The effect of magnetic field advection on turbulent magnetohydrodynamic flow in a square duct

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In the field of magnetohydrodynamics, turbulent duct flows in a transverse magnetic field are of considerable interest. Their velocity fields exhibit electromagnetic boundary layers and anisotropy due to damping of turbulent fluctuations by the Lorentz force. The weak induced magnetic field is slaved to the velocity field (quasistatic limit) in laboratory experiments since the magnetic Reynolds number is small. Here we consider flow conditions with finite magnetic Reynolds number in direct numerical simulations, whereby the full induction equation needs to be solved. The modification of the imposed transverse field by the flow and other differences to the quasistatic results are briefly discussed.

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1 Introduction

Interactions of conducting flows and magnetic fields constitute the field of magnetohydrodynamics (MHD). An important topic in MHD is the self-excitation of magnetic fields in the liquid cores of planets, which is known as dynamo effect [1]. Liquid metal flows are also important in metallurgy. Due to their much smaller dimensions, self-excitation of a magnetic field does not occur, i.e. an imposed magnetic field is required. Static fields are used for flow control (open loop) in metal casting and crystal growth [1]. They are also used in novel techniques for contactless flow measurement based on induction. Highly conducting flows in strong magnetic fields will also occur in liquid metal cooled blankets for future fusion reactors of the Tokamak type. These blankets are systems of ducts surrounding the plasma chamber in order to absorb the neutron flux for further energy conversion and breeding of tritium. The Lorentz forces lead to strong increase of hydraulic resistance. MHD duct flows are not only important for technological reasons but are also central to fundamental MHD research [2]. In fact, MHD originally developed from investigations of duct flows by Hartmann in the 1930s [3]. MHD duct flows exhibit magnetic damping of turbulence in combination with mean flow modification and formation of electromagnetic boundary layers. The interplay of these effects gives rise to flow phenomena not encountered outside MHD. Numerical simulations have contributed to the understanding of turbulent MHD duct flows in the quasistatic case when the induced magnetic field is weak [4]. Here we focus on the modification of the flow and the magnetic field that would appear in high-speed flows in large ducts.

2 Numerical simulations and results

We perform direct numerical simulations of turbulent conducting flow in a square duct with insulating walls and vertical field. The induced magnetic field must be solved in the whole space. In order to account for the exterior domain, a boundary element method is used that leads to non-local boundary conditions for the magnetic field on the duct walls. The induction equation is

Fig. 1: Contours of the mean axial velocity $\langle u_x \rangle_{x,t}$ in the cross-section at $Rm = 0$ (left), $Rm = 400$ (middle). The contour range spans from $\langle u_x \rangle_{x,t} = 0$ at the light end to $\langle u_x \rangle_{x,t} = 1.35$ at the dark end. Difference $\langle u_x \rangle_{x,t}(Rm = 400) - \langle u_x \rangle_{x,t}(Rm = 0)$ (right).

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solved with explicit time-stepping. Magnetic field advection is characterized by the magnetic Reynolds number \( Rm = UL/\lambda \) where \( U \) is the mean velocity, \( L \) is the half width of the duct cross-section and \( \lambda \) is the magnetic diffusivity of the fluid. The other parameters are the hydrodynamic Reynolds number \( Re = UL/\nu \) and the Hartmann number \( Ha = B_0L(\sigma/\rho\nu)^{1/2} \) where \( B_0, \sigma, \rho, \nu \) represent the magnetic field scale, electrical conductivity, mass density and kinematic viscosity. In the quasistatic case \( Rm = 0 \) it is known that turbulence is sustained only when \( Re/Ha > 200 \). Anisotropy effects become apparent when the interaction number \( Ha^2/Re \) is not too small. Achieving both conditions necessitates \( Re \gtrsim 10^5 \), which is challenging even with \( Rm = 0 \) [5]. To keep the numerical effort manageable, we therefore consider a lower \( Re = 14500 \) and \( Ha = 43.5 \) in agreement with [6] for the quasistatic case. For similar reasons, we choose values of \( Rm \) considerably larger than in metallurgical applications.

Fig. 1 shows mean streamwise velocity distributions for \( Rm = 0 \) and \( Rm = 400 \). Both distributions show characteristic boundary layers, i.e. side layers near \( y = \pm 1 \) and thinner Hartmann layers at \( z = \pm 1 \). For \( Rm = 400 \) the velocities are reduced near the corners, and increased in the central region. The boundary layers are not visibly changed. Statistical averaging was necessary for much longer times at \( Rm = 400 \) due to slow modulations, e.g. in the kinetic and magnetic energy of the fluctuations. Fig. 2 shows that the mean secondary flow driven by the turbulence is also modified at \( Rm = 400 \). Although it remains of the same order of magnitude as for \( Rm = 0 \), it is weaker in the central region. Nonetheless, it has a profound effect on the mean magnetic field that can be seen in Fig. 2 (right). The corner eddies advect the magnetic field towards the midplane \( y = 0 \) near the Hartmann walls, whereby it is partly expelled from the corner regions. Near the center of the duct, they advect the magnetic field towards the side walls, whereby it is also partly expelled from the bulk. The increased magnetic field near the middle of the Hartmann layers causes a stronger flow acceleration there. As a result, the mean streamwise velocity is increased, which causes the change in the velocity distribution in Fig. 1. A detailed discussion of the numerical method and of other flow statistics are provided in our recently accepted paper [7].

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References