Observer Designs for a Turbocharger System of a Diesel Engine

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Abstract: We consider observer designs for an exhaust-gas turbocharger based on a third order nonlinear model of the air path of a diesel engine. To propose an observer, the system is analyzed in view of different observability properties for nonlinear systems. We derive explicit constraints for the system state guaranteeing observability. Based on that we design a high-gain observer and a sliding mode observer for the system. The performance of both designs has been assessed through a simulation study.

Keywords: Nonlinear Systems, Observability Analysis, High-Gain Observer, Sliding-Mode Observer, Turbocharger Diesel Engine

1. INTRODUCTION

Over the last decades, diesel engines have gained interest because they provide higher fuel efficiency than gasoline engines. However, diesel engines have the disadvantage to require a complicated exhaust gas treatment. One way to achieve a better power utilization and an improved treatment of exhaust gases is the use of turbochargers. Fig. 1 shows the structure of such turbocharger system.

![Fig. 1. Schematic diagram of a turbocharged diesel engine with EGR (Herceg et al., 2006).](image)

The function of the turbocharger is to increase the power density of the engine by pushing additional fresh air into the cylinders of the engine in order to burn more fuel without smoke generation, allowing for the injection of more fuel without reaching the smoke bound. The turbine has a variable geometry (VGT) so as to adapt the turbine efficiency based on the operating point of the engine, driven by the energy in the exhaust gas.

The second feedback path to the intake manifold from the exhaust manifold is due to exhaust gas recirculation which is controlled by an exhaust gas recirculation (EGR) valve. The oxygen in the intake is replaced by recirculated exhaust gases, thereby reducing the temperature profile of the combustion and hence the emissions of nitrogen oxides. The interactions are relatively complex. The VGT actuator is usually used to control the intake manifold absolute pressure (MAP) while the EGR valve controls the mass air flow (MAF) into the cylinders. Both the EGR and VGT paths are driven by the exhaust gases. However, both the EGR and VGT actuator are equipped with position sensors. For a detailed description refer to Ladommatos et al. (1996) and references therein.

Vast research has been dedicated to the control of diesel engines. Many controllers were proposed in the literature, e.g., Lyapunov controller design (Janković and Kolmanovsky, 2000), controllers based on a Linear Parameter-Varying (LPV) model for turbocharged diesel engine (Jung and Glover, 2003), indirect passivation (Larsen et al., 2000), predictive control (Ferreau et al., 2007), feedback linearization (Dabo et al., 2009) and sliding mode control (Utkin et al., 2000). Other approaches for air-to-fuel ratio control of turbocharged diesel engines were also proposed in the literature (Fredriksson, 1999; Guzzella and Onder, 2013; Janković et al., 1998). The different solutions are mostly nonlinear and model-based. Usually it is assumed that the state variables be measurable or somehow computable.

However, full state information is not the case for many systems in practice, as was explained in Fredriksson and Egardt (2002). The intake manifold pressure or boost pressure is measurable. The compressor power may be
computed in principle. But the sensors used for computing the compressor power are quite expensive, and, therefore, it is often assumed to be unmeasurable. So as to relax this assumption the use of observers for estimating the unmeasurable states may be considered a solution; first designs have been proposed only in the recent past (Glenn et al., 2011; Salehi et al., 2013; Qu et al., 2015).

The focus of this contribution is on estimating the states of this turbocharger model. To this end, the model is analyzed in terms of observability first. Within the observability analysis we shall consider two concepts of observability for nonlinear systems, namely, weak observability and uniform observability. Local weak observability is an indispensable condition for the design of observers, see Hermann and Krener (1977). The uniform observability, as introduced by Gauthier and Bornard (1981), is a stronger concept and allows for the stable design of High-Gain observers. Afterwards, we investigate the application of two different observer approaches on the turbocharger system: a high-gain observer and a sliding-mode observer. The performances of both are compared in terms of design, complexity and robustness.

The paper is structured as follows: The following section recalls the nonlinear model of the turbocharger system, based on a simplification of a more complete physical model. Section 3 presents the observability analysis of the nonlinear system using the observability rank condition analytically and more extensive numerical calculations. In Sections 4 and 5 we present the high-gain observer and sliding-mode observer design, respectively. Both observers are simulated and results are presented in Section 6. Conclusions are drawn in Section 7.

2. MODELING

The modeling of the air path system of a diesel engine, which is composed of a turbocharger with variable turbine geometry, exhaust gas recirculation and intercooler, is not an easy task and consists largely of identification and map creation, as Wahlström and Eriksson (2011) had presented in detail. Moreover, complicated functional relationships arise, for example, by attaching flow functions.

Through versatile measurements describing the couplings, a comprehensive information flow exists for the system which then can be used for observation. However, this results in a very complex mathematical model to be mastered. Fortunately, simplifications and approximations have been made.

Against this background, here we use a fully parameterized nonlinear model of the turbocharger which is based on the approach from Janković and Kolmanovsky (2000). This is a simplified third-order nonlinear model derived from an eighth-order nonlinear mean-value model of the air path of a turbocharged diesel engine with EGR and VGT. The full-order air path model is of eighth order with states: intake and exhaust manifold pressure (\(p_i\) and \(p_e\)), oxygen mass fractions in the intake and exhaust manifolds, turbocharger speed and the two states describing the actuator dynamics for the two control signals.

For more simplicity the model is reduced to a third-order model subject to the following assumptions: The oxygen mass fraction variables are ignored since they are not coupled with other engine dynamics and are difficult to measure. The temperature variables are omitted because temperature dynamics are slow when compared with pressure and flow dynamics.

The detailed derivation of the eighth-order nonlinear model of the engine under investigation may be found in Jung et al. (2002). In Table 2 all variables and parameters are listed. Reducing the consideration to the third-order model, we may refer to just three states \(p_i\), \(p_e\), and \(P_t\) that represent the intake manifold pressure, the exhaust manifold pressure, the power transferred by the turbocharger, respectively. Then the following system description is obtained:

\[
\begin{align*}
\Sigma: & \quad \begin{cases}
\dot{p}_i = \frac{RT_i}{V_i} (W_{ci} + W_{xi} - W_e) \\
\dot{p}_x = \frac{RT_x}{V_x} (W_{xe} + W_f - W_{xi} - W_{xt}) \\
\dot{P}_c = \frac{1}{\tau} (-P_c + P_t)
\end{cases}
\end{align*}
\]

Following the analysis of the turbine mass-flow presented by Schollmeyer (2010), the flow rates as well as the compressor and turbine power are given by

\[
W_{ci} = \frac{\eta_{ci}}{c_p T_a} \left( \frac{p_c}{p_a} \right)^{\gamma - 1},
\]

\[
W_{xi} = \frac{A_{egr} (x_{egr}) p_{x}}{2 \sqrt{RT_x}} \left( \frac{2p_x}{p_i (1 - \frac{p_e}{p_x})} \right),
\]

\[
W_{te} = \frac{\eta_{te} p_t NV_g}{120 RT},
\]

\[
W_{xt} = (ax_{vgt} + b) \left( \frac{c}{p_a} \left( \frac{p_a}{p_x} - 1 \right) + d \right) \frac{p_x}{pv_{ref}}
\]

\[
\times \sqrt{\frac{T_{ef}}{T_x}} \frac{2p_i}{p_x} \frac{1 - \frac{p_x}{p_i}}{\left( \frac{p_a}{p_x} \right)^{\gamma}},
\]

\[
P_t = W_{xt} c_p T_x \eta_{t} \left( 1 - \left( \frac{p_a}{p_x} \right)^{\gamma} \right)
\]

with the constant parameters: compressor and turbine efficiency \(\eta_c\) and \(\eta_t\), volumetric efficiency \(\eta_{v}\), intake manifold temperature \(T_i\), exhaust manifold temperature \(T_x\), and time constant of the turbocharger power transfer \(\tau\).

The control inputs of the diesel engine model based on (1), are the EGR position \(x_{egr}\) and the VGT position \(x_{vgt}\). For simplifying the considerations of the model, however, the effective areas of the valves, i.e. \(A_{egr} = ax_{egr} + b\) and \(A_{vgt}\) are used as control inputs. Furthermore, the control inputs \(A_{egr}\) and \(A_{vgt}\) are constrained due to the minimal and maximal EGR and VGT positions. The intake manifold pressure or boost pressure, is measurable and forms the system output.

It should be noted that the model has a singularity when the pressure in the intake manifolds equals the ambient pressure, that is, if \(p_i = p_a\) then the compressor flow becomes infinite. There exists also another singularity when the turbine stalls. The simplified model can not handle this situation and, as explained in (Fredriksson and Egardt, 2002), the system is only valid on the set \(\Omega = \{ (p_i, p_e, P_c) : p_i > p_a, p_x > p_a, P_c > 0 \}\). Fortunately,
it may be demonstrated that \( \Omega \) is invariant, thus, every trajectory starting in \( \Omega \) stays in \( \Omega \) (Janković et al., 1998).

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N ) Engine Speed</td>
<td>2000</td>
<td>rpm</td>
</tr>
<tr>
<td>( W_f ) Fuel rate</td>
<td>5</td>
<td>kg/h</td>
</tr>
<tr>
<td>( R ) Specific gas constant</td>
<td>287</td>
<td>J/paK</td>
</tr>
<tr>
<td>( p_i ) Intake manifold pressure</td>
<td>-</td>
<td>h/ Pa</td>
</tr>
<tr>
<td>( p_a ) Exhaust manifold pressure</td>
<td>-</td>
<td>h/ Pa</td>
</tr>
<tr>
<td>( T_i ) Reference pressure</td>
<td>101.3</td>
<td>K</td>
</tr>
<tr>
<td>( P_a ) Ambient pressure</td>
<td>101.3</td>
<td>K</td>
</tr>
<tr>
<td>( P_c ) Compressor Power</td>
<td>-</td>
<td>kW</td>
</tr>
<tr>
<td>( T_e ) Exhaust manifold temperature</td>
<td>599</td>
<td>K</td>
</tr>
<tr>
<td>( T_{ref} ) Referent Temperature</td>
<td>298</td>
<td>K</td>
</tr>
<tr>
<td>( T_a ) Ambient Temperature</td>
<td>298</td>
<td>K</td>
</tr>
<tr>
<td>( V_i ) Volume of the intake manifold</td>
<td>0.006</td>
<td>m(^3)</td>
</tr>
<tr>
<td>( V_e ) Volume of the exhaust manifold</td>
<td>0.001</td>
<td>m(^3)</td>
</tr>
<tr>
<td>( V_d ) Displacement Volume</td>
<td>0.002</td>
<td>m(^3)</td>
</tr>
<tr>
<td>( \eta_m ) Turbo mechanical efficiency</td>
<td>98</td>
<td>[%]</td>
</tr>
<tr>
<td>( \eta_c ) Compressor isentropic efficiency</td>
<td>61</td>
<td>[%]</td>
</tr>
<tr>
<td>( \eta_t ) Turbine isentropic efficiency</td>
<td>76</td>
<td>[%]</td>
</tr>
<tr>
<td>( \eta_v ) Volume efficiency</td>
<td>87</td>
<td>[%]</td>
</tr>
<tr>
<td>( c_F ) Specific heat at constant pressure</td>
<td>104.4</td>
<td>[J/paK]</td>
</tr>
<tr>
<td>( c_v ) Specific heat at constant volume</td>
<td>727.4</td>
<td>[J/paK]</td>
</tr>
<tr>
<td>( \mu ) constant</td>
<td>0.286</td>
<td>–</td>
</tr>
<tr>
<td>( a ) Parameter ( a )</td>
<td>0.136</td>
<td>–</td>
</tr>
<tr>
<td>( b ) Parameter ( b )</td>
<td>0.176</td>
<td>–</td>
</tr>
<tr>
<td>( c ) Parameter ( c )</td>
<td>0.4</td>
<td>–</td>
</tr>
<tr>
<td>( d ) Parameter ( d )</td>
<td>0.6</td>
<td>–</td>
</tr>
</tbody>
</table>

For our analysis we may refer to system (1) in terms of system

\[
\Sigma: \begin{cases} 
  \dot{x} = F(x, u), \\
  y = h(x),
\end{cases}
\]

where \( F : \mathbb{R}^3 \times \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) and \( h : \mathbb{R}^3 \rightarrow \mathbb{R} \) differentiable in \( x \) sufficiently often. The function expressions of \( F(x, u) \) correspond to the right side of the system (1), that is \( x = (p_i, p_a, P_c^*) \) and \( h(x) = x_1 \).

### 3. OBSERVABILITY

For nonlinear systems there exist various observability concepts. In this contribution, in particular, we consider weak observability and uniform observability which are significant for the observer approaches to be applied later, following Besançon (2007).

Consider a single-output system of class \( \Sigma \) from (2). In general, the observability map is

\[
o_a(x) := \left( \begin{array}{c}
  h(x) \\
  \mathcal{L}_F^1 h(x) \\
  \vdots \\
  \mathcal{L}_F^{k-1} h(x)
\end{array} \right),
\]

where \( \mathcal{L}_F^k \) describes the \( k \)-th Lie derivative with respect to \( F(\cdot, u) \). Observability may then be characterized by the solvability of the resulting system of nonlinear equations for the state \( x \). In view of the implicit function theorem, the Jacobian of this map may be used for showing local

solvability. The Jacobian of the observability map is called observability matrix, given by

\[
o_a(x) := \left( \begin{array}{ccc}
  \frac{\partial}{\partial x} h(x) \\
  \frac{\partial}{\partial x} \mathcal{L}_F h(x) \\
  \vdots \\
  \frac{\partial}{\partial x} \mathcal{L}_F^{k-1} h(x)
\end{array} \right),
\]

where \( \frac{\partial}{\partial x} \cdot \cdot \cdot \) denotes the gradient.

Here it is important to note that the observability map and its Jacobian will actually also depend on the input. Excluding that these inputs may cause conflicts with observability, naturally, leads to the stronger concept of uniform observability. A system is uniformly observable if the observability map may be solved uniquely for the state, whatever the input. In this case, every input is a so-called universal input.

The analytical expression of the observability matrix for the turbocharger system is rather lengthy, and not easy to analyze. In order to get an impression of observability, the analysis is made first for the free system, i.e., with inputs indetical to zero. In this case the observability matrix is

\[
o_3(x) = \left( \begin{array}{ccc}
  1 & 0 & \beta_1(x_1) \\
  0 & 0 & \beta_2(x_2) \\
  * & * & \beta_3(x_2)
\end{array} \right)
\]

where

\[
\beta_1(x_1) = \frac{RT_i}{V_i} \frac{1}{P_a p_a^\mu},
\]

\[
\beta_2(x_2) = \frac{c_p \eta_c T_e}{p_a} \left( 1 - p_a p_a^\beta \right) b \left( c \left( p_a/p_x - 1 \right) + d \right) \frac{p_x}{p_{ref}},
\]

In order to probe whether the autonomous system is at least locally weakly observable. For showing this, we use the observability rank condition presented by Hermann and Krener (1977), i.e., we check whether rank\(O_3(x) = 3\). Thus, it is only necessary to verify that its diagonal elements are different from zero.

In the region of interest \( \Omega \) we have that \( \beta_1(x_1) \neq 0 \). Fig. 2 shows that the values of the third element of the diagonal will also never be zero. Therefore we conclude that the system is at least locally weakly observable on the set \( \Omega \).

The remaining analysis is concerned with the evaluation of the inputs to determine whether these are universal. The complete model in affine form is given by

\[
F(x, u) = \left( \begin{array}{ccc}
  \ast & \ast & u_1 + \ast \\
  \ast & \ast & \ast \\
  \ast & \ast & \ast
\end{array} \right) \left( \begin{array}{c}
  0 \\
  u_2
\end{array} \right),
\]

in this way it is easy to see that the inputs are not coupled. Then for simplifying the analysis, we analyzed each input independent from each other. That is, analyzing \( u_1 \) we set \( u_2 \equiv 0 \) and vice versa. It turns out that the
The rationale of the high-gain (HG) observers is to find a linear output injection which is able to dominate the nonlinear terms for stabilizing the estimation error dynamics, which is governed by a constant gain matrix. Due to their inherent simple structure, HG observers are frequently used in practical applications. This also makes the analysis, each input turns out an universal input and we may conclude that the turbocharger system is uniformly observable in the operation area of validity Ω.

4. HIGH-GAIN OBSERVER

The respective observability matrices $O_3(x, u_1)$ do not show triangular structure. Therefore, the analytic calculation of its determinant results in a complex task even with the help of computational software. However, for getting an impression we may plot the determinant as a function of states, considering the dynamics of the system and the entire range of input values. Note that the range of operation was obtained fixing two states maximizing the other, Fig. 3 shows that the determinant of $O_3(x, u_1)$ is never zero for the relevant range of states, analogously Fig. 4 shows the same result for the second input.

Therefrom we may assume that there is no relevant states in which the determinant becomes zero. In view of this analysis, each input turns out an universal input and we may conclude that the turbocharger system is uniformly observable in the operation area of validity Ω.

Fig. 2. Values of $\frac{\partial^2 \nu(x)}{\partial x_2}$ in the region of interest

In this equation, function $\varphi$ describes the nonlinearities of the system. Furthermore, we will take benefit from

$$\dot{x} = \left( \frac{\partial O_3(x)}{\partial x} \right)^{-1} \dot{z}$$

which is very useful for the observers design in original coordinates. This way, only the observability matrix $O_3(x)$ needs to be inverted which is much simpler than determining the complete inverse transformation of a nonlinear mapping.

In view of the global observability of the system in ONF (4) we design the observer

$$\dot{\hat{z}} = A \hat{z} + b \varphi(\hat{z}, u, \hat{u}) - l(\epsilon)(\hat{y} - y)$$

with

$$l(\epsilon) = \begin{pmatrix} \epsilon^{-1} & 0 & 0 \\ 0 & \epsilon^{-2} & 0 \\ 0 & 0 & \epsilon^{-3} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \Lambda(\epsilon)k$$

for some $\epsilon > 0$ and $k_i > 0$ such that they correspond to the coefficients of a Hurwitz polynomial. Choosing $\epsilon$
sufficiently small, the observers will have a stable state estimation error dynamics. It is clear that for small $\epsilon$ the diagonal entries of $\Lambda(\epsilon)I$ will be large, thus the name high-gain observer.

In original coordinates this state observer takes the form

$$\begin{align*}
\dot{x} &= F(\hat{x}, u) - \left( \frac{\partial \theta_3(\hat{x})}{\partial \hat{x}} \right)^{-1} l(\epsilon)(\hat{y} - y) \\
\dot{y} &= \hat{x}_1
\end{align*}$$

which shows just the necessity to invert the Jacobian.

5. SLIDING MODE OBSERVER

Continuous observers usually converge to the origin only in the absence of disturbances. Sliding-mode (SM) observers have been designed to resolve this issue by using discontinuous or non-Lipschitz continuous injection terms. SM observers, generally speaking, are known to show a remarkable robustness against disturbances and may further show a finite convergence time, at least theoretically. Different designs for SM observers were proposed, as for example, the generalized super-twisting observer (GSTO) and homogeneous SM observers Moreno (2013). Those approaches may guarantee different properties, but are well-studied only for second order systems. The SM observer that we adopt here for the third order system (1) was first proposed in (Moreno and Dochain, 2013). For a system in ONF this observer takes the form

$$\begin{align*}
\dot{z}_1 &= -L k_1 |\hat{y} - y|^{\frac{5}{2}} \text{sign}(\hat{y} - y) + \dot{z}_2 \\
\dot{z}_2 &= -L^2 k_2 |\hat{y} - y|^{\frac{3}{2}} \text{sign}(\hat{y} - y) + \dot{z}_3 \\
\dot{z}_3 &= -L^3 k_3 \text{sign}(\hat{y} - y) + \phi(\hat{z}, u, \hat{u}) \\
\hat{y} &= \hat{z}_1
\end{align*}$$

The gains $k_1$, $k_2$ and $k_3$ again are all positive, selected in order to guarantee the convergence of the observer in absence of external perturbations. The choice of gains may follow the condition $k_1k_2 > k_3$. It may be motivated by imposing a Hurwitz polynomial for a linear error dynamics. Thus, one is left with choosing some appropriate $L > 0$.

In the original coordinates, the SM observer for system (1) again only requires the inversion of $\partial_3(x)$. In order to have a representation similar to the HG observer let $L = \frac{1}{\epsilon}$. Then the observer in original coordinates reads

$$\begin{align*}
\dot{x} &= F(\hat{x}, u) - \left( \frac{\partial \theta_3(\hat{x})}{\partial \hat{x}} \right)^{-1} l(\epsilon) \left( |\hat{y} - y|^{\frac{5}{2}} \text{sign}(\hat{y} - y) \right) \\
\dot{y} &= \hat{x}_1
\end{align*}$$

Stability and convergence of the associated state estimation error dynamics was shown by Moreno and Dochain (2013).

6. RESULTS

The performance of the observers designed for the turbocharger system was studied by simulations performed in MATLAB. The three states of the turbocharger model shall be estimated from the input manifold pressure $p_i$ as the output and the inputs. The values of the normalized inputs for the scenario of simulation were the EGR position $x_{egr} = 0.7$ and the VGT position $x_{vgt} = 0.6$.

The state estimation errors obtained with the HG observer, using different $\epsilon$ values are shown in Fig. 5. Furthermore, the values of constants design $k_1$, $k_2$ and $k_3$ were set considering three poles placed at -10. It can be seen that the one with smallest $\epsilon$ yields a more pronounced peaking, as expected. Besides the peaking, as known from the literature, for smaller $\epsilon$ we also expect

- faster convergence of the observation error,
- amplification of the measurement noise,
- better attenuation of process disturbances.

These trade-offs always have to be taken into account in the design procedure.

The state estimation errors using the SM observer with different values of $L = \frac{1}{\epsilon}$ where $L_1 < L_2 < L_3$ are shown in Fig. 6. For simulating comparable dynamics, the remaining
Fig. 5. Estimation errors using the high-gain observer

design values have been kept as in the same. No particular parameter tuning was made.

It can be seen that the one with largest $L$ yields a faster convergence of the observation error. Furthermore, this method improves the HG observer since, remarkably, the peaking phenomenon turns out not present in the results. The performance of the designed observers has been assessed by imposing measurement noise. More precisely, in the simulations the measured output was contaminated by additive normally distributed noise with zero mean and standard deviation $3.4832 \times 10^4$. The transient performance of the estimation errors is shown in Figures 7–9. The convergence velocity of the observation error increases while the gain of the nonlinear part increases. There exists a trade-off for the HG observer in the selection of this gain. That is, for high gains the perturbation is dominated better, but to the price that the measurement noise is amplified and the peaking phenomenon is more pronounced.

The observability analysis for a model of turbocharger in a diesel engine has been performed using weak and uniform observability concepts. Operating the system in the invariant set of the usual operation range we have shown that the system is uniformly observable. Based on this, we have designed a high-gain observer and a sliding-mode observer so as to obtain estimates for the turbocharger states. Imposing a similar estimation error dynamics, simulations with experimental testbed-data show that both observers are able to reconstruct suitably the desired states. How-

On the other hand, the Figures 8–9 show a slower convergence of the error for the Sliding Modes observer, however, in this case, the measurement noise is not present in the results.

For the two observer approaches taken in consideration the estimated intake and exhaust manifold pressure as well as the compressor power are very close to the state variables of the real system.

7. CONCLUSION
ever, simulations also show that the sliding-mode observer shows no peaking and is less prone to noise.

Future work will concentrate on the design of output-feedback controllers for the turbocharger system, based on the presented observer studies.

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