Speed-sensorless tracking control of a DC-motor via a double Buck-converter

J. Linares-Flores[†⋆], J. Reger[‡], and H. Sira-Ramírez[†]

***Abstract*— This article deals with the sensor-less trajectory tracking control of a double-Buck-converter/dc-motor combination. We simultaneously regulate, both, the output voltage of the first buck-converter and the angular velocity of the dc-motor shaft so that it tracks a pre-specified reference trajectory. The main result of our proposed control scheme is a linear time-varying controller that is based only on measurements of converter currents and voltages. The control law is derived using the exact tracking error dynamics from which a static linear passive output feedback is derived. All variables of the reference trajectories are generated exploiting the differential flatness property of the combined multi-variable system. The discrete switching control realization of the designed continuous feedback control law is accomplished by means of a $\Sigma-\Delta$-modulation scheme. Experimental results are provided.**

I. INTRODUCTION

The combination of dc-to-dc power converters with dc-motor drives is very common in industry. These power converters apply the required supply voltage to the dc-motor in accordance with the demanded task [7], represented, for example, by the desired angular velocity profile or angular position reference trajectory. Customarily, the proposed feedback controllers are devised under very restrictive assumptions, e.g., ramps and constant values acting as reference trajectories for the specification of a desired angular velocity profile. This policy often results in unfavorable transient behavior and, impulse-like functions for the controller output, with the consequence that the mechanical part has to absorb the undesired discontinuities [3].

Nonlinear average models replacing the discrete nature of the switching control in the transistor are commonly used in the feedback control design for dc-to-dc power converters [6], [2], [5]. Switched implementations of average dynamic output feedback control laws by means of a $\Sigma-\Delta$-modulator are widely known in the classic communications and analog signal encoding literature; for novel applications see [3], [9]. In this paper, we present a tracking control for a smooth stationary set-point change of the output voltage of a buck-converter loaded by a second buck converter feeding the dc motor. The angular velocity of dc-motor, driven by the second buck-converter, is also independently regulated. We call this cascade arrangement of buck converters a "double Buck-converter". The main task is to achieve the regulation of the angular velocity of the dc-motor shaft. A simple linear-time-varying state feedback controller, based on exact tracking error dynamics passive output feedback, is shown to semi-globally stabilize the state trajectory tracking error to zero while requiring only the measurement of the converter currents and voltages for the feedback law synthesis. The dynamic average multi-variable model of the "double-Buck-converter/dc-motor" combination is shown to conform to a special energy managing structure which is suitable for passivity-based feedback techniques. The reference signals of the multi-variable system are generated exploiting the differential flatness property of the overall system. We use a polynomial interpolation of Bézier type for the calculation of the two smooth reference trajectories corresponding to the initial and final desired voltage of first buck converter and the angular velocity of dc-motor, on the output side of the second buck-converter. These trajectories, in turn, completely define the reference trajectories of the input currents and average controls of the double Buck-converter. The reference trajectories can all be computed in an off-line fashion.

Our contribution is organized as follows: In Section 2 a linear time-varying feedback controller is derived which is based on elementary passivity considerations on the Exact Tracking Error Dynamics Passive Outputs Feedback (ETEDPOF), see [8], Section 3 deals with the average model of the multi-variable system "double-Buck-converter/dc-motor", with the converters in a cascaded arrangement. Section 4 is concerned with the flatness-based off-line trajectory generation required by the proposed linear time-varying feedback controller. In Section 5 the experimental setup is briefly described and used for validating the performance of the proposed control policy via actual laboratory experiments. The last section presents some conclusions and perspectives for further work.

II. EXACT TRACKING ERROR DYNAMICS PASSIVE OUTPUT FEEDBACK

Consider the following passive system in matrix notation

$$A\,\dot{x} = (J(u_{av}) - R)\,x + B\,u_{av} + \varepsilon(t) \tag{1}$$

where x is an n-dimensional vector, A is a symmetric, positive definite, constant, matrix, $J(u_{av})$ is a skew symmetric matrix, for all u_{av}, of the form

$$J = J_0 + \sum_{i=1}^{m} J_i\,u_{i,av}\,. \tag{2}$$

⋆ Author for correspondence

† J. Linares-Flores and H. Sira-Ramírez are with the Sección de Mecatrónica, Centro de Investagación y Estudios Avanzados del IPN (CINVESTAV-IPN), 07300 México D.F., México, email: jlinares@cinvestav.mx, hsira@cinvestav.mx

‡ Institute of Automation and Control (EIT 8.1), Universität der Bundeswehr München, Werner-Heisenberg-Weg 39, 85579 Neubiberg, Germany, email: reger@ieee.org

Here the matrices J_i, $i = 0,,1\ldots,m$ are constant skew symmetric matrices. R is a symmetric, positive semi-definite constant matrix. B is a constant $n\times m$ matrix. In terms of its n-dimensional column vectors, the matrix B is given by $B = [b_1, b_2, \ldots, b_m]$. The vector u_{av} is the average control input vector assumed to be m-dimensional, with each component $u_{i,av}$ taking values in the closed set $[0,1]$ of the real line. In any case, $u_{i,av}$ represents a *bounded* average control input function. $\varepsilon(t)$ is an n-dimensional smooth vector function of the time t or, sometimes, a vector of constant entries.

Note that the matrix R represents the *dissipative field* of the system while $J(u_{av})$ represents the, possibly control input dependent, *conservative field* of the system. The *control input channels* are represented by the constant matrix B while $\varepsilon(t)$ represents *external input sources*, such as batteries or ac line voltages.

For abbreviation, we let $B^\star(t)$ represent the time varying matrix

$$B^\star(t) = (J_1 x^\star(t) + b_1, \ldots, J_m x^\star(t) + b_m)\ . \tag{3}$$

In the sequel, we adhere to the following assumptions:

1) We assume that $u_{i,av} \in [0,1]$ for all t.
2) The system (1)-(2) is differentially flat (see [8],[10]). This property guarantees that given an arbitrary smooth and bounded reference trajectory $x^\star(t) \in \mathbb{R}^n$, there exists a smooth open loop bounded control input, $u^\star_{av}(t) \in [0,1]^m$, such that for all trajectories starting at time t_0 at state $x(t_0) = x^\star(t_0)$ the tracking error $e(t) = x(t) - x^\star(t)$ is identically zero for all $t \in [t_0, \infty[$. In other words the nominal system

   $$A\dot{x}^\star = (J(u^\star_{av}) - R)\, x^\star + B\, u^\star_{av} + \varepsilon(t) \tag{4}$$

   is valid for all times with a given $x^\star(t_0)$ which renders the components of the open loop nominal control $u^\star_{av}(t)$ bounded in the interval $[0,1]^m$, for all $t \geq t_0$.
3) The *dissipation matching* condition is satisfied, that is, for any constant, positive definite, symmetric matrix Γ we have uniformly valid the following relation

   $$R + B^\star(t)\,\Gamma\, {B^\star}^{\mathrm{T}}(t) > 0\,. \tag{5}$$

   Roughly speaking, this condition assures that if the natural dissipation of the system is not given in some subspaces it may nevertheless be achieved by an appropriate control action.

We state the following theorem from [8]:

Theorem 1 *Under the previous assumptions on the system (1)-(2), the tracking error vector $e(t) = x(t) - x^\star(t)$ is semi-globally asymptotically exponentially stabilized to zero whenever a linear time-varying tracking error feedback controller of the form*

$$u_{av} = u^\star_{av} - \Gamma\, B^\star(t)^{\mathrm{T}} e(t) \tag{6}$$

is used.

Note that the presented controller is a *linear* time-varying feedback controller that enforces an Exact Tracking Error Dynamics by Passive Outputs Feedback (ETEDPOF); furthermore called ETEDPOF-controller.

Remark 1 *The weakening of condition 3 to a positive semi-definite condition, still produces an asymptotic stability result, provided Krasovskii-LaSalle's invariance condition is satisfied [12]. In other words, whenever the set*

$$\left\{ e \in \mathbb{R}^n \,|\, \exists t \geq t_0 \text{ such that } e^{\mathrm{T}} \left(R + B^\star(t)\,\Gamma\, {B^\star}^{\mathrm{T}}(t) \right) e = 0 \right\}$$

contains only the element $e = 0$ of the tracking error space.

Proof We set $e_u = u_{av} - u^\star_{av}$ and note that due to the linearity of equation (2) the expression $J(u_{av}) - J(u^\star_{av})$ may be written as

$$J(u_{av}) - J(u^\star_{av}) = \sum_{i=1}^{m} J_i \left(u_{i,av} - u^\star_{i,av} \right) = \sum_{i=1}^{m} J_i\, e_{u,i}\,. \tag{7}$$

We may obtain the tracking error dynamics using (1) and (4), accordingly

$$\begin{aligned} A\,\dot{e} \;&=\; A\,(\dot{x} - \dot{x}^\star) = \\ &=\; J(u_{av})\,(x - x^\star) - R\,(x - x^\star) + B\,e_u + \\ &\quad (J(u_{av}) - J(u^\star_{av}))\, x^\star\,. \end{aligned} \tag{8}$$

Substitution of equation (7) in (8) results in

$$\begin{aligned} A\,\dot{e} \;&=\; J(u_{av})\,e - R\,e + B\,e_u + \sum_{i=1}^{m} J_i\, e_{u,i}\, x^\star \\ &=\; J(u_{av})\,e - R\,e + (B + (J_1 x^\star, \ldots, J_m x^\star))\, e_u \end{aligned} \tag{9}$$

which now with equation (3) is equivalent to

$$A\,\dot{e} = J(u_{av})\,e - R\,e + B^\star(t)\,e_u \tag{10}$$

that is, the open loop tracking error dynamics.

We choose the Lyapunov function candidate $V(e) = \frac{1}{2} e^{\mathrm{T}} A\, e$. The symmetry of the matrix A and the antisymmetry of the matrix $J(u_{av})$, for any u_{av}, then implies that

$$\dot{V}(e) = e^{\mathrm{T}} A\,\dot{e} = -e^{\mathrm{T}} R\,e + e^{\mathrm{T}} B^\star(t)\,e_u$$

In view of equation (6), we may express $e_u = u_{av} - u^\star_{av} = -\Gamma\, {B^\star}^{\mathrm{T}}(t)\, e(t)$, which shows that

$$\dot{V}(e) = -e^{\mathrm{T}} \left(R + B^\star(t)\,\Gamma\, {B^\star}^{\mathrm{T}}(t) \right) e < 0\,.$$

The exponential stability follows from the uniform boundedness of the positive definite symmetric matrix $R + B^\star(t)\,\Gamma\, {B^\star}^{\mathrm{T}}(t)$ by a constant symmetric matrix, provided by assumption 2, and the fact that A and R are constant symmetric matrices. However, since the average control input $u^\star_{av}(t)$ is bounded in the interval $[0,1]^m$, for all $t \geq t_0$, the result cannot be global. The stability of the origin of the error space will be semi-global due to the fact that control input saturations will depend on the values of the initial states. We have, as a consequence that, semi-globally,

$$\lim_{t\to\infty} e = 0 \quad \Longrightarrow \quad \lim_{t\to\infty} x = x^\star\,.$$

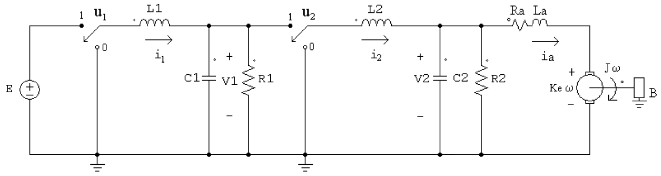

Fig. 1. Model of a cascaded double-Buck/dc-motor combination

III. Multi-variable Model of the Combined dc-motor/double-Buck-converter

We consider a cascaded combination of a dc-to-dc "double-Buck" power converter and a dc-motor, as depicted in Fig. 1. Using Kirchhoff's current and voltage laws and Newton's second law of mechanics, we obtain the following average model of the multivariable system:

$$L_1 \frac{di_1}{dt} = -v_1 + E u_1 \tag{11}$$

$$C_1 \frac{dv_1}{dt} = i_1 - \frac{v_1}{R_1} - i_2 u_2 \tag{12}$$

$$L_2 \frac{di_2}{dt} = -v_2 + v_1 u_2 \tag{13}$$

$$C_2 \frac{dv_2}{dt} = i_2 - i_a - \frac{v_2}{R_2} \tag{14}$$

$$L_a \frac{di_a}{dt} = v_2 - R_a i_a - K_e \omega \tag{15}$$

$$J \frac{d\omega}{dt} = K_m i_a - B\omega + \tau_L \tag{16}$$

Note that for the dc-motor $K_m = K_e = K$. In light of the denotations introduced in the past section we may then write

$$A\,\dot{x} = (J(u_{av}) - R)\,x + B\,u_{av} + \varepsilon(t) \tag{17}$$

where

$$A = \mathrm{diag}(L_1, C_1, L_2, C_2, L_a, J)\,,$$

$$J(u_{av}) = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -u_{2,av} & 0 & 0 & 0 \\ 0 & u_{2,av} & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -K \\ 0 & 0 & 0 & 0 & K & 0 \end{pmatrix},$$

$$R = \mathrm{diag}\left(0, \frac{1}{R_1}, 0, \frac{1}{R_2}, R_a, B\right),$$

$$B = \begin{pmatrix} E & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \varepsilon(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tau_L \end{pmatrix}, \quad x^\star = \begin{pmatrix} i_1^\star \\ v_1^\star \\ i_2^\star \\ v_2^\star \\ i_a^\star \\ \omega^\star \end{pmatrix}.$$

Thus, we obtain

$$B^\star(t) = (J_1\,x^\star + b_1, J_2\,x^\star + b_2) = \begin{pmatrix} E & 0 \\ 0 & -x_3^\star \\ 0 & x_2^\star \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{18}$$

by means of which the control reads

$$u = u_{av}^\star - \Gamma B^{\star\mathrm{T}}(t)\,(x - x^\star)\,. \tag{19}$$

We may use the diagonal matrix $\Gamma = \mathrm{diag}(\Gamma_1, \Gamma_2)$, with Γ_1 and Γ_2 both positive constants. It can easily be shown that the dissipation matching condition, see equation (5), is satisfied as long as the reference $x_2^\star > 0$, for all $t > t_0$, which is a reasonable assumption in a practical setup because the second Buck-converter needs an input voltage, in any case.

Finally, we obtain the control inputs

$$u_{1,av} = u_{1,av}^\star - \Gamma_1\,E\,(i_1 - i_1^\star) \tag{20}$$

$$u_{2,av} = u_{2,av}^\star + \Gamma_2\,i_2^\star\,(v_1 - v_1^\star) - \Gamma_2\,v_1^\star\,(i_2 - i_2^\star) \tag{21}$$

IV. Trajectory planning

The system is differentially flat, with flat outputs given by the output voltage v_1 and the angular velocity of the dc-motor ω. In terms of reference trajectories we may now write

$$F_1 = v_1^\star\,, \tag{22}$$

$$F_2 = \omega^\star\,. \tag{23}$$

Indeed, using the nominal (loadless) model of the system equations (11) – (16) all system reference variables (states and controls) are parameterizable in terms of the flat output references F_1 and F_2, that is

$$v_1^\star = F_1 \tag{24}$$

$$i_1^\star = C_1 \dot{F}_1 + \tfrac{1}{R_1} F_1 + i_2^\star u_2^\star \tag{25}$$

$$u_1^\star = \tfrac{L_1 C_1}{E} F_1^{(2)} + \tfrac{L_1}{E R_1} \dot{F}_1 + \tfrac{1}{E} F_1 + \tfrac{L_1}{E} \tfrac{di_2^\star}{dt} u_2^\star + \tfrac{L_1}{E} i_2^\star \tfrac{du_2^\star}{dt} \tag{26}$$

$$\omega^\star = F_2 \tag{27}$$

$$i_a^\star = \tfrac{J}{K} \dot{F}_2 + \tfrac{B}{K} F_2 \tag{28}$$

$$v_2^\star = \tfrac{L_a J}{K} F_2^{(2)} + \tfrac{L_a B + R_a J}{K} \dot{F}_2 + \tfrac{R_a B + K^2}{K} F_2 \tag{29}$$

$$\begin{aligned} i_2^\star = {} & \tfrac{C_2 L_a J}{K} F_2^{(3)} + \tfrac{C_2 L_a B R_2 + C_2 J R_a R_2 + L_a J}{R_2 K} F_2^{(2)} \\ & + \tfrac{C_2 B R_a R_2 + C_2 R_2 K^2 + L_a B + R_a J + J R_2}{K} \dot{F}_2 \\ & + \tfrac{R_a B + K^2 + B R_2}{R_2 K} F_2 \end{aligned} \tag{30}$$

$$\begin{aligned} u_2^\star = {} & \tfrac{L_2 C_2 L_a J}{K} \tfrac{F_2^{(4)}}{F_1} + \tfrac{L_2 C_2 L_a B R_2 + L_2 C_2 J R_a R_2 + L_2 L_a J}{R_2 K} \tfrac{F_2^{(3)}}{F_1} + \\ & \tfrac{L_2 C_2 B R_a R_2 + L_2 C_2 R_a K^2 + L_2 J R_2 K + L_2 L_a B + L_2 J R_a + L_a J R_2}{R_2 K} \tfrac{F_2^{(2)}}{F_1} \\ & + \tfrac{L_2 B R_2 + L_2 B R_a + L_2 K^2 + L_a B R_a + J R_a R_2}{R_2 K} \tfrac{\dot{F}_2}{F_1} + \tfrac{R_a B + K^2}{K} \tfrac{F_2}{F_1} \end{aligned} \tag{31}$$

Once the references trajectories $v_1^\star$ and $\omega^\star$ are specified, all reference trajectories may be generated in view of the control law (20)-(21) — in particular, $i_1^\star$ and $i_2^\star$, and moreover, the nominal controls $u_1^\star$ and $u_2^\star$ which are the average controls $u_{1,av}^\star$ $u_{2,av}^\star$, respectively.

Note that the trajectory planning is an off-line task. Consequently, it may be verified in advance that a certain choice of flat outputs does result in control inputs in saturation or not. Should the situation arise, imposing reduced slopes on the references trajectories of the flat outputs in most cases solves the problem.

V. EXPERIMENTAL SET-UP FOR THE "DOUBLE-BUCK" CONVERTER-DC MOTOR SYSTEM

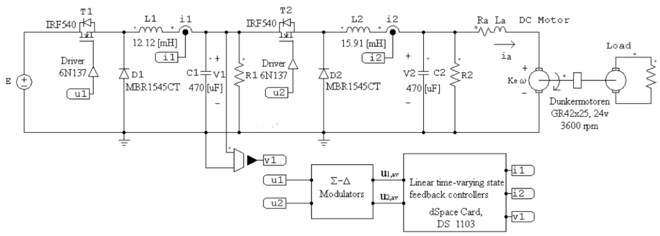

Fig. 2. Experimental setup of the double-Buck-converter/dc-motor with a ETEDPOF-controller commanded by a dSPACE DS1103 controller card

The design parameters of the experimental setup of the "double-Buck"-power converter were chosen to be: $E = 55$V, $C_1 = 470\mu$F, $C_2 = 470\mu$F, $L_1 = 12.12$mH, $L_2 = 15.91$mH, while the switching frequency was set to 45kHz. A resistance of value $R_1 = 100\Omega$ on the output of the first buck-converter is used to simulate a constant load, the second resistance was chosen to be $R_2 = 10\text{k}\Omega$, implying just little losses. The parameters identified by standard means on a real dc-motor (Dunkermotoren GR42x25) were found to be: $R_a = 8.13\Omega$, $L_a = 8.9$mH, $K = 43.15\text{V}\sec/\text{rad}$, $J = 7.95 \times 10^{-6}\text{kgm}^2$, $B = 47\mu\text{Nm}\sec/\text{rad}$. The load on the dc-motor is represented by an identical motor (same inertia, resistance, etc.) acting as a generator; it is connected to a resistance of $R_{\text{load}} = 100\Omega$. The sampling time of the digital controller was set to be $50\mu\sec$. We have chosen moderate controller gains of $\Gamma_1 = 0.02$ and $\Gamma_2 = 0.01$ in recognition of the noise on the current and voltage measurements that enter the control law unfiltered. By this choice in addition, the control inputs are kept away from the saturation bounds. A DS1103 dSPACE controller card was used for the implementation of the ETEDPOF-controller (see Figure 2). The switched control law was implemented by means of two $\Sigma - \Delta$ modulators (see [4]) which command the switch position functions of the converters. We devised the average continuous time-varying, linear, feedback control laws as the computed duty ratio functions.

A nominal desired angular velocity profile $\omega^\star$ that exhibits a rather smooth start for the dc-machine was specified using an interpolating Bézier polynomial of 11th order where the initial angular velocity was set to be $\omega_{ini} = 0\text{rad}/\sec$ valid until $t_{ini,\omega} = 3\sec$ and the final desired value of the angular velocity was specified as $\omega_{fin} = 450\,\text{rad}/\sec$ to be reached at $t_{fin,\omega} = 4.5\sec$, that is, we used

$$\omega^\star(t) = \begin{cases} \omega_{ini}, \; t < t_{ini,\omega} \\ \omega_{fin}, \; t > t_{fin,\omega} \\ \omega_{ini} + (\omega_{fin} - \omega_{ini})\, b_\omega\left(\dfrac{t - t_{ini,\omega}}{t_{fin,\omega} - t_{ini,\omega}}\right), \; \text{else} \end{cases}$$

with

$$b_\omega(\tau) = 462\,\tau^6 - 1980\,\tau^7 + 3465\,\tau^8 - 3080\,\tau^9 + 1386\,\tau^{10} - 252\,\tau^{11}\,.$$

The nominal desired output voltage profile $v_1^\star$ for the first buck converter was specified to also exhibit a rather smooth rest to rest trajectory. To this end, we used a fifth order Bézier polynomial. The initial output voltage was set to be $v_{1,ini} = 0.1$mV valid until $t_{ini,v} = 0.5\sec$ and the final desired value of the output voltage was specified as $v_{1,fin} = 28$V to be reached at $t_{fin,v} = 1\sec$, that is

$$v_1^\star(t) = \begin{cases} v_{1,ini}, \; t < t_{ini,v} \\ v_{1,fin}, \; t > t_{fin,v} \\ v_{1,ini} + (v_{1,fin} - v_{1,ini})\, b_v\left(\dfrac{t - t_{ini,v}}{t_{fin,v} - t_{ini,v}}\right), \; \text{else} \end{cases}$$

with

$$b_v(\tau) = 10\,\tau^3 - 15\,\tau^4 + 6\,\tau^5\,.$$

The reference trajectories employed in the control law (20) and (21) follow from the above-given reference trajectories $\omega^\star$ and $v_1^\star$ as a consequence of the differential parametrization stated in equation (25), (26), (30), and (31).

In the measurement, initially, the input current results in $i_{1,ini} = 0.03$A, reaches a value of 0.21A at about $1\sec$. At $4.5\sec$ it takes the final value of $i_{1,fin} = 0.52$A. Concerning the current i_2, initially, it is $i_{2,ini} = 0.01$A and finally reaches $i_{2,fin} = 0.44$A at about $4.5\sec$, as depicted in Fig. 3. The output voltage v_1 starts at the initial value of $v_{1,ini} = 0$V and reaches a temporary stationary value of 29V at $1\sec$. When the acceleration of the motor shaft sets in, v_1 increases up to the final value of $v_{1,fin} = 35$V, (see Figure 3).

The currents i_1 and i_2 show significant differences to the corresponding reference currents. Those differences are due to un-modeled losses, as for example resistances in the input inductor, diode resistances and polarization sources, as well as resistances in the transistors. However, the tracking of the current reference trajectories and of the voltage v_1 are satisfactory as long as the motor shaft does not accelerate. This is the cause of load effects which are imposed by the generator inertia and the load resistance of $R_{\text{load}} = 100\Omega$. Consequently, the corresponding average control input signals $u_{1,av}$ and $u_{2,av}$ generated by the ETEDPOF-controller of the "double-Buck-converter/dc-motor" system vary from their nominal controls $u_{1,av}^\star$ and $u_{2,av}^\star$, respectively; the more when the motor accelerates. In any case, the control action remains in the bounds of $[0, 1]$. The final values taken by both average control inputs are $u_{1,av}^{fin} = u_{2,av}^{fin} = 0.7$V. The tracking of the angular velocity, however, is very satisfactory (See Reinoza *et al.* [11]).

VI. CONCLUSION

In this article we have presented a sensor-less trajectory tracking controller for the cascade combination of two Buck-converters, the first of them possessing a resistive load and the second converter loaded by a dc-motor. We simultaneously regulate, both, the output voltage of the first buck-converter and the angular velocity of the dc-motor shaft by means of switched controlled policies synthesizing the duty ratio functions commanding the switched "actuators" of the cascaded converters. The control objectives are 1) to regulate the motor angular velocity so that it tracks a pre-specified reference trajectory and 2) to regulate the output voltage of the first converter to track a smooth rest-to-rest voltage

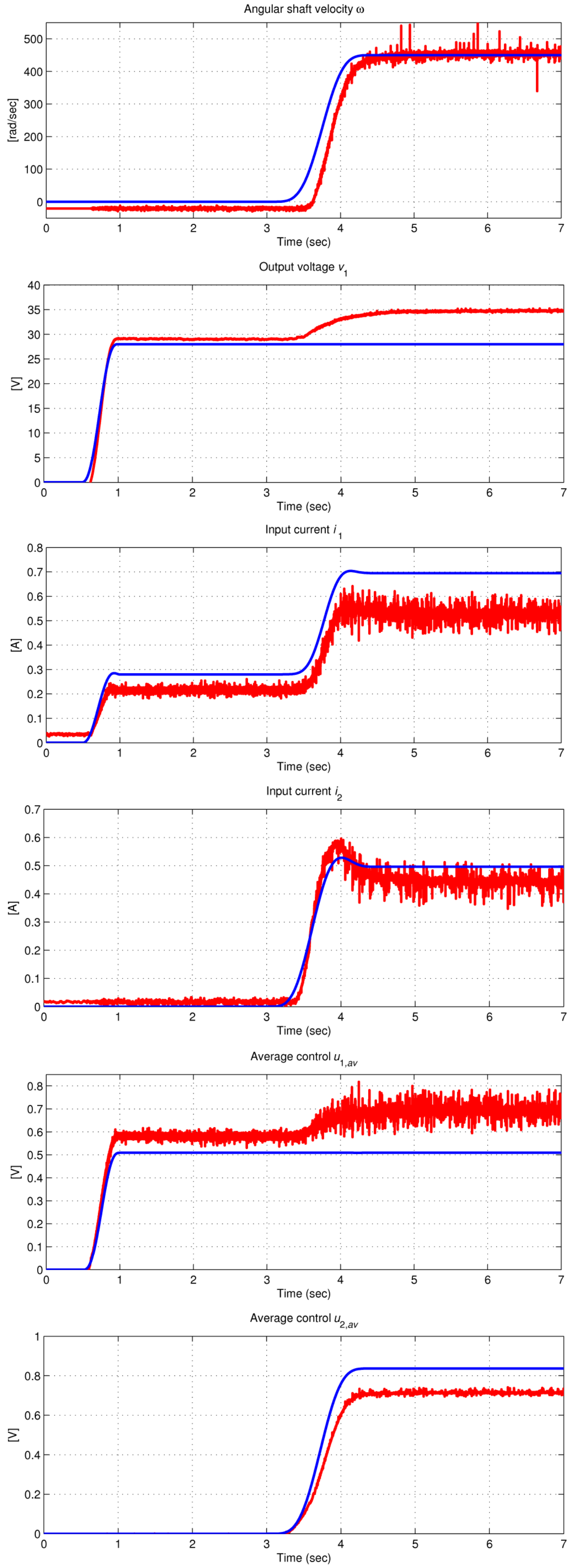

Fig. 3. Experimental results for the "double-Buck" converter-dc motor with ETEDPOF-controller — reference (blue line), measurement (red line)

reference trajectory on a known resistive load. The main result of the paper is the introduction of a linear time-varying feedback controller which is based only on measurements of the converter inductor currents and the output voltage of the first Buck-converter. The derived feedback control law is, in fact, a static passivity based controller that uses the exact tracking error dynamics passive output for feedback purposes. Such a passive output demands the use of electric variables alone. Therefore no mechanical sensors, such as tacho-meters, are needed for the implementation of the controller. All nominal reference trajectories demanded by the linear feedback controller are generated exploiting the differential flatness property of the combined multi-variable system. The discrete switching control realization of the designed continuous feedback control law is accomplished by means of a $\Sigma-\Delta$-modulation scheme. In this article we present actual experimental results depicting the closed loop performance of the system under the proposed multi-variable linear time-varying controller.

Further experimental studies are possible and other challenging problems remain to be solved. Suitable modifications of the proposed controller to achieve robustness with respect to resistive load variations and unforseen load torques affecting the motor shaft are being pursued at the present time. These entitle adaptive feedback control schemes and also algebraic parameter estimation options. Furthermore, the resistive load on the first buck-converter will be replaced with a dc-motor of appropriate nominal input voltage in order to validate the performance of the controller in an application on two real dynamic systems.

Acknowledgements

The authors gratefully acknowledge the constructive comments of the reviewers. Moreover, the authors express their gratitude to Dipl.-Phys. Bernd Schmidt for his valuable contributions to the experimental realization.

References

[1] M. Fliess, J. Lévine, P. Martin, and P. Rouchon, Flatness and defect of nonlinear systems: introductory theory and examples, *Int. J. Control*, Vol. 61, No. 6, 1995, pp. 1327–1361

[2] B. Lehman and R.M. Bass, Extensions of Averaging Theory for Power Electronic Systems, *IEEE Transactions on Power Electronics*, Vol. 11, No. 4, 1996, pp. 542–553.

[3] J. Linares-Flores and H. Sira-Ramírez, DC Motor Velocity Control through a DC-to-DC Power Converter, *43rd IEEE Conference on Decision and Control*, Vol. 5, 2004, pp. 5297–5302.

[4] G. Luckjiff and Ian Dobson, Hexagonal $\Sigma-\Delta$ Modulators in Power Electronics, *IEEE Transactions on Power Electronics*, Vol. 20, No. 5, 2005, pp. 1075–1083.

[5] V. Hagenmeyer, P. Kohlrausch, and E. Delaleau, Flatness-based control of the separately excited DC drive, In: A. Isidori, F. Lamnabhi-Lagarrigue, and W. Respondek (Eds.), *Nonlinear Control in the Year 2000*, Lecture Notes in Control and Information Sciences, Vol. 258, 2000, pp. 439–451.

[6] A. Kugi and K. Schlacher, Nonlinear H-infinity Controller Design for a DC-to-DC Power Converter, *IEEE Transactions on Control Systems Technology*, Vol. 7, No. 2, 1999, pp. 230–237.

[7] M. H. Rashid, *Power Electronics — Circuits, Devices, and Applications*, Pearson, Prentice Hall, 2004, London.

[8] H. Sira-Ramírez and S. K. Agrawal, *Differentially Flat Systems*, Marcel Dekker, New York, 2004.

[9] H. Sira Ramírez and R. Silva-Ortigoza, Sliding Mode $\Sigma - \Delta$ Modulation Control of the Boost Converter, *Asian Journal of Control*, Vol. 7, No. 4, 2005, pp. 349–355.

[10] H. Sira Ramírez, Are nonlinear controllers really necessary in power electronics devices?, *11th European Power Electronics Conference (EPE-2005)*, Dresden, Germany, 2005.

[11] M. Reinoza-Angulo, M. Spinetti-Rivera, and H. Sira-Ramírez, *Comparative analysis of different mathematical models for the boost converter (extended semi-conductor models)*, Spanish, Congress on Automation and Control (CAC-2005), Univ. Simón Bolívar, Caracas, Venezuela, November 2005.

[12] M. Vidyasagar, *Nonlinear Systems Analysis*, 2^{nd} ed., SIAM, 2002.