Eddy-current braking of a translating solid bar by a magnetic dipole

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The paper addresses the problem of a conducting rectangular bar of square cross-section which is moving with constant velocity in the field of an arbitrarily oriented magnetic dipole. The braking Lorentz force on the bar is obtained by FEM and compared with the analytical solution for a moving infinite plate in the field of a magnetic dipole [2]. The computation of the induced currents requires solution of a Laplace equation with mixed boundary conditions for the electric potential inside the moving bar.

1 Introduction

The induction of currents in a conducting solid or liquid body moving through a magnetic field provides the basis for electromagnetic measurements of its velocity. Usually, an induced voltage is measured, which requires electric contact [1]. Lorentz Force Velocimetry (LFV) and Eddy Current Testing (ECT) eliminate this contact by using the braking effect of an external magnetic field on a moving conducting body [2]. The magnetic system which produces the primary magnetic field is also influenced by the moving conductor and experiences an equal but opposite force, whose measurement provides information about the conductor velocity or flowrate. Simple semi-analytical models can be used for the basic analysis and better understanding of the physics of the interaction between a moving bar and a magnetic dipole. The obtained reference results provide a basis for verification of more complex computer simulations. We have performed such a semi-analytical investigation for a translating solid bar with an arbitrarily oriented magnetic dipole. This approach can be extended to laminar fluid flow as well.

The magnetic Reynolds number \( Re = \mu_0 \sigma vh \) (\( \sigma \) is the electrical conductivity of the bar, \( \mu_0 = 4\pi \cdot 10^{-7} \, \text{H/m} \) is the magnetic constant, \( v \) is the velocity of the bar, \( h \) is the distance between the dipole and the bar) is less than unity for the selected ranges of \( v, h \) and \( \sigma \) and our problem can be treated in the quasistatic approximation. In this case, the secondary magnetic field which arises from the induced eddy currents \( \vec{j} \) in the moving conducting bar is small compared with the primary field of the dipole. We are interested in the force on the translating bar, which is equal in magnitude to the force on the dipole due to the secondary magnetic field.

2 Mathematical description of the problem

The induction of the magnetic dipole can be expressed as \( \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \left( \frac{3(\vec{m} \cdot \vec{r})\vec{r} - \vec{m} \vec{r}^2}{r^5} \right) \), where \( \vec{m} = m \cdot \vec{e}_i \) \((i = x, y, z)\) is the magnetic dipole moment, \( m = \sqrt{\vec{m} \cdot \vec{m}} \), \( k_i = (\vec{m} \cdot \vec{e}_i)/m \) and \( \vec{r} \) is the vector from the dipole location to the observation point. The solid infinite bar with cross-section dimensions width \( \times \) height \( = 2a \times 2b \) is moving with a constant velocity \( \vec{v} = (v, 0, 0) \) (Fig. 1). In the quasistatic approximation, the induced currents are given by Ohm’s law \( \vec{j} = \sigma (\nabla \phi + \vec{v} \times \vec{B}) \) for a moving conductor. The electric field is represented as gradient of an electric potential \( \phi \). The induced currents are solenoidal (\( \nabla \cdot \vec{j} = 0 \)) since the conductor is electrically neutral. This condition leads to the Laplace equation \( \nabla^2 \phi = 0 \) since the velocity is constant. The induced currents have no normal component on all surfaces of the bar, which gives rise to the following Neumann boundary conditions (BC) \( \frac{\partial \phi}{\partial z} \bigg|_{z=0, z=-2b} = v B_y \) and \( \frac{\partial \phi}{\partial y} \bigg|_{y=\pm a} = -v B_z \).

The currents and potential should also vanish at \( x = \pm \infty \) for an infinite bar: \( \lim_{x \to \pm \infty} \phi = 0 \). The finite bar will have \( \frac{\partial \phi}{\partial x} \bigg|_{x=\pm b} = 0 \) at the ends instead. A general analytical solution of the problem with the described BC cannot be obtained. For this reason, an automated Matlab\textsuperscript{TM} script coupled with the Comsol\textsuperscript{TM} Laplace solver is used to solve the problem numerically. The motion of the finite bar is described by setting appropriate BC with modified \( \vec{r} \). The force on the bar is found by integrating the Lorentz force density \( \vec{F} \times \vec{B} \).

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Fig. 1 Sketch of the studied problem.
3 Results for square bars and discussion

A first study has been performed to check the dependence of results on the bar length (because zero potential BC should be applied for far ends) and on the numerical grid. The computed $x$ component of the Lorentz force $F_x$ is compared with the analytical force expression for the infinite plate $[2]$ $F_{\text{plane}} = \frac{1}{128\pi} \frac{\mu_0 l^2 \sigma v}{h^3} \left(1 - \frac{h^3}{(h + 2b)^3}\right)$. It is found, that the accuracy of the results is almost independent of the length $2l$ of the bar (Fig. 2), however, it is very sensitive to the mesh quality, especially for small distances between the bar and the dipole when the computed value $F_x$ should converge to $F_{\text{plane}}$. Numerical simulations for an essentially infinite bar have shown that the maximal force $F_x$ is obtained when the dipole is oriented in vertical direction (Fig. 3) provided it is located above the middle of the bar. It is also found that $F_x \sim 1/h^3$ for any dipole orientation. Figs. 4, 5 show results for a finite bar with $h/b = 0.2$ and $l = 5b$ with $h = 0.01m$. The displacement $\delta x$ is measured from the center of the bar. We find that the moving bar with finite length produces a smaller maximal value of the Lorentz force when compared with the infinite bar or the infinite plate (Fig. 4). We ascribe this effect to the reduced bar volume under the influence of the magnetic field for the selected value of $h$. For the finite bar the computations demonstrate characteristic peaks for the force component $F_z$ (Fig. 5) when the bar ends cross the $x$ position of the vertically oriented dipole. However, the magnitude of this lift force $F_z$ is still several times smaller than corresponding drag force $F_x$ on the dipole and its measurement could therefore be difficult.

The presented numerical approach will be used as a base for further analysis of the magnetic dipole interaction with moving conducting solid bodies of cylindrical cross-section and laminar flow in a duct.

Acknowledgements MK, TB and AT acknowledge financial support from the Deutsche Forschungsgemeinschaft in the framework of the RTG Lorentz Force Velocimetry and Lorentz Force Eddy Current Testing (grant GRK 1567/1).

References