Performance and robust stability trade-off in minimax LQG control of vibrations in flexible structures

Allahyar Montazeri *, Javad Poshtan, Amir Choobdar

Electrical Engineering Department, Iran University of Science and Technology, Narmak 16846, Tehran, Iran

A R T I C L E   I N F O

Article history:
Received 1 August 2007
Received in revised form 8 May 2009
Accepted 14 May 2009
Available online 16 June 2009

Keywords:
Minimax LQG control
Robust stability
Robust performance
Active vibration control

A B S T R A C T

An optimal robust Minimax LQG control of vibration of a flexible beam is studied in this paper. The first six modes of the beam in the frequency range of 0–800 Hz are selected for control purposes. Among these modes, three modes in the frequency range of 100–400 Hz are used for control, while the other three modes are left as the uncertainty of modeling. Both the model and the uncertainty are measured based on experimental data. The nominal model is identified from frequency response data and the uncertainty is presented by a frequency weighted multiplicative modeling method. For the augmented plant consisting of the nominal model and its accompanied uncertainty, a Minimax LQG controller is designed. A trade-off between robust stability and robust performance is shown by selecting two different choices of uncertainty modeling. Simulation results show that the proposed robust controller increases performance index by reducing the residual vibrations at some specific points on the structure. These on-line methods have much more computational complexity, and they will exhibit good performance, especially when the aim is to eliminate narrowband disturbances [9,10]. A well-known adaptive algorithm in the field of active noise and vibration control is the so-called FxLMS algorithm [11]. This algorithm has been used and tested in many applications because of its simplicity and effectiveness [12–14].

The use of robust controllers for active suppression of vibrations has also been studied and developed in recent years [15–18]. Spatially robust $H_2$ and $H_\infty$ controllers for control of vibrations in an experimental beam are designed and tested in [19,20]. To eliminate vibration on the entire beam, an appropriate cost functional, indicating the vibration energy of the structure, is selected and an optimal controller, considering limits on the actuator signals, is designed. The use of $H_\infty$ norm in the design of robust controllers will lead to a conservative control system, and the performance of such controllers is not often satisfactory. To alleviate this problem, and obtain a controller with robust performance and robust stability properties, $\mu$-synthesis technique has been introduced. The use of $\mu$-synthesis in the robust control of vibration in flexible structures was investigated in [21,22]. A more suitable criterion for minimization is the $H_2$ norm of performance index. However, design of robust $H_2$ controllers is computationally intractable, and there is no analytic solution for them [23]. Since LQG controllers are special cases of $H_2$ controllers for Gaussian signals, considering robustness issues in LQG controllers will result in a controller with good nominal and robust performance. A Minimax LQG controller in which the worst-case LQG performance index is minimized is one
such controller [24]. The use of Minimax LQG control in active vibration control applications was reported in [25]. This controller is designed in the worst case and can withstand two kinds of uncertainties: modeling uncertainties as well as measurement noise. Unlike \( H_2 \) and \( H_{\infty} \) controllers in which a non-convex performance index is optimized, the Minimax LQG controller is obtained by minimizing a convex optimization problem and solving two steady-state algebraic Riccati equations [24].

Model-based control design, such as for robust controllers, requires an appropriate model of the plant. One approach to having a model of a flexible structure is to use analytical modeling [26]. In this method, the governing partial differential equations of the structure are solved using the specified boundary conditions, and the transfer function between two arbitrary points of the structure is achieved. This method will be effective only when the structure has simple geometry and the governing equations of the physical plant (for example the Euler–Bernoulli equation of the beam) are known in detail. Finite Elements method is a good choice for more complex shapes when analytical equations are hard to achieve. This method will result in the estimates of mass, stiffness and damping matrices as a good approximation of the true system [27,28]. Since, in both cases, high order models are obtained for control design purpose, it is necessary to truncate or reduce the order of the obtained model [29]. Another efficient way of modeling a structure is the use of identification techniques. In these methods, the transfer function of the system is calculated based on time or frequency domain data, gathered from an experimental setup. An important point in the design of a robust controller is the assumed bound of uncertainty. This uncertainty bound can be calculated by computing the difference between the identified and measured frequency response of the structure. In [30] several robust identification techniques for identifying the model of structure and its accompanied uncertainties are evaluated and compared.

In this paper, performance and robustness trade-offs in designing a Minimax LQG controller for active control of vibration in a flexible beam is investigated. For this purpose, two kinds of uncertainty are selected to model the out-bandwidth modes of the flexible beam. One uncertainty model has a tight fit of unmodeled dynamics, while the other will describe the uncertainties of truncated modes in a more conservative manner. For each of these uncertainty models, a Minimax LQG controller is designed, and the performances of controllers are compared in terms of the ability in increasing the damping of lightly damped modes of the flexible beam. For this purpose, in Section 2 the experimental beam for which a controller is to be designed is introduced. In Section 3, the model of the beam as well as the corresponding uncertainty bound is identified. In Section 4, a brief description of the Minimax LQG control algorithm is given, and then robust stability and performance of the controller with two different uncertainty bounds are compared, based on simulation results, in Section 5. Finally, in Section 6, the problems of this design method and conclusions are discussed.

2. Experimental setup

The flexible beam, electro-dynamic shaker (B&K4810), power-amplifier (B&K2706), accelerometer (B&K 4394), and pulse analyzer (B&K 3560c) used for data acquisition are shown in Fig. 1. The effective bandwidth of the shaker is in the frequency range of 50–18 kHz, and no distortion will occur in this range. The maximum force that may be generated by this shaker is 10 N. The beam is made of aluminum with dimensions 100 cm by 5 cm by 0.4 cm. The properties of the experimental beam with clamped boundary conditions and its first six measured resonant frequencies, are listed in Tables 1 and 2. The two ends of the beam are clamped on a heavy table to implement clamped boundary conditions and to isolate it from undesirable vibrations which may affect it during the test. The frequency response of the beam was measured between two points on the surface of the beam where the first six modes are measurable (Fig. 1). This was carried out by sending an impulse signal to the shaker, and measuring the output of the accelerometer. The input signal is the force measured by the force transducer at the tip of the shaker, and the output signal is the acceleration measured by the accelerometer. The sampling time of all signals used in data acquisition is 0.2 ms. These inputs and outputs were sent to a pulse analyzer, and the corresponding frequency response was computed by using its built-in functions.

In order to have a more accurate model of the beam and to reduce the effect of measurement noise, this procedure was repeated ten times, and the average frequency response was obtained, as shown in Fig. 2. The frequency range of 0–800 Hz which, according to Fig. 2 contains the first six modes of the beam, is considered as the control bandwidth. Based on this frequency

![Fig. 1. General scheme of experimental setup (a) Shaker and its power-amplifier (b) PULSE analyzer.](image-url)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (cm)</td>
<td>100</td>
</tr>
<tr>
<td>Width (cm)</td>
<td>5</td>
</tr>
<tr>
<td>Thickness (cm)</td>
<td>0.4</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>10 N/m²</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>2700</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>103.5</td>
</tr>
<tr>
<td>3</td>
<td>211</td>
</tr>
<tr>
<td>4</td>
<td>344.5</td>
</tr>
<tr>
<td>5</td>
<td>519.5</td>
</tr>
<tr>
<td>6</td>
<td>699</td>
</tr>
</tbody>
</table>

Table 1: Properties of the beam.

Table 2: Resonant frequencies of the beam.

*Fig. 1.* General scheme of experimental setup (a) Shaker and its power-amplifier (b) PULSE analyzer.
response function (FRF), the second, third, and fourth modes of the beam are selected for control purposes, and the other three modes are used as the undermodeling uncertainty. By fitting a transfer function to the second, third, and fourth modes of the beam, using the Least-Square method, a model of the beam is calculated in (1). The frequency range of 0–700 Hz, which contains the first six modes of the beam, is considered as the target bandwidth for control purpose.

3. Model and uncertainty estimation

To be able to consider the effect of unmodeled dynamics in the design of controller, approximately half of the bandwidth in the frequency range of 0–340 Hz is selected for control purposes. By fitting transfer functions of various orders to the measured FRF in this frequency range (using MATLAB “invfreqs” command), it can be seen that the highest order stable transfer function with a reasonable fit to the measured FRF in the selected bandwidth is a sixth order transfer function. It should be noted that by increasing the model order, MATLAB least-square fit will result in unstable transfer functions, making it necessary to use a more advanced technique such as the Chebychev polynomial method. In fact, because of the low order denominator of this transfer function, it is only possible to model the second, third and fourth modes of the beam. Hence, the fifth and sixth modes are considered, as the uncertainty comes up due to the truncation of the high order dynamics of the beam, and the first mode is regarded as the uncertainty related to the low order transfer function selected for its model. In this way the bandwidth of interest for control is the frequency range of 100–400 Hz, and the out-bandwidth modes may degrade the performance or even cause instability if not considered. Fig. 3 shows the amplitude and phase of the measured and fitted transfer functions. The order of numerator and denominator of the identified model are chosen as 4 and 6 respectively. This is in agreement with the analytical model of the beam from which a second order transfer function is dedicated to each mode.

\[
G(s) = \frac{-2.555s^4 - 0.0615s^3 - 1.628s^2 + 0.00352s - 0.01232}{s^4 + 0.056s^3 + 3.013s^4 + 0.106s^3 + 2.077s^2 + 0.0350s + 0.267}
\]

(1)

The transfer function in (1) is chosen as the nominal model of the beam in the rest of the paper. By considering the uncertainty of this nominal model as a multiplicative uncertainty, the real model of the beam can be written as:

\[
G(s) = G(s)(1 + W(s)\Delta(s)).
\]

(2)

Here, \(G(s)\) represents the measured (actual) frequency response of the beam, and \(W(s)\) is the weighting function used to normalize the uncertainty in different frequencies. The normalized uncertainty \(\Delta(s)\) is assumed to satisfy the following condition:

\[
\|\Delta(s)\|_{\infty} \leq 1.
\]

(3)

A smaller norm will increase the stability margin but the performance may be decreased. To guarantee the inequality in (3), the weighting function \(W(s)\) must be calculated such that the following inequality holds:

\[
|W(j\omega)| \geq \frac{G_A(j\omega) - G(j\omega)}{G(j\omega)}.
\]

(4)

To achieve a good trade-off between stability robustness and performance of the controller, this part of design procedure is crucial in a robust controller design. The weighting function \(W(s)\) in the multiplicative uncertainty of the plant is modeled with two filters, as shown in Figs. 5 and 6. The first one is a Chebychev type-I filter of order 11, and the second one is chosen to be a pth-norm filter. The first filter is achieved by subtracting bode plots of two high-pass analog filters with different cut-off frequencies adjusted to make a hole in middle frequencies. The characteristics of the second filter, such as the cut off frequency, order, and pass-band ripples are adjusted after some trial and error using the FDA toolbox of MATLAB. The stop-band filter is converted to an analog filter using Tustin method. The resulting analog filter will set an upper bound on the uncertainties. As is clear from Fig. 5, the fit is not tight to unmodeled dynamics, and the uncertainty bound for the first mode, which is part of unmodeled dynamics of plant, is lower than its FRF. In Fig. 6 a pth-norm filter of order 10 is fitted and a tighter fit is achieved. It will be shown that this loose/tight fitting will result in better robust performance and robust stability, respectively. The ripple of the pass-band of these filters has to be chosen so that the desired shape of the weighting function \(W(s)\) is obtained. In fact this can change the ups and downs of the weighting function at some frequencies in a manner that a suitable performance and stability trade-off is achieved for controller design. It must be noted that although increasing the order of the filter will result in a better fitting, it will cause the designed controller to become unstable.

4. Minimax LQG control theory

A significant idea in robust control design is to solve the problem with a minimax optimization procedure. This is a game-type problem in which the design of a controller is performed...
in a worst case situation and in the presence of uncertainties. In this game-type problem, the designer may be considered as a minimizing player who tries to find an optimal control strategy to maintain a certain level in robust performance of the closed-loop system when it is faced with some degrees of uncertainty. In contrast, the uncertainty in the underlying plant may impair the performance of the closed-loop system (or even destabilize it). Thus, one may think of the uncertainty as a maximizing player in this game. The advantage of this method is that it may allow one to convert the issue of robustness into a mathematically tractable game-type minmax optimization framework. This may be called a robust LQG or Minimax problem. In this method, a cost functional, as in the LQG problem, is introduced and then a robust controller is designed by defining constraints on the magnitude of the model uncertainties and on the maximum level of noise uncertainty. This problem will be more complicated when the noise affecting the system is not a white Gaussian noise. In the LQG control, it is assumed that a white Gaussian noise is the input to the plant. It is proved that if the real input noise to the plant is not Gaussian, this algorithm can result in robust stability of the closed-loop system by appropriate tuning of some parameters [24]. However, the system may show poor robust performance.

As is shown in Fig. 4, the disturbance $w$ is applied to the output of the system. For the case when the disturbance is assumed in the input of the plant, the same approach can be used [24]. It should be noted that the general plant in (5) includes the states of the main system $G(s)$, as well as its uncertainty weight $W(s)$. If the state-space realization of the main system is shown by $(A_m, B_m, C_m, D_m)$, and the state-space realization of the uncertainty weight is shown by $(A_d, B_d, C_d, D_d)$, the augmented plant can be written as follows:

$$
\dot{x} = \begin{bmatrix} A_m & 0 \\ 0 & A_d \end{bmatrix} x + \begin{bmatrix} B_m \\ 0 \end{bmatrix} u + \begin{bmatrix} B_d \\ 0 \end{bmatrix} p + \begin{bmatrix} 0 \\ 0 \end{bmatrix} w
$$

$$
q = \begin{bmatrix} 0 & C_m \end{bmatrix} x + D_m u
$$

$$
y = \begin{bmatrix} C_m & 0 \end{bmatrix} x + D_m u,
$$

where $x = [x_m \quad x_d]^T$ is the state of the perturbed system, and

$$
A = \begin{bmatrix} A_m & 0 \\ 0 & A_d \end{bmatrix}, \quad B_1 = \begin{bmatrix} B_m \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} B_m \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & C_m \end{bmatrix}, \quad D_1 = D_m.
$$

$C_2 = [C_m \quad 0], D_2 = D$. In (5), $D$ is a tuning parameter. As is clear from Fig. 4, this parameter is a weighting parameter for the white Gaussian noise applied to the output of the plant. The relationship between input and output of the uncertainty model may be written in the general integral quadratic constraint form as follows:

$$
\lim_{T \to \infty} \frac{1}{T} E \left[ \int_0^T \| p(t) \|^2 dt - \int_0^T \| q(t) \|^2 dt \right] \leq d
$$

(6)

where $d$ is a parameter that restricts the size of total disturbance acting on the system. Choosing $d$ a small positive real number is equivalent to a large infinity norm of the normalized uncertainty $\Delta$, and hence a better robust performance for the controller is expected. In contrast, the existence of large uncertainties in the disturbance signal (such as non-Gaussian noise) is equivalent to a large $d$, and this will lead to the design of a controller with good robust stability.

The performance index for the design of a Minimax LQG controller given in (7) is similar to the standard LQG problem, unless there are some additional constraints on uncertainties, like the one in (6):

$$
J = \lim_{T \to \infty} \frac{1}{2T} E \int_0^T (x(t)^T Q x(t) + u(t)^T R u(t)) dt
$$

(7)

In (7), $Q$ is the state weighting matrix and is equal to $C_2^T C_2$, $R$ is the weighting matrix on the input and limits the control effort. This weight is usually selected large enough to prevent the input saturation of the control signal. The solution of the Minimax LQG problem can be obtained by solving two steady state algebraic Riccati equations, as follows:

$$
(\bar{A} - \bar{B}_2 \bar{D}_2^T (\bar{D}_2 \bar{D}_2^T)^{-1} \bar{C}_2) Y_{\infty} + Y_{\infty} (\bar{A} - \bar{B}_2 \bar{D}_2^T (\bar{D}_2 \bar{D}_2^T)^{-1} \bar{C}_2)^T - Y_{\infty} \left( \bar{C}_2^T (\bar{D}_2 \bar{D}_2^T)^{-1} \bar{C}_2 - \frac{1}{\tau} Q_{\infty} \right) Y_{\infty} + \bar{B}_2 (I - \bar{D}_2^T (\bar{D}_2 \bar{D}_2^T)^{-1} \bar{D}_2) \bar{B}_2^T = 0
$$

(8)
where the new matrices are defined as:

\[ Q_r = Q + \tau C_1^T C_1 \]
\[ R_r = R + \tau D_1^T D_1 \]
\[ \Pi_r = \tau C_1^T D_1. \]

Besides, the solutions of (8) and (9) are required to satisfy the following conditions:

\[ Y_\infty > 0, \]
\[ X_\infty > 0, \]
\[ I - \frac{1}{\tau} \bar{Y}_\infty X_\infty > 0. \]

As is clear from the preceding equations, parameter \( \tau \) has a dominant effect in the design of the controller. In order to be able to choose the value of \( \tau \) appropriately, a new cost functional derived from (7) can be written as:

\[
W_\tau = \frac{1}{2} \times \text{trace} \left[ Y_\infty Q_r + \left(Y_\infty C_2^T + \bar{B}_r \bar{D}_1 \right) \left(\bar{D}_2 \bar{D}_2^T \right)^{-1} \bar{C}_2 \right]
+ \text{rd.} \quad (12)
\]

Since parameter \( \tau \) does not depend linearly on \( X_\infty \) and \( Y_\infty \), minimizing the resulted cost functional is not a straightforward task. The key problem in designing a good Minimax LQG controller is to choose a suitable value for \( \tau \). It can be shown that this cost functional has an infimum at \( \tau_1 > 0 \), and this value can be found by trial and error [24]. The Minimax LQG controller is obtained in state-space representation by the set of equations below:

\[
A_c = \bar{A} + \frac{Y_\infty Q_r}{\tau} - (Y_\infty C_2^T + \bar{B}_r \bar{D}_1) \left(\bar{D}_2 \bar{D}_2^T \right)^{-1} \bar{C}_2
- \left( \bar{B}_1 + \frac{Y_\infty \Pi_r}{\tau} \right) \bar{R}_r^{-1} \bar{B}_1^T X_\infty + \Pi_r \left( I - \frac{1}{\tau} Y_\infty X_\infty \right)^{-1}
\]
\[
B_c = (Y_\infty C_2^T + \bar{B}_r \bar{D}_1) \left(\bar{D}_2 \bar{D}_2^T \right)^{-1}
\]
\[
C_c = \bar{R}_r^{-1} \bar{B}_1^T \left( X_\infty + \Pi_r \right)
\left( I - \frac{1}{\tau} Y_\infty X_\infty \right)^{-1}
\]
\[
D_c = 0.
\]

There are several parameters in this algorithm that must be tuned properly to have a good closed-loop performance. The performance of the closed loop system must be checked in every step of tuning these parameters. The interested reader can refer to [24,25,31] for more description on Minimax LQG control problem.

5. Controller design for flexible beam

In this section the aim is to evaluate the designed minimax LQG controller for the identified model in Section 3, and its associated uncertainties. In fact the performances of the designed controllers are simulated using experimental data, and no explicit experimental implementation of the control system was attempted due to the lack of necessary actuators. As a matter of fact, in this study, the model of the beam in the frequency range of 100–400 Hz is measured and estimated for the design of the controller, and the other three modes are considered as the uncertainty to take into account the inappropriateness of the control model. If the intent is to design a controller for a practical active control application, the weighting functions must be tuned by trial and error to achieve a reasonable performance for the designed controller in the presence of other sources of uncertainties that are not known beforehand. However, the results presented here are sufficient to demonstrate the difficulties which may occur in the design of a controller in real applications such as how the spillover will degrade the performance of the closed-loop system or may even cause instability.

The identified model in Section 3 is used to design a robust controller using Minimax LQG technique. As mentioned earlier, selecting a large \( d \) in (6) may result in good robust stability in the presence of uncertainties, but on the other hand it will deteriorate the performance of the closed-loop system. Besides, increasing \( d \) can increase the cost functional linearly. After some trial and error, \( d \) was chosen 10–10. For our SISO plant, \( R \) is a scalar and was selected 10–8. \( D \) which relates noise to the output has a great effect on the value of the cost functional and was chosen 10–3.

By plotting the performance index \( W_\tau \) for different values of \( \tau \), the interval where the global infimum of the cost functional may be found was extracted. Then, with a more exact search using a nonlinear optimization algorithm, the fine tune of \( \tau \) was performed. It should be noted that a small change in \( \tau \) may impair the performance of the system. This search was performed for two types of uncertainty filters designed in Section 2, and hence two different values of \( \tau \) were obtained. Having these values, the robust controller can be designed using (13). The values of \( \tau \) and their corresponding cost functions are:

\[ \tau_1 = 197.2 \quad W_{\tau_1} = 18883.77 \]
\[ \tau_2 = 931.6 \quad W_{\tau_2} = 42026.41. \]

In comparison with the estimated model of the beam which is of order 6, the designed controllers are of order 14. This is due to the augmentation of the model of the beam with the designed weighting functions. For real applications, this controller must be discretized and the control signal will be applied to a PZT or the shaker power-amplifier using a D/A card installed on the computer. The amplified signal then will be applied to the structure by the PZT actuator or the shaker. The frequency responses of two designed controllers in the bandwidth of 0–700 Hz are plotted in Figs. 7 and 8 respectively. It can be inferred from these figures that the robust controller has the same nature as the plant, and the resonant peaks of the controllers occur near the resonant peaks of the plant. As stated earlier, the aim of these two controllers is to decrease the resonant peaks of the frequency response of the beam, while holding the robust stability condition with respect to unmodeled dynamics.

Open-loop and closed-loop frequency responses of the plant for uncertainty weights \( W_1 \) and \( W_2 \) are shown in Figs. 9 and 10, correspondingly. As can be seen in these figures, the damping of the modes for the first controller is greater than that of the second controller. This can be described as a result of a tighter uncertainty bound in the design of this controller. In fact, at low and high frequencies of Fig. 6 where the magnitude of the weighting function is large, the optimizer will force the magnitude of the controller to be reduced. This can be clearly seen by comparing the magnitude of the controllers in Figs. 7 and 8. On the other hand, reducing the loop gain as a result of the reduction of the control effort will deteriorate the disturbance rejection performance of the closed-loop control system. The bad performance of the controller for the first two modes in Fig. 10 is the result of this phenomenon. Certainly this bad performance is traded off by large robust stability margins that may occur even in the controller bandwidth.
In order to have a better view of the properties of the designed controller, its performance against a broadband disturbance (a multi-sinusoid in the frequency range of 0–700 with random phase) is shown in Fig. 11. As can be seen from this figure, the amplitude of the time response of the closed-loop system is reduced, especially at resonance frequencies. The control effort of the controller is plotted in Fig. 12. It can be seen in this figure that the amplitude of the control effort is in a range which is comparable.
to those in real life applications, and hence reasonable for implementation using PZT actuators, for example. The comparison of magnitude of both controllers with the magnitude of the FRF of the beam in Fig. 9, also confirms that the generated control signals can be applied experimentally using, for example, a PZT actuator or electro-dynamic shaker.

6. Conclusions

Robust control and performance trade-off in the design of an active vibration control system for a flexible beam is investigated in this paper. By fitting a continuous transfer function to the second, third and fourth modes of the beam based on experimental data, the nominal plant of the system is obtained. The other three modes of the beam are assumed as the uncertainty model. Two different weighting functions with loose and tight fitting of uncertainty are selected to show the robustness and performance trade-offs in the Minimax LQG control technique. A multiplicative type of uncertainty is used in modeling, and the Minimax LQG controller is designed for the plant augmented by uncertainty model. The aim is to design a controller which must have good robust stability and performance against model and noise uncertainties. The controller is obtained in a worst-case situation and an optimal problem is solved to achieve good performance for the closed-loop system.

Since uncertainties have undesired effects on the stability of the system, selecting a conservative weighting function in the uncertainty model may lead to robust stability. However, the designer should make a good trade-off between stability margin and performance of the closed-loop system. Moreover, the LQG controller design was based on white Gaussian noise disturbance. Therefore, there are other uncertainties such as non-Gaussian noise or Gaussian noise with varying variance, the performance of the closed-loop system may be impaired. However, by the Minimax LQG controller, the robust stability against these types of uncertainty is attained. As there are many parameters which can affect the performance of the closed-loop system, it must be remembered that by choosing some suitable parameters, the desirable performance can be achieved. It was also shown that the Minimax LQG controller may not eliminate the vibrations perfectly; however, since the controller decreases the resonance peaks of the frequency response of the beam, the compensated structure will exhibit better damping.

References