NUMERICAL CALIBRATION OF A MULTICOMPONENT LOCAL LORENTZ FORCE FLOWMETER

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Local Lorentz force velocimetry is a local velocity measurement technique for liquid metals. Due to the interaction between an electrically conducting liquid and an applied magnetic field, eddy currents and flow-braking Lorentz forces are induced in the fluid. Due to Newton’s third law, a force of the same magnitude acts on the source of the applied magnetic field, which is a permanent magnet in our case. The magnet is attached to a gauge that has been especially developed to record all three force and three torque components acting on the magnet. This new-generation local Lorentz force flowmeter (L2F2) has already been tested in a test stand for continuous casting with a 15 mm cubic magnet providing an insight into the three-dimensional velocity distribution of the model melt GaInSn near the wide face of the mold. For better understanding of these results, especially regarding torque sensing, we propose dry experiments which consist in replacing a flowing liquid by a moving solid. Here, as the velocity field is fixed and steady, we are able to decrease considerably the variability and the noise of the measurements providing an accurate calibration of the system. In this paper, we present a numerical study of this dry calibration using a rotating disk made of aluminum and two different magnet systems that can be shifted along the rotation axis as well as in the radial direction.

1. Introduction. In metallurgical industry the in-situ measurement of local velocity is still an unsolved problem. The liquid metals are typically aggressive and, usually at temperatures over 1000°C, making the classic contact techniques like Pitot tubes and hot-wire probes, useless. Even optical techniques like Particle Image Velocimetry fail due to the opaqueness of liquid metals. In order to overcome these challenges, a non-contact measurement technique has been developed called Lorentz Force Velocimetry (LFV) [1, 2]. It works according to the principles of magnetohydrodynamics: when an electrically conducting liquid flows through a static magnetic field, eddy currents are induced and interact with the applied magnetic field generating flow-braking Lorentz forces in the fluid. On the other hand, due to Newton’s third law, a counter force of the same magnitude and proportional to the velocity acts on the permanent magnet system. Depending on the volume subset of the flow that is exposed to the magnetic field, we can have access to the flow rate or to the local velocity of the metal melt. In the first case, the magnetic field penetrates the entire cross-section, and in the second one, a localized magnetic field is desired. Owing to the rapid decay of magnetic fields, a localized magnetic field can be generated by using a single magnet or magnet arrangements which are considerably smaller than the fluid domain. The magnet system is connected to a force sensor and altogether this device is called the local Lorentz force flowmeter (L2F2). A new-generation L2F2 is able to measure all three force and three torque components acting on the magnet system. With this sensor attached to a 15 mm cubic magnet, we were able to obtain information on the three-dimensional velocity structure of GaInSn near the wide face of the mold.
of a continuous casting model, namely, the Mini-LIMMCAST facility at Helmholtz-Zentrum Dresden-Rossendorf [3]. The known double-roll structure, typical for continuous casting of steel, was clearly identified by either the force or the torque signal. Additionally, the force perpendicular to the wall gave information on the velocity in this direction. On the contrary, we were not able to measure this component of the torque as it was expected to be much smaller that the two principal components.

In order to make this torque accessible, we propose a cross-shaped permanent magnet arrangement which would produce a favorable magnetic field distribution in the liquid metal enhancing the torque component perpendicular to the wide face of the mold. This torque is expected to give information about the local gradient or curl of the velocity field. Before using this magnet system at the Mini-LIMMCAST facility, we propose a dry calibration procedure with a rotating disk made of aluminum.

The rotating disk with a permanent magnet concept was patented in 1903 by Otto Schulze in his work “Improvements in Speed Indicators” [4]. His invention was connected to the part of the machine whose velocity one wants to measure. This principle was later used as the basis of the eddy current velocity indicator. In our dry calibration proposal for the multicomponent force/torque sensor, we are going to adapt this idea for the calibration of the torque component \( T_z \) which points in the direction of rotation of the disk. This torque is expected to be constant across the surface of the disk except near the edge and proportional to the angular velocity.

To predict the force and torque magnitude, we performed simulations of the rotating disk setup, which are reported in the present work. They allow us to quantify the effects of the magnet shape and the distance between the magnet and the disk.

The structure of the paper is as follows. In section 2, we present briefly the basics of Lorentz force velocimetry followed by some theory for a rotating disk. In section 3, we discuss the different magnet shapes and magnetic field distributions. In section 4, we present the numerical model and its corresponding results giving a quantitative assessment of the expected magnitude of the torque during experiments. Finally, section 5 provides the main conclusions.

2. Mathematical model.

2.1. Basic equations. When a liquid with a velocity \( \mathbf{v} \) and a conductivity \( \sigma \) moves through a static magnetic field \( \mathbf{B}_0 \), the current density

\[
\mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

is induced according to the Ohm’s law for moving conductors, where \( \mathbf{E} \) is the electric field and \( \mathbf{B} \) is the sum of the applied \( \mathbf{B}_0 \) and induced \( \mathbf{b} \) magnetic fields. For the low magnetic Reynolds number \( R_m \ll 1 \), which implies that \( \mathbf{B} \approx \mathbf{B}_0 \), the Lorentz force density is \( \mathbf{f} = \mathbf{j} \times \mathbf{B}_0 \). By integrating \( \mathbf{f} \) in the fluid domain, we obtain the total force and torque acting on the magnet by [5]

\[
\mathbf{F} = \int_{Vol} \mathbf{j} \times \mathbf{B}_0 \, dV
\]

\[
\mathbf{T} = \int_{Vol} \mathbf{r} \times \mathbf{f} \, dV,
\]

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Fig. 1. Dry calibration of the multicomponent force/torque sensor. Test principle: a permanent magnet system is placed in front of an electrically conducting disk which rotates with a constant angular velocity \( \omega_0 \). In the figure, the cross-shape permanent magnet arrangement (CSMA) is shown and its center is placed at \((0, -50\, \text{mm})\) leaving an air gap \( a_g \).

respectively. Here, \( r \) is the distance vector between every subvolume of the fluid and a given reference point which could be either the center of the magnet or the center of the sensor coordinate system which is defined by calibration.

The velocity field of the rotating disk shown in Fig. 1 is a solid body rotation around the \( z \)-axis, i.e.

\[
v = \omega_0 e_z \times x,
\]

where the vector \( x \) denotes the position. The vorticity of this movement is

\[
\omega_z = (\text{rot} \, v) \cdot e_z = 2\omega_0.
\]

In case of low magnetic Reynolds number (\( R_m \approx 0.1 \) or smaller), the electric field \( E \) from Eq. 1 can be written as \(- \text{grad} \, \phi\), where \( \phi \) is the electric potential.

Finally, we end up with the low-\( R_m \) approximation [6]

\[
j = \sigma (- \text{grad} \, \phi + v \times B_0).
\]

In order to obtain \( \phi \) and, since \( v \) and \( B_0 \) are already given, we take the divergence of Eq. 6, where \( \text{div} \, j = 0 \) due to charge conservation, and solve the Poisson equation

\[
\nabla^2 \phi = \text{div}(v \times B_0).
\]

The boundary conditions on \( \phi \) are obtained by observing that the normal component of the electric current density vanishes on the surface of the disk. In this way, the current density is found from Eq. (6). The magnetic field will be specified later.

2.2. Two-dimensional limit. A particular feature of the velocity field (4) is that the curl of the electromotive force \( v \times B_0 \) is curl-free. This can be shown by direct calculation of \( \text{rot}(v \times B_0) \) using \( \text{div} \, B_0 = 0 \) and \( \text{rot} \, B_0 = 0 \). The result is

\[
\text{rot}(v \times B_0) = \text{grad} \, (\omega_0 e_z \cdot (B_0 \times x)).
\]

Since the curl of the current density \( j \) differs from Eq. (8) only by the prefactor \( \sigma \), it follows that \( \text{rot} \, j \) is also curl-free, i.e.

\[
\text{rot}(\text{rot} \, j) = \text{grad} \, \text{div} \, j - \Delta j = 0.
\]
Because \( \text{div} j = 0 \), it follows from Eq. (9) that each component of the current density satisfies the Laplace equation, in particular, the axial component \( j_z \). On the surfaces \( z = \text{const} \) of the disk we have \( j_z = 0 \) as boundary condition. At the (outer) rim of the disk there can be a component \( j_z \). However, this component should become negligible when the magnet is sufficiently far away from it such that there is no significant electromotive force at the rim. In this case, \( j_z \) satisfies the Laplace equation with uniform Dirichlet boundary conditions. The solution is then \( j_z = 0 \) throughout the disk, i.e. the current density is strictly two-dimensional.

When this limit applies, one can represent the solenoidal current density by a stream function \( \psi(x, y) \), i.e.

\[
j = \text{rot} \psi(x, y) e_z.
\]

With this representation, it then follows from Ohm’s law that

\[
-e_z \cdot (\text{rot} j) = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \sigma \omega_0 [e_z \cdot (x \times \nabla)] B_z.
\]

This two-dimensional Poisson equation depends parametrically on \( z \). At the outer boundary the current density is tangential to the rim, i.e. the rim is a streamline \( \psi = \text{const} \).

The effects of the magnet position are fairly obvious in the two-dimensional formulation when the disk has infinite radius. Assuming that the center of the magnet is at \( x_0 = (x_0, y_0, z_0) \), one can decompose the right-hand side of Eq. (11) as

\[
\sigma [\omega_0 e_z \cdot ((x - x_0) \times \nabla)] B_z + \sigma [\omega_0 e_z \cdot (x_0 \times \nabla)] B_z,
\]

where the first term corresponds to a rotation about \( x_0 \) and the second one to a translation with a velocity \( \omega_0 e_z \times x_0 \). The solution of Eq. (11) is then a superposition of the current distributions corresponding to the rotation and translation, respectively. The translation can be decomposed further into a translation along \( x \) and one along \( y \). In conclusion, the solution is

\[
\psi = \sigma \left( \omega_0 \psi^{(r)} + \omega_0 x_0 \psi^{(y)} - \omega_0 y_0 \psi^{(z)} \right),
\]

where \( \psi^{(r)} \) is the solution for a pure rotation about \( z \) and \( \psi^{(x)}, \psi^{(y)} \) are the solutions for pure translations along \( x \) and \( y \), respectively.

From the decomposition (12) it is obvious that an axisymmetric magnetic field (about the \( z \)-axis) does not generate a contribution \( \psi^{(r)} \) since the axial field component is then given by \( B_z (\sqrt{(x - x_0)^2 + (y - y_0)^2}) \), whereby the source term cancels. For symmetry reasons, this contribution is the only source of the axial torque. A pure translation will only cause a drag force in the same direction. The translational contributions \( \psi^{(x)} \) and \( \psi^{(y)} \) clearly differ by a rotation of coordinates through a right angle about the \( z \)-axis. As a result of Eq. (13), the resulting planar force components satisfy

\[
F_x = -\omega_0 y_0 F_x^{(0)}, \quad F_y = \omega_0 x_0 F_y^{(0)},
\]

where the value \( F_x^{(0)} \) corresponds to a translation with unit velocity along \( x \), and \( F_y^{(0)} \) to a translation with unit velocity along \( y \).

3. Magnet systems. Since an axisymmetric field does not lead to an axial torque component, the magnetic field should have a pronounced non-axisymmetric structure. In view of previous and future experiments, we consider two non-axisymmetric basic magnet shapes shown in Fig. 2. For the cubic magnet with
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Fig. 2. Permanent magnet systems for the dry calibration proposal for the multicomponent force/torque sensor: (a) cross-shaped magnet arrangement (CSMA), (b) cross-shaped magnet (CSM), and (c) 10 mm cubic permanent magnet (CUBIC). All magnet systems share the same volume (1000 mm$^3$) and material (NdFeB) for comparison, and their magnetization points orthogonal to the surface of the disk.

Table 1. Parameters of numerical investigation for the dry calibration of the multicomponent force/torque sensor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet systems</td>
<td>CSMA, CUBIC</td>
</tr>
<tr>
<td>Material</td>
<td>NdFeB</td>
</tr>
<tr>
<td>Grade</td>
<td>N52</td>
</tr>
<tr>
<td>Remanent magnetization</td>
<td>$B_r = 1.43$ T</td>
</tr>
<tr>
<td>Magnetization density</td>
<td>$M_0 = 1.138 \times 10^6$ A/m</td>
</tr>
<tr>
<td>Electrical conductivity</td>
<td>$\sigma = 25.4$ MS/m</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>$\omega_0/2\pi = 1$ Hz</td>
</tr>
<tr>
<td>Air gap $a_g$</td>
<td>$1...5$ mm</td>
</tr>
</tbody>
</table>

uniform magnetization, there is an analytical solution for the magnetic field distribution [7]. This model can be used further for obtaining analytically the magnetic field of cross-shaped magnets by the superposition principle of magnetic fields, or in other words, by adding simultaneously the magnetic field of five $5 \times 5 \times 8$ mm$^3$ rectangular magnets that share the same magnetization direction pointing orthogonal to the surface of the disk.

The parameters of the magnetic material are given in Table 1.

During the fabrication of the CSMA composed of smaller magnets glued together (Fig. 2a), we have experienced problems. For bigger CSMA, the bonding through an adhesive joint is far more challenging due to the higher forces and it may not hold the magnets together. In order to avoid this problem, we will also investigate the alternative of having a single magnet with this cross-shaped form (Fig. 2b). Fig. 3 shows the contour plot of the magnetic field component $B_z$ of the CSMA and CSM (Figs. 2a and 2b) measured by a 3D Hall probe with a 1 mm resolution in comparison with the analytic solution of a CSMA.

According to Fig. 3, our numerical model can predict the magnetic field produced by a cross-shaped permanent magnet system accurately. Additionally, our measurements of the magnetic field component $B_z$ show that the CSM exhibits a stronger and more symmetric magnetic field distribution than the CSMA. This situation can be explain due to intrinsic positioning errors of the small-size magnets during the process of bonding into the cross-shaped form. Hence, in the future
for bigger cross-shaped magnet systems, we are going to consider a single magnet in a cross-shaped form and not a cross-shaped magnet arrangement which has not shown any noticeable advantage in the present study.


4.1. Computational approach. The disk in our model is made of aluminum AlMgSi1, as in the future experiments, having an electrical conductivity of $\sigma = 25.4 \text{MS/m}$. A permanent magnet system (Fig. 2) is placed in front and scans the surface of the disk leaving a constant air gap $a_g$. Due to the relative movement of the rotating disk subjected to an external static magnetic field, eddy currents are generated inducing forces and torques acting on the magnet system.

As explained in section 3, the two magnet systems taken into consideration are a 10 mm cubic permanent magnet (CUBIC) and a cross-shaped permanent magnet arrangement (CSMA). Both magnet systems share the same material (NdFeB, N52, $B_r = 1.43 \text{T}$) and volume (1000 mm$^3$). The magnetization direction is the same for all, including the ones in the CSMA, and points orthogonal to the surface of the disk. The parameters are given in Table 1. In the $x$-$y$ plane, the magnets are always aligned with these coordinate axes (in particular, the “arms” of the cross).
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In our numerical study, we first determine the magnetic field from the analytical solution for a cubic magnet [7]. In the full three-dimensional approach we solve Eq. (7) and then obtain $\mathbf{F}$ and $\mathbf{T}$ through Eq. (2) and Eq. (3), respectively. Here, the torques are calculated in the center of the magnet system at $x_0$. The software used for this numerical study is COMSOL Multiphysics 5.2, whereby the mesh is composed of nonuniform 162750 second-order hexahedral elements (1.33e6 DoF) that take into account the decay of the magnetic field with the distance in the axial direction of the disk.

We have also implemented the two-dimensional problem based on the stream function $\psi$. In this case, we solve Eq. (11) for equidistantly spaced layers $z = \text{const.}$ in COMSOL and compute the current and Lorentz force densities from $\psi$. The

![Lorentz force contour and vector plots](image)

*Fig. 4.* Lorentz force contour and vector plots for $F_x$, $F_y$ and $T_z$. The vector and contour plot (a) were calculated based on $F_x$ and $F_y$, where the contour plot corresponds to the magnitude and the vectors to the direction of the total force at the magnet position $(x_0, y_0)$. The torque component perpendicular to the surface of the disk $T_z$ (b) is constant in the region where the magnet is sufficiently far away from outer boundaries. Parameters: CSMA, $\omega_0/2\pi = 1$ Hz, $a_\theta = 3$ mm.
Fig. 5. Torque component $T_z$ acting on two different magnet systems moving from $(0, -50 \text{ mm})$ to $(0, 50 \text{ mm})$ relative to a rotating disk made of aluminum. The CSMA as well as the CUBIC are placed in front of the disk leaving an air gap of 3 mm. The torque $T_z$ is proportional to the angular velocity of the disk and constant in the region between $y_0 \approx -20 \text{ mm} \ldots 20 \text{ mm}$. The CSMA experiences about 9 times higher torque than the 10 mm cubic magnet. Parameters: CSMA and CUBIC, $\omega_0/2\pi = 1 \text{ Hz}$, $a_g = 3 \text{ mm}$.

4.2. Variation of horizontal magnet position. We scan the surface of the disk from $(x_0 = -50 \text{ mm}, y_0 = -50 \text{ mm})$ to $(x_0 = 50 \text{ mm}, y_0 = 50 \text{ mm})$ having a grid resolution of 5 mm and a constant air gap like in the future experiments. The results are shown in Figs. 4 and 5. It is observed in Fig. 4 that the force is tangential and grows linearly with the distance from the center. Near the rim, it decreases significantly and decays to zero outside the disk. The torque component $T_z$ is indeed constant in a region inside the disk sufficiently away from the rim. In this case, the results of three- and two-dimensional simulations with COMSOL are also in a very good agreement with relative errors far below $10^{-2}$.

Fig. 5 shows that there is an important increase of about 9 times higher torque using the CSMA when compared with the reference CUBIC magnet system in the inner region with constant $T_z$. However, its magnitude (0.53 $\mu$Nm) is smaller than the resolution of our multicomponent force/torque sensor ($\pm 1.4 \mu$Nm) for a rotation frequency of 1 Hz. But, as $T_z$ is proportional to the angular velocity $\omega_0$ of the disk, we can consider higher rotation frequencies in the future experiments. We have to keep in mind that with high angular velocities, the magnet system should be placed near the center of rotation of the disk in order to avoid finite Rm effects by high relative velocities.

4.3. Variation of the air gap. Due to the rapid decay of magnetic fields, the force and torque acting on the magnet are very sensitive to the distance $h$ to the liquid metal. For a magnetic dipole next to a moving plate, the force scales $\sim h^{-3}$ and the torque $\sim h^{-2}$ [1]. Therefore, we want to investigate in this subsection the effect of the air gap $a_g$ variation on the forces and torques which act on the magnet. The CSMA is going to be placed on the $y$-axis in order to have a single force $F_x$ in the $x$-$y$ plane and the corresponding torque component $T_z$. As the force distribution causing $F_x$ is generated inside the disk at an axial distance comparable to $a_g$ from the center of the magnet, it induces additionally a torque component $T_y$. In this sense, the magnet is placed at $(0, 10 \text{ mm})$ and...
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The effect of the air gap $a_g$ variation on $T_y/T_y, \@a_g=1\, \text{mm}$, $F_x/F_x, \@a_g=1\, \text{mm}$, and $T_z/T_z, \@a_g=1\, \text{mm}$. The center of the magnet system is placed at $x_0 = 0\, \text{mm}$ and $y_0 = 10\, \text{mm}$.

Fig. 6 shows the effect of the air gap $a_g$ variation on $T_y/T_y, \@a_g=1\, \text{mm}$, $F_x/F_x, \@a_g=1\, \text{mm}$, and $T_z/T_z, \@a_g=1\, \text{mm}$. The center of the magnet system is placed at $x_0 = 0\, \text{mm}$ and $y_0 = 10\, \text{mm}$.

The results show the corresponding decay of $F_x$, $T_y$, and $T_z$ when the air gap $a_g$ is increased. Here, a fitting model is proposed for a quantitative assessment of this decay

\[
C_{\text{fit}} = a \cdot (a_g + l)^b = a \cdot h^b,
\]

where $C_{\text{fit}}$ corresponds to the force or torque value and $l$, $b$, and $a$ to the model parameters, respectively. The quantity $h$ is the distance between the center of the magnet and the surface of the disk $h = a_g + l$, where $l$ is constant and equal to 4 mm for the CSMA. The results of our fitting model show that the force $F_x$ and the torques $T_y$ and $T_z$ scale with the distance from the center of the magnet to the surface of the disk as $\sim h^{-2.4}$, $\sim h^{-1.5}$ and $\sim h^{-7.4}$, respectively. $T_z$ decays approximately 5 times faster than $T_y$, which appears to be the least sensitive component with respect to air gap variations. In order to explain this finding, we depict in the following figure the magnetic field distribution of $B_z$ at a distance of 1 and 5 mm away from the surface of the magnet.

Fig. 7 shows that the cross-shaped type of the magnetic field distribution is no longer visible above an air gap bigger than 5 mm, which corresponds to an almost zero value of $T_z$ (Fig. 6). In conclusion, an optimal magnetic field distribution in the disk is mandatory for measuring the torque $T_z$ acting on the magnet arrangement. When the magnet is sufficiently away from the surface of the disk, the field approaches an axisymmetric shape that does not provide the axial torque component $T_z$.

5. Conclusions. In our numerical study, we have confirmed that by using a cross-shaped permanent magnet system we can considerably enhance the torque perpendicular to the surface of a rotating disk when compared with the reference cubic permanent magnet. This torque gave us information about the local curl of the velocity field, which for a rotating disk is constant and proportional to the angular velocity. However, its magnitude for $\omega_0/2\pi = 1\, \text{Hz}$ is in the range of $0.5\, \mu \text{Nm}$, which is three times lower than the resolution of our multicomponent force/torque sensor ($\pm 1.4\, \mu \text{Nm}$). But, as $T_z$ is proportional to the angular velocity,
we could in our future experiments increase \( \omega_0 / 2\pi \) from 1 Hz to 50 Hz covering a range of \( T_z = 0.53 \ldots 26.9 \mu \text{Nm} \). According to preliminary simulations of our results in [3], the expected order of magnitude of \( T_z \) at the Mini-\textsc{limm}CAST facility is in this range using a larger CSMA than the one addressed in this paper.

From the theoretical point of view, it could be interesting to study the effect of the air gap on the axial torque component for generic non-axisymmetric fields within the two-dimensional approach.

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References


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