.5 \).

Evolution of the spread of passive tracer particles obtained for the time-periodic electrode switching case is given in Fig. 15. Panels shown in Fig. 15 show that the material line is stretched by the left and right electrodes alternatively. As the time proceeds, repeated stretching and folding result in a well-mixed glass melt except near the boundaries. Fig. 16 compares the local stretch ratios obtained for the time-periodic electrode switching with that of the spatially periodic electrode switching. It shows that material stretching in both cases are in good agreement. This clearly suggests that the electromagnetically driven pipe mixer with physically realizable time-periodic electrode switching can effectively homogenize the glass melt flow through it. Desired degree of mixing can be achieved by selecting appropriate pipe length. Multiple units with different electrode orientations are also possible to obtain the required extent of mixing.

5. Conclusions

An electromagnetically driven pipe mixer comprising of a pipe with two inner electrodes was conceptualized to homogenize highly viscous and weakly electrically conducting liquids. One possible application is glass melt homogenisation to produce high quality optical glasses. The mixer works based on the principle of Aref’s blinking vortex model. The inner electrodes are energized alternatively while the pipe is kept at zero electric potential invariably. An externally applied magnetic field along the flow direction interacts with the local electric currents inducing a spatially varying electromagnetic body force in the flowing fluid. These Lorentz forces make the flow to turn around the left and right electrodes alternatively while traveling axially. This gives rise to transversely intersecting streamline portraits, which in turn lead to the chaotic advection.

The Lagrangian motion of passive tracer particles was numerically simulated using an isothermal, non-dimensional model to study the effect of various governing parameters on the chaotic mixing inside the electromagnetically driven pipe mixer. The present study shows that the modified Hartmann number, the electrode switching frequency and the electrode separation distance are the governing parameters characterizing the chaotic mixing in the system. For the present problem, the modified Hartmann number is a measure of the cross-sectional stretching and, therefore, strongly influences the chaotic mixing. In the present model, the electrode switching frequency is responsible for the blinking action and, therefore, controls the transverse intersection of the successive streamline portraits in a given system. There exists an optimum electrode switching frequency away from which the chaotic mixing effect starts deteriorating. Based on the model predictions, an electrode separation distance of \( s = 0.5 \) is recommended as the optimum value of the electrode separation distance.

In order to validate the applicability of the results obtained using the non-dimensional model to glass melt homogenisation, numerical simulations were carried out employing a dimensional model with glass melt as the model fluid. Results obtained show that the chaotic mixing behavior of the glass melt is very close to the non-dimensional model predictions. A time-periodic electrode switching, in which the two inner electrodes were switched alternatively, was numerically simulated using the dimensional model. Model predictions show that the material stretching in the spatially periodic and the time-periodic switching cases are in good agreement. The numerical predictions are expected to be useful for developing an efficient glass melt homogenisation system.

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