Identification of a stable model for a flexible plate using frequency domain

subspace methods

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Abstract: Having a suitable model is one of the most important steps in active noise and vibration control systems. There are many applications in which the derived numerical model does not show the system behavior appropriately. In such cases, obtaining a model using system identification methods can play a special role. To this aim, in this paper the problem of identification of a flexible plate which is attached to upper side of an enclosure is addressed. The frequency response of the plate is measured between the two specific points and the data are applied to the several subspace-based frequency domain identification algorithms with and without Instrumental Variable method. Since the real system, by nature, is stable and the models identified in all cases are unstable, in the next step stabilization algorithm with different initial values is used to extract the stable part of the system. Finally, the qualities of the stabilized identified models are evaluated by calculating the RMS error between the measured and identified models.

Keywords: System Identification, Subspace methods, Vibration measurement, Active control.

1. INTRODUCTION

Recently, scientists have devoted a wide attention to the active noise and vibration control problems, and these control methods have been used in many applications (Elliott, 1999). During the recent decades, much research has been done on the flexible plate which is the main element of most mechanical systems such as enclosure, airplane, elevator and etc. In fact in most of the cases, the main reason of the produced noise in enclosure is vibration of flexible plate, and hence control of vibrations will be a remedy (Pota et al., 2002). Since feedback control methods, such as μ-control, \( H_\infty \) and \( H_1 \) are model based controllers, attaining a model of the flexible plate is the first step in designing a suitable controller. Having an analytical model of the plate-enclosure system is appropriate for preliminary analysis and design of the controller, and in practical applications identification methods have a valuable place (Fang et al., 2004, Al-Basyouni et al., 2005). Since in most experiments, frequency response functions are gathered, frequency identification methods are suitable techniques to attain a proper model of the system.

Various frequency identification methods are explained and investigated in (Pintelon et al., 2001). Subspace identification is one of the most common methods which estimates the state space matrices and bring some advantages than other methods. One of the advantages of subspace methods is no need of nonlinear optimization. The second advantage is that in this method the identification of multivariable systems is as simple as identification of single input-single output systems (Ljung, 1999, Mckelvey, 2004). In (Mckelvey et al., 2002), the subspace method has been used to identify the acoustic model of an enclosure. Although subspace methods have some advantages, it requires some methods to adjust the parameters to improve the quality of the identified model. Maximum Likelihood (ML) method proposed in (Wills et al. 2009) provides the possibility to increase the quality of the identified model in a few steps by decreasing the error between the model and validation data. It has been shown that the proposed ML method is numerically robust and can reliably deal with high order models over a wide bandwidth.

Identification of a stable model is important because most of the systems used in practice are stable. In frequency domain identification methods stability constraint is imposed by weighting the cost function with the inverse of noise variance in the desired frequency band, which leads to no controllable systematic errors. To tackle this problem, one possible solution is to search for appropriate model in the restricted parameter space. The solution find by this method is locally optimal; however it is stable (Ellacott et al., 1976). In (Mckelvey et al., 1996) the projection idea was used to identify a stable model. The problem of this method is that fitness of data is reduced in comparison with the original unstable identified model. The method proposed in (D’haene et al., 2006) generates a stable model from noisy data by using an iterative algorithm which includes an unconstrained optimal noise removal step.
In this paper the aim is to identify a stable model for a flexible plate installed on the topside of an enclosure. One approach to solve this problem is to use robust identification methods with Kautz orthonormal basis functions for resonance functions. The uncertainties accompanied with the identified nominal model may be captured using stochastic (Montazeri, 2006) or deterministic (Esmail Sabzali, 2008) approaches. Another way to identify a more exact nominal model is to fit a transfer function directly to the measured data. Preliminary results in (Montazeri, 2009) show that fitting a transfer function to the measured FRF of the plate-enclosure system with least square method when the model is parameterized based on the ascending power of Chebychev polynomials, will result in an unstable transfer function. Hence, here as another approach frequency domain subspace methods along with some stabilization techniques are applied to find a stable model for the system.

To this end in Section 2, first the current know-how of frequency domain subspace identification techniques and also stabilization methods are reviewed. After that the results of different identification techniques applied to the measured data are discussed in Section 3. Finally, conclusions will be presented in Section 4.

2. IDENTIFICATION METHODS

2.1 Subspace Method

Frequency domain subspace identification methods have been developed both in continuous and discrete time domains. To identify high-order systems in continuous-time methods and also to deal with the condition number problem in such systems, recursive Forsyth polynomials (Pintelon et al., 2001, Van Overschee et al, 1996) and s-operator (Yang et al., 2000) approaches have been proposed. However, to alleviate this problem, for identification of high-order systems, often discrete-time methods have been used. In order to apply the discrete-time identification algorithms, the measured FRF data should be transformed from continuous-time domain to the discrete-time domain. Bilinear transformation, as defined in (1), may be a good choice. In (1), s, z are Laplace and z- transform operators and T can be seen as a sort of sampling period (Pintelon et al., 2002).

\[ s = \frac{2(z-1)}{T(z+1)} \]  

Using N-point discrete-time Fourier transform (DFT), state space model of system can be written as shown in (2), where N is the number of validation data, \( m \) is the number of inputs and \( r \) is the number of outputs.

\[ Y(k) = CX(k) + DU(k) \]

\[ \psi_p X(k) = AX(k) + BU(k) \]  

In this equation, \( A \in R^{n \times n} \), \( B \in R^{n \times m} \), \( C \in R^{r \times n} \) and \( D \in R^{r \times m} \) are supposed to be a minimal realization of system and \( \psi_p = e^{-(j2\pi p/T)} \).

The main idea of subspace method is to estimate the extended observability matrix \( O \) using (3), which is computed by recursive formulation of second and first equation of (2). In (3), \( Y \in C^{p \times r} \) and \( U \in C^{p \times m} \) are defined by (4) and (5) respectively.

\[ Y = OX + \Gamma U \]  

\[ \Gamma = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C(A)^{-2}B & C(A)^{-3}B & \cdots & D \end{bmatrix} \]  

\[ Y = \begin{bmatrix} \Psi_1 Y(1) & \Psi_2 Y(2) & \cdots & \Psi_p Y(N) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_1^{p-1} Y(1) & \Psi_2^{p-1} Y(2) & \cdots & \Psi_p^{p-1} Y(N) \\ U(1) & U(2) & \cdots & U(N) \end{bmatrix} \]  

\[ U = \begin{bmatrix} \Psi_1 U(1) & \Psi_2 U(2) & \cdots & \Psi_p U(N) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_1^{p-1} U(1) & \Psi_2^{p-1} U(2) & \cdots & \Psi_p^{p-1} U(N) \end{bmatrix} \]  

The parameter “p” is selected in a way that \( p \geq n \). The interested readers may refer to (Mckelvey et al., 1996) for more details about estimation of system matrices. Neglecting the noise property will increase the variance of estimation parameters; as a result, it makes the identification method to be inconsistent. It would be desired to consider the covariance matrix in the identification algorithm to be able to reduce the difference between the identified model and the real plant. Since having the exact information about the noise covariance matrix is not possible, the so-called “Instrumental Variable” concept has been proposed which annihilates the effect of noise by adding some data which are uncorrelated with noise (Mckelvey 1997). It has been shown in (Pintelon et al., 2002) that when the true noise covariance matrix is replaced by the sample noise covariance matrix, obtained from small number of independent repeated experiments, consistent identification result will be achieved considering a few assumptions. Typically, “Instrumental Variable” concept makes it possible to deal with noise effect without having any information about the noise covariance matrix.

2.2 Frequency Domain Maximum Likelihood (ML)

In the literature, ML method is regarded as an identification method exploited the noise information in a theoretical frame work. In this method, the estimated parameters are calculated by maximizing the probability density function of outputs. Since the cost function is parameterized in a quite complicated way by estimation parameters, calculating the parameters are done using numerical methods, such as Error Jacobian Matrix (Pintelon et al., 2001), and Exception Maximization method (EM) (Wills et al. 2009). It should be mentioned that all of these methods use Gauss-Newton search algorithm to optimize the cost function.

In this paper ML method proposed by (Wills et al. 2009) is used to estimate the state matrices based on (2) and assumes \( V(k) \) and \( E(k) \) as the process and measurement noises respectively. In addition, it is presumed that \( V(k) \) and \( E(k) \) are independent zero mean variable with complex normally distribution. The cost function for estimation of parameters is defined as...
In this equation \( G(K) \) is system transfer function computed in \( \psi_k \). \( \theta \) is defined by (7) and (8) where in these equation matrices \( Q \) and \( R \) are covariance matrices of \( V(k) \) and \( E(k) \) respectively. Also, the matrix \( P_k \) is defined by (9)

\[
\theta = \text{arg max}_{\theta} \log \left( P_s(Y(1), ..., Y(N)) \right)
\]

\[
L(\theta) = -\left( \sum_{k=1}^{N} \log |P_k| + \left\| P_k^{-1/2} (Y(K) - G(K)) \right\|^2 \right)
\]  (6)

To find \( \theta \) that maximize (9), it is sufficient to provide conditions that satisfy \( \Xi(\theta, \theta') > \Xi(\theta, \theta) \), where

\[
\Xi(\theta, \theta') = E_{\theta'} \left( \log P_0(X,Y) | Y \right)
\]  (10)

Therefore, an algorithm in \( \theta \) iteration, first compute \( \Xi(\theta, \theta') \) (E-step) and then compute \( \theta_{i+1} \) by (11) (M-Step). For more details on estimation of system matrices, an interested reader may refer to (Wills et al. 2009).

\[
\hat{\theta}_{i+1} = \text{arg max}_{\theta} \Xi(\theta, \theta')
\]  (11)

2.3 Stabilization

For stabilization of the unstable model, in most of techniques the penalty cost function is used. For example in (Balough et al., 2008), the proposed method stabilizes the model by adding delay to model and solve some related PDE equations, however adding delay is not appropriate for closed loop applications. The algorithm proposed in (D’haene et al., 2006) decreases the estimation error in the desired frequency band; but increases error in other frequency band. In this algorithm, the index function is defined by (12) and it is optimized using an iterative algorithm.

\[
J_{\text{stable}} = \sum_{i=1}^{N} \left[ \tilde{G} \left( \psi_i - G_{\text{est}}(\psi_i, \theta) \right) \right]^2
\]  (12)

Initial stable model \( \tilde{G}(\psi_0, \theta_0) \) to start the iterative algorithm is obtained from the unstable identified model. There are some concepts to choose a stable model form an unstable one.

According to (D’haene et al., 2006) there is a transformation matrix which decomposes the state matrix of system in two different parts, one is related to the stable part of the state matrix and another is related to the unstable part. So the transfer function of system can be written by some parts of two transfer function, \( G_{su}(\psi) \) and \( G_{su}(\psi) \) which are related to the stable and unstable parts of main transfer function respectively.

By considering this issue, the first concept to select the initial value of the iterative algorithm takes into account only \( G_{su}(\psi) \). This results in a simple part of the system, which is optimal \( L_{2\text{-norm}} \) approximation of the system transfer function. The second concept is replacing \( \tilde{A}_{su} \) (state matrix of unstable part \( G_{su}(\psi) \) with \( \tilde{A}_{su}^{-1} \) in \( z \)-domain or flipping unstable poles around imaginary axis in \( s \)-domain. The third concept is replacing \( G_{su}(z^{-1}) \) with \( G_{su}(z) \) in \( z \)-domain or \( G_{su}(s) \) with \( G_{su}(s^{-1}) \) in \( s \)-domain.

The other possibility to choose an initial stable model has been proposed in (Mckelvey et al., 1996) which imposes stability constraint during the identification process using discrete subspace method. In this method during the estimation of \( A \) matrix, unstable eigenvalues of matrix have been mapped into unit circle.

3. SIMULATION RESULTS

This section presents the identification results of a flexible plate located on the topside of the enclosure (Fig. 1). The details of implementation, analytical modelling and modal analysis of this enclosure can be found in (Montazeri, 2009) and is briefly described here. As can be shown in Fig. 1, the input-output data are measured and transmitted to the pulse analyzer. In this way it is possible to measure the frequency response function (FRF) between the force point and accelerometer using internal functions of pulse analyzer. In order to have a more accurate model of the plate and to reduce the effect of measurement noise, this procedure has been repeated ten times, and the average frequency response has been used for identification. To alleviate the instability problem reported in (Montazeri, 2009), here frequency domain subspace identification method is applied to the measured FRF data.

The identification results using subspace method is shown in Fig. 2. In these figures, continuous lines and dash lines show the real system and the identified system frequency responses respectively. The value of RMS error between the identified and measured FRF is labelled in each plot. The order of the model, selected for identification is 22 using cross validation method. The results in Fig. 2(a) are obtained by discrete subspace method with instrumental variable (Mckelvey et al., 1996, Mckelvey, 1997). According to this figure, the fitness is good up to the frequency 350 Hz. Figures 2(b) and 2(c) show the identification results by \( w \)-operator, but in Fig. 2(c) the zero constraint is imposed on matrix \( D \) (Yang et al., 2000). It can be seen that the fitness is good up to the frequency 350 Hz, however their fitness are generally less than that of Fig 2(a). In addition, in Fig. 2(c) a resonance peak can be seen at frequency 188 Hz; but there is no resonance peak at this frequency in the measured frequency response of the system. The identification result using MATLAB system identification Toolbox (Ljung, 2007) is presented in Fig. 2(d). In contrast to Fig 2(c), the fitness of the identified model in Fig 2(d) is better, but it is missing the 5th and 7th modes and two other peaks instead have been appeared at frequencies 318.5 and 343.5 Hz.

The identified models in Fig. 2 are improved by ML method (Wills et al., 2009). This method has been developed as a MATLAB Toolbox as can be found in (Wills et al). The result of this method when initial condition is obtained by \( w \)-operator approach is shown in Fig. 3. The reason for using \( w \)-operator method is that it shows the best fitness among others after using the ML method. It can be seen that the fitness of the identified model is improved, but it could not identify the
6th and 7th modes. The reason for this improvement is that in ML method the value of cost function in (12) in each step is reduced. Also, in this algorithm the information of noise variance is updated which in turn helps that the identified model remains consistent. It should be mentioned that all identified models have unstable poles.

In order to stabilize the identified models, the iterative algorithm introduced in Section 2.3 is applied. Since the quality of fitness by this algorithm depends on the initial condition, diverse initial values are selected for the start of algorithm (D’haene et al., 2006).

The frequency responses of the stabilized identified models with different initial values are shown in Figs. 4-7. These figures illustrate, stabilize identified models when the initial condition is selected by discrete subspace method, w-operator without constraint, w-operator with constraint and ML method respectively. In all these figures, the initial condition of algorithm have been obtained by first, second and third concepts. In addition, in Fig. 4(d) projection method is used for stabilization. The RMS errors of different identified models and the corresponding model order before and after stabilization are listed in Table 1.

As can be seen in Table 1, the fitness of the identified model in Figs. 4(a) and 4(d) are better, and the order of model in Figs. 4(a) is shorter. In both Figs. 4(a) and 4(d), the 5th resonance frequency has a shift.

The identified models in Figs. 5(a) to 5(c) have approximately the same fitness, and the one in Fig. 5(a) has less order and higher fitness especially at low frequencies. All models have a shift in 4th mode and they are not able to identify 6th and 7th modes.

The fitness value of the identified model in Fig. 6 is almost the same as in Fig. 5. But they have undershoot at the frequency 340 Hz. According to Fig. 7, it can be seen that identified stable system using initial value obtained by ML method has a fewer fitness rather than other methods, although the ML method has the best fitness without any stabilization.

Fig. 1: Experimental setup for measuring frequency response of the plate by modal analysis.

Fig. 2: Frequency responses of the identified system using (a) discrete subspace method with instrumental variable, (b) w-operator without constraint, (c) w-operator with constraint and (d) MATLAB system identification Toolbox.

Fig. 3: Frequency response of the identified system using ML method.

Fig. 4: Frequency response of the identified system after stabilization when initial value is chosen by discrete subspace method and (a) first concept, (b) second concept, (c) third concept and (d) projection method.

Generally based on the results summarized in Table 1 and shown in Figs. 4 to 7, it can be concluded that combination of stabilization method and discrete subspace method by projection gives the best identified model in term of the RMS error.
Table 1. RMS of error for stable identified system.

<table>
<thead>
<tr>
<th>Identification method</th>
<th>First concept</th>
<th>second concept</th>
<th>third concept</th>
<th>projection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete subspace method</td>
<td>0.1812e-3</td>
<td>0.2468e-3</td>
<td>0.2465e-3</td>
<td>0.1733e-3</td>
</tr>
<tr>
<td>w-operator without constraint</td>
<td>0.203e-3</td>
<td>0.1959e-3</td>
<td>0.2023e-3</td>
<td>-</td>
</tr>
<tr>
<td>w-operator with constraint</td>
<td>0.2173e-3</td>
<td>0.1973e-3</td>
<td>0.1988e-3</td>
<td>-</td>
</tr>
<tr>
<td>ML method</td>
<td>0.2e-3</td>
<td>0.2757e-3</td>
<td>0.3e-3</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 5: Frequency response of the identified system after stabilization when initial value is chosen by w-operator method without constraint and (a) first concept, (b) second concept, (c) third concept.

Fig. 6: Frequency response of the identified system after stabilization when initial value is chosen by w-operator method with constraint and (a) first concept, (b) second concept, (c) third concept.

Fig. 7: Frequency response of the identified system after stabilization when initial value is chosen by ML method and (a) first concept, (b) second concept, (c) third concept.

4. CONCLUSION

Most of the time, designing a successful controller is highly dependent on the suitable model with good fitness in the desired frequency band. In this paper, different subspace frequency domain identification methods are evaluated for the purpose of identification of a flexible plate attached to the upper side of an enclosure. The identification results by discrete subspace, w-operator, and ML methods are presented. It can be seen that the discrete subspace method has a better fitness due to powers of $e^{jw}$ from a natural orthogonal basis, however the fitness of models which obtained by w-operator is improved by ML method. Because the outputs of the identification algorithms are not stable model, an iterative method with different initial values used to make them stable. Simulations show that the initial values affect the quality of fitness. The results confirm that using the iterative algorithm with initial value obtained by the projection method will show the best result for this application.

REFERENCES


