Electromagnetic interaction of a conducting cylinder with a magnetic dipole caused by steady rotation

Sonja Engert\textsuperscript{1,2,*}, Thomas Boeck\textsuperscript{1}, and André Thess\textsuperscript{1}

\textsuperscript{1} Dept. of Electrothermal Energy Conversion, Ilmenau University of Technology, P.O. Box 100565, Ilmenau, Germany
\textsuperscript{2} Dept. of Thermodynamics and Magnetofluiddynamics, Ilmenau University of Technology, P.O. Box 100565, Ilmenau, Germany

The motion of a conductor in a magnetic field induces eddy currents whose interaction with the field produces Lorentz forces opposing the motion. One can determine the velocity of the conductor from the force on the magnet system since the latter is equal but opposite to the Lorentz force on the conductor. This contactless method is known as Lorentz force velocimetry (LFV). We study an idealized configuration of LFV, i.e. a rotating solid cylinder interacting with a point dipole. The understanding of parameter influences in this setup can be helpful for more realistic configurations. We use a purely kinematic approach appropriate for low magnetic Reynolds numbers. Numerical results for small and large distances between dipole and cylinder have been obtained with the commercial software COMSOL Multiphysics.

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{fig1.png}
\caption{Rotating cylinder near a magnetic point dipole.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{fig2.png}
\caption{Computational grid used in the COMSOL simulations: y-z-view (left), x-y-view (right).}
\end{figure}

1 Problem formulation

We consider an infinitely long, electrically conducting, rotating cylinder with conductivity $\sigma$, radius $R$ and velocity $\vec{v} = (0, -\Omega z, \Omega y)^T$, where $\Omega$ is the angular velocity, under the influence of the inhomogeneous magnetic field of a point dipole. The dipole is located above the cylinder at a distance $h$. The magnetic field of the dipole is

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\left(3\vec{r}\vec{m}\vec{r}/r^5 - \vec{m}/r^3\right),$$

where $\vec{r}$ denotes the position in a shifted coordinate system with the dipole at the origin. The vector $\vec{m} = (m_x, m_y, m_z)^T$ is the magnetic moment. The geometry is illustrated in figure 1.

The movement induces eddy currents in the cylinder. Assuming that the magnetic Reynolds number $R_m = \mu_0\sigma\Omega R^2$ is small, the eddy current density is determined by Ohm’s law $\vec{j} = \sigma(-\nabla\phi + \vec{v} \times \vec{B})$ and the condition $\nabla \cdot \vec{j} = 0$. This formulation based on the electric potential $\phi$ and $\vec{B}$ as applied magnetic field, is known as quasistatic approximation. The induced currents are confined to the conductor, i.e. the normal component of $\vec{j}$ vanishes on the surface. The Lorentz force on the cylinder is the integrated density $\vec{f}_L$, i.e. $\vec{F}_L = \int_{\text{cyl}} \vec{F} \times \vec{B} dV$. The torque on the cylinder is $\vec{T} = \int_{\text{cyl}} \vec{r} \times \vec{f}_L dV$. The problem is made dimensionless using the characteristic length $R$, the characteristic velocity $\Omega R$ and the characteristic magnetic induction $\mu_0 R^3$. The nondimensional variables are

$$j_{\text{new}} = \frac{j}{\sigma\Omega R \mu_0 m R^2}, \quad F_{\text{new}} = \frac{F}{\sigma\Omega R \mu_0 m^2 R^2}, \quad T_{\text{new}} = \frac{T}{\sigma\Omega R \mu_0 m^2 R^2}, \quad h_{\text{new}} = \frac{h}{R}.$$

2 Numerical simulations

We solve the nondimensional problem $\nabla^2 \phi = 2B_z\phi$ resulting from Ohm’s law and $\nabla \cdot \vec{j} = 0$ with the FEM software COMSOL. A typical FEM mesh is shown in figure 2. Starting from a homogeneous grid in the middle of the cylinder the round structure was discretized by angular elements, which become thinner near the boundaries. Moreover, the mesh is very fine directly under the dipole. Depending on the distance $h$ the mesh could be adjusted in several properties, like the cylinder length, the number of grid elements, the ratio of the element size in the center and on the boundary (in x-direction). The cylinder length

\* Corresponding author: e-mail sonja.engert@tu-ilmenau.de, phone +49 3677 69 1187
Section 13: Flow control

Fig. 3: Nondimensional Lorentz force (a) and torque (b) on a rotating cylinder for a wide range of dipole distances. The torque vanishes when the magnetic moment points along the cylinder axis.

Fig. 4: Comparison of simulation results and analytical calculation of the Lorentz force (a) and torque (b) for small dipole distances.

of the simulation model was adapted in order to approximate the force in the infinite case with 95% accuracy. The results for Lorentz force and torque for the three main orientations of the dipole are illustrated in figure 3.

3 Analytical approximations

For very small dipole distances the problem becomes equivalent to that of a thick translating infinite plate with velocity \( v = \Omega R \) in the \( y \)-direction. The Lorentz force and torque in this case can be calculated analytically [1,2]. For a vertical dipole the results for the plate are

\[
F_y = 128 \pi \mu_0 m^2 \sigma v \frac{h^3}{m^2}, \quad T_x = 128 \pi \mu_0 m^2 \sigma v \frac{h^2}{m^2}.
\] (1)

Figure 4 shows that the simulation results are in very good agreement with eq. (1) for dipole distances \( h < 0.1 \).

In case of large \( h \) the dependencies on the distance change because of the finite width of the cylinder. In Figure 3 the force decays as \( h^{-6} \) and the torque as \( h^{-5} \). In analogy with the translating conducting square bar studied in [3], we have formulated and solved an analytical approximation for this case by asymptotic analysis in the small parameter \( \epsilon = R/h \). It allows us to exploit the slow variation of the induced currents along the cylinder axis by scaling the coordinate \( x \) with \( \epsilon \). A regular perturbation expansion leads to two-dimensional Poisson problems for the electric potential at the different orders of approximation. Force and torque are zero for the leading order of approximation \( \epsilon^0 \). In the next order of approximation \( \epsilon^1 \) the force is non-zero and the result is in good agreement with simulation results for dipole distances \( h > 100 \). In this way we have confirmed the force decay rate \( h^{-6} \) and the fact that \( F_y \) has its maximum for the vertical dipole orientation. The details of this analysis and further numerical results will be published elsewhere.

Acknowledgements We are grateful to DFG for financial support in the framework of Research Training Group “Lorentz Force Velocimetry and Lorentz Force Eddy Current Testing” (grant GRK 1567/1).

References