AN EXAMPLE OF FLATNESS BASED FAULT TOLERANT CONTROL USING ALGEBRAIC DERIVATIVE ESTIMATION

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Abstract: A new method of fault tolerant control is proposed, which is based on the flatness of the system. An algebraic derivative estimation method, which is robust to noise, is used to estimate actuator faults. This fast estimation permits an accomodation of the control to the fault. Taking into account control saturations, an online adaptation of the reference trajectory is investigated.

Keywords: Fault tolerant control, derivative estimation, control saturation, actuator fault accomodation, trajectory replanning.

1. INTRODUCTION

The interest of model based fault diagnosis has increased more and more in the last years (see for example (Chen, 1999; De Persis and Isidori, 2001; Gertler, 1998; Frank, 1990; Staroswiecki and Comtet-Varga, 2001)). This paper is the first of a series of papers in which we focus on the problem known as *Fundamental Problem of Residual Generation* (FPRG), firstly introduced by (Jones, 1973; Willsky, 1976), which aims at fault detection and isolation in the case of multiple faults. The methods developed to tackle this problem have to be distinguished into linear and nonlinear methods.
Considering methods based on linear system models, an abundant literature is available, while we would like to highlight the approaches developed in (White and Speyer, 1987; Massoumnia, 1989). Alternatively, a recent approach based on algebraic tools (Fliess *et al.*, 2004) deals with the residual generation without the need to synthesize any filters and/or observers.
Diagnosis in the context of nonlinear systems is a subject of fundamental interest, since nonlinear methods are able to extend the domain of FDI applications - see for example (Chen, 1999; De Persis and Isidori, 2001; Gertler, 1998; Join, 2002; Join *et al.*, 2003).
In ((Fliess *et al.*, 2005)), encouraging results in the field of fault tolerant control were obtained applying innovative methods. Usually, in order to minimize the effect of a fault on the control activity, the control law is modified by taking into account the result of the fault diagnosis procedure. This might lead to an additive control action, see for instance ((Lunze *et al.*, 2001; Theilliol *et al.*, 2002)). However, the saturation limits of the control signal, that are always present in practice, may require to change the reference trajectory in order to keep up a sucessful fault accomodation scheme (Tarbouriech and Garcia, 1997; Kapila and Grigoriadis, 2002). This particular aspect is one of the main contributions of this work.
In this paper, our ideas are illustrated on a simple

first order linar system. The subsequent extension of these concepts to nonlinear MIMO systems does not represent any difficulty, and respective works will are about to be published by the authors.
This paper is organized as follows. Firstly flatness based control and algebraic derivative estimation are shortly recalled. Afterwards, the system model and its control is explained in sections 4 and 5. The paper's main contributions are given in sections 6 and 7, where fault accommodation and trajectory replanning is proposed. Section is devoted to describe some forthcoming works.

2. FLATNESS

We recall basic notions from Fliess et al. (Fliess *et al.*, 1995). According to these lines, an n-th order single input single output system

$$\dot{x} = f(x, u) \tag{1}$$

with state $x \in \mathbb{R}^n$ and input $u \in \mathbb{R}$ is (differentially) flat if there exists a scalar, probably ficticious output

$$y = \phi(x)\,, \tag{2}$$

called flat output, that allows to differentially parameterize the state and the input as per

$$x = \psi_x(y, \dot{y}, \ldots, y^{(n-1)})\,, \tag{3}$$

$$u = \psi_u(y, \dot{y}, \ldots, y^{(n)})\,. \tag{4}$$

An important consequence, of the parametrization given in (4) is that having chosen a nominal desired reference trajectory $y^\star$ for the flat output determines the necessary nominal control $u^\star$, that is

$$u^\star = \psi_u(y^\star, \dot{y}^\star, \ldots, y^{\star(n)}) \tag{5}$$

which is a feature we are going to exploit in the subsequent paragraphs.

3. ALGEBRAIC DERIVATIVE ESTIMATION

Consider a real-valued polynomial function in time

$$y_N(t) = \sum_{j=0}^{N} \frac{y_N^{(j)}(t_\mathrm{i})}{j!}\,(t - t_\mathrm{i})^j \tag{6}$$

of degree N, $t \geq t_\mathrm{i}$, which may be considered as an N-th order approximation of a time signal $y(t)$. We may derive $y_N(t)$ at least $(N+1)$-times with respect to time to obtain zero. Using usual operational calculus notation, this may be rephrased in the operator domain, which reads

$$s^{N+1} Y_N(s) = \sum_{i=0}^{N} s^{N-i} y_N^{(i)}(t_\mathrm{i}) \tag{7}$$

and involves standard rules from operational calculus, only. Differentiating both sides α times with respect to the operator s and premultiplying with $1/s^\nu$, $\nu \geq 0$, yields

$$\frac{1}{s^\nu}\frac{d^\alpha}{ds^\alpha}\left(s^{N+1} y_N(s)\right) = \frac{1}{s^\nu}\frac{d^\alpha}{ds^\alpha}\sum_{i=0}^{N} s^{N-i} y_N^{(i)}(t_\mathrm{i}) \tag{8}$$

The choice of $\alpha = 0, \ldots, N$ generates a triangular system of linear equations for determining the values of the derivatives. Hence, these quantities are *linearly identifiable*. So as to remove all time derivatives wrt. $y_N(t)$ we need to choose $\nu \geq N+1$.

For simplicity, we may refrain from exposing further steps of resolution this system of linear equations. By setting $N = 1$ and $\nu = 3$ we obtain the following formulae for $\dot{y}_N(t_\mathrm{i})$

$$\dot{y}_1(t_\mathrm{i}) = -\frac{6}{T_\mathrm{win}^3}\int_0^{T_\mathrm{win}} (2\sigma - T_\mathrm{win})\, y(t_\mathrm{i} - \sigma)\,\mathrm{d}\sigma \tag{9}$$

where T_win is an arbitrary window time. It turns out in the sequel, that we will not have to estimate higher order derivatives that the first one. Of course other choices of N and ν are possible. Thanks to the noise attenuating effect of the integration in eq.(9), this expression can be quite effectively used to get a quasi-instantenous estimation of the first derivative of a noise signal $y(t)$, when T_win is chosen small enough, so that the linear approximation of $y(t)$ is valid. Also, the numeric implementation of eq.(9), in order to get a valid estimate at any instant t, proves no difficulty when realizing it via a slinding window approach. In this case, T_win has to be chosen by compromising between two influences: the larger T_win is chosen, the better the noise attenuation will be, whereas the validity of the polynomial approximation of $y(t)$ depends on the size of N, and limits T_win to some upper bound. Applications and details are given in (Fliess *et al.*, 2004; Fliess *et al.*, 2005a).

4. MODEL PRESENTATION

In this paper, our ideas are presented on an first order LTI SISO system, whose transfer function is given by

$$G(s) = \frac{1}{1 - Ts}, \tag{10}$$

where $T > 0$ is the known system time constant. Due to the system pole at $s = \frac{1}{T}$, it is unstable. The corresponding differential equation with input $u(t)$ and ouput $y(t)$ is given by

$$\dot{y}(t) = \frac{1}{T}[y(t) - u(t)]. \tag{11}$$

In order to include a command saturation function and an additive actuator fault $f_a(t)$, which enters after the saturation block, into the model, we introduce the so called 'free control signal' $u'(t)$, by which we mean the desired control output before the saturation block. Thus,

$$u(t) = sat_S\Big(u'(t)\Big) + f_a(t), \tag{12}$$

where $sat_S(x)$ denotes the saturation function parametrized by the saturation limit S,

$$sat_S(x) = \begin{cases} S & \text{for} \quad x \geq S \\ x & \text{for} \ -S < x < S \\ -S & \text{for} \quad x \leq -S \end{cases} \tag{13}$$

We consider that the actuator faults only occur in a steplike form, thus

$$f_a(t) = F_a \sigma(t - T_a), \tag{14}$$

where $\sigma(t)$ denotes the unit step, T_a corresponds to the instant where the fault occurs and F_a defines its amplitude. By limiting our analysis to steplike faults, the eventual adaptation of the reference trajectory can be based on the estimated amplitude $\hat{F}_a$ of $f_a(t)$, which then represents a constant 'offset' of $u(t)$ for $t > T_a$. To give a realistic setting of our ideas, we added equally distributed white measurement noise in all simulations, denoted by $n(t)$, onto the output signal. Also, an unkown initial condition $x(t=0) = x_0$ should be admitted. The system's state space description is then

$$\begin{aligned} \dot{x}(t) &= \tfrac{1}{T}[x(t) - u(t)],\ x(0) = x_0 \\ y(t) &= x(t) + n(t). \end{aligned} \tag{15}$$

5. FLATNESS BASED CONTROL AND OFFLINE TRAJECTORY PLANNING

For the theoretical analysis of this chapter, which deals with the flatness based control and offline trajectory planning problem, we neglect the effect of noise, which is then included in the subsequent simulations. In the first subsection, the flatness based offline trajectory planning of the system, subject to the saturation limitations, is investigated, whereas the second subsection deals with the flatness based specification of the error dynamics.

5.1 Flatness based offline trajectory planning

The instability of the system makes a feedback control necessary. Since the cutting of $u'(t)$ by the saturation function would turn the system nonlinear, which is clearly undesired because it leads to instablility problems, we presume that the nominal control signal, denoted by $u^\star(t)$, should remain within the interval $[-S+\epsilon, S-\epsilon]$, when the trajectory is planned offline. Here ϵ denotes a security margin that we prefer our free control command $u'(t)$ and the reference command $u^\star(t)$ to keep away from the saturation limits and which accounts for measurement noise, unkown initial conditions, and robustness to small actuator faults. Thanks to the flatness property of the system, which exhibits the flat output y , the trajectory planning is straightforward. As x_0 is unknown, but will necessarily lead to a different control activity $u'(t) \neq u^\star(t)$, whenever $x_0 \neq 0$, it is a good policy to calculate, right after the system has been switched on and x_0 has been measured, whether the saturation will be hit later on, suggesting a replanning of trajectory. This is covered in section 7.

Let $y^\star(t)$ be the reference trajectory that we would like to track. In our case, we would like to drive the system from the origin to a stationary final regime $y_f > 0$, starting at time t_0 and arriving at t_f. An appropriate reference trajectory is given by a first order Bezier polynomial, which is defined by

$$y^\star(t) = y_f \left(3\left(\frac{t-t_0}{\Delta T}\right)^2 - 2\left(\frac{t-t_0}{\Delta T}\right)^3\right), \tag{16}$$

for $t_0 \leq t \leq t_f$, $y^\star(t) = 0$ for $t < t_0$ and $y^\star(t) = y_f$ for $t > t_f$. The transfer time ΔT is given by $\Delta T = t_f - t_0$. The nominal control signal, $u^\star(t)$, can be directly found from the differential parametrization of the system, and is equal to

$$u^\star(t) = y^\star(t) - T\dot{y}^\star(t). \tag{17}$$

In order to comply with the saturation limitation, the maximum and minimum values of $u^\star(t)$, denoted by $u^\star_{\max}$ and $u^\star_{\min}$ are of interest. Moreover, the corresponding time instants, $t_{\max}$ and $t_{\min}$, are important. Short calculations using Cardano's formula show that $u^\star_{\max} = y_f$ and $t_{max} = t_f$, while $t_{\min}$ is given by

$$t_{\min} = \frac{\Delta T}{2} + t_0 + 1 - \frac{\sqrt{\Delta T^2 + 4}}{2} \tag{18}$$

and

$$u_{\min} = y_f \frac{8 + \Delta T^3 - 4\sqrt{\Delta T^2+4} - \Delta T^2\sqrt{\Delta T^2+4}}{2\Delta T^3}. \tag{19}$$

Since $u_{\min} \to 0$ for $\Delta T \to \infty$, expression (19) can be used to determine the minimun value for ΔT, denoted by $\Delta T_{\min}$, such that $u^\star(t) \in [-S+\epsilon, S-\epsilon]$, once y_f is fixed.

5.2 Specifying the error dynamics

Let $e(t) = y(t) - y^\star(t)$ be the tracking error. If we define the control law to be

$$u'(t) = u^\star(t) + K_P \cdot e + K_I \int_0^t e(\sigma)d\sigma \tag{20}$$

and assume $u'(t) = [-S, S]$ is valid, the error dynamics under the occurence of a steplike actuator fault is given by

$$\dot{e}(t)+\frac{K_P-1}{T}e(t)+\frac{K_I}{T}\int_0^t e(\sigma)d\sigma = -\frac{F_a}{T}\cdot\sigma(t-T_a), \tag{21}$$

which gives

$$\ddot{e}(t)+\frac{K_P-1}{T}\dot{e}(t)+\frac{K_I}{T}\cdot e(t) = -\frac{F_a}{T}\cdot\delta(t-T_a) \tag{22}$$

after differentiation. For a choice of $K_P > 1$, $K_I > 0$, the solution of eq. (22) will converge to zero asymptotically. Thus the control law eq. (20) ensures asymptotically stable tracking, even in the case of unkown steplike actuator faults, which is of course a known fact. Furthermore, the solution of eq.(22) in the fault free case is important for the later retracking issue. If we choose $\left(\frac{K_P-1}{2T}\right)^2 = \frac{K_I}{T}$, it is given by

$$e(t) = \left\{e(T_0) + \left(\dot{e}(T_0) + \frac{K_P-1}{2T}e(T_0)\right)(t-T_0)\right\} \cdot \exp\left(-\frac{K_P-1}{2T}(t-T_0)\right), \tag{23}$$

where $T_0 > 0$ is any instant at which the first initial condition $e(T_0) = y(T_0) - y^\star(T_0)$ can be measured, whereas the second initial condition $\dot{e}(T_0) = \dot{y}(T_0) - \dot{y}^\star(T_0)$ has to be computed with the help of the derivative estimation technique explained in section 3, since $y(t)$ is a noisy measurement signal from which derivatives cannot be directly obtained. The utility of the derivative estimation method is crucial in this respect. Then, still assuming $u'(t) = [-S, S]$, $u'(t)$ can be calculated using the control law (20), which then permits to verify whether the assumption $u'(t) = [-S, S]$ was valid or whether the saturation will be hit later under the current system conditions.

The control sheme which was just presented is already robust to small steplike actuator faults. In the next section, we present a fault estimation and control accomodation sheme by which the effect of the fault can be ruled out much faster, as the subsequent simulations will show.

6. FAULT IDENTIFICATION AND ACCOMODATION

In this chapter, we would like to present how the derivative estimation sheme can be used to identify the actuator fault. This estimation can then be used to accomodate the control law to the fault. In fact, from eq.(11) and (12), an estimate $\hat{f}_a(t)$ of $f_a(t)$ is clearly given by:

$$\hat{f}_a(t) = y(t) - sat_S\Big(u'(t)\Big) - T\hat{\dot{y}}(t), \tag{24}$$

where $\hat{\dot{y}}(t)$ is the estimate of $\dot{y}(t)$. It is now logical to modify the control law (20), accomodating it to the actuator fault:

$$u'_{\text{acc}}(t) = u^\star(t) + K_P\cdot e + K_I\int_0^t e(\sigma)d\sigma - \hat{f}_a(t). \tag{25}$$

By this policy, the effect of the actuator fault is compensated very fast and in a very direct way. A comparison of tracking by control law (20) and (25) is given in fig. 1. It shows how fast the output regains track when the control law is actively accomodated to the fault. The efficiency of the fault estimation is schown in fig.2, where the actuator fault versus its estimation is plotted. The estimation $\hat{f}_a(t)$ of eq.(24) was smoothened by a first order low pass filter with time constant equal to 0.1 - note the smoothened flank of the fault estimation. The saturation limit was ignored, for it will be delt with in the next section. Also, the initial condition was set to zero. Measurement noise was chosen to be equally distributed between -0.01 and 0.01. System time constant T was set to be equal to 1 sec. Figure 3 shows the reference control command $u^\star(t)$ together with the actual control signals, when the accomodation is not applied ($u'(t)$) or is applied ($u'_{\text{acc}}(t)$). The offset to $u^\star(t)$ of the actual control signals clearly corresponds to the amplitude of $f_a(t)$. As it can be seen, the accomodating strategy leads to a control signal that actually needs less negative amplitude in order to control the system, which is due to the integrator in the control law and the fact that by the accomodation sheme the tracking error is driven to zero much faster.

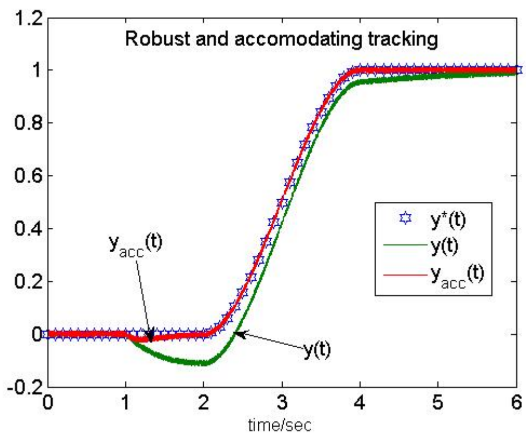

Fig. 1. Comparison of nominal output trajectory $y^\star(t)$, output signal when no accomodation is applied ($y(t)$), and accomodated output $y_{acc}(t)$.

7. DYNAMIC TRAJECTORY REPLANNING

We now want to introduce the idea of a dynamic replanning of trajectories. This might become desirable whenever the control signal is limited by the saturation block or whenever we can predict that it will be limited by the saturation block in

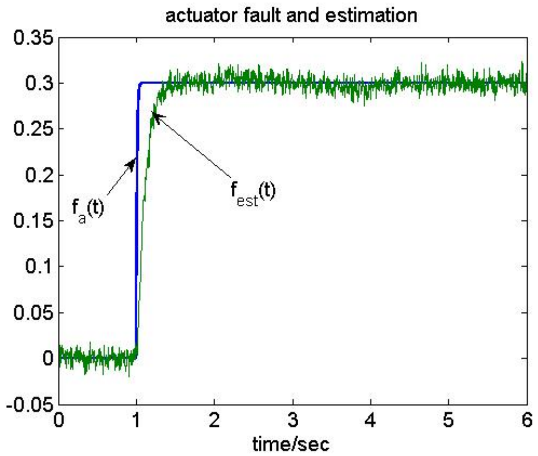

Fig. 2. Actuator fault and its estimation

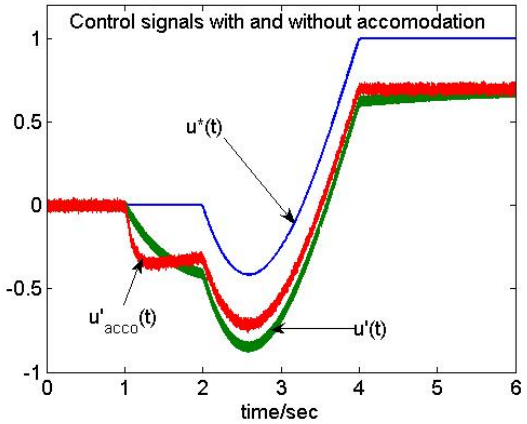

Fig. 3. Comparison of nominal control signal $u^{\star}(t)$, control signal when no accomodation is applied ($u'(t)$), and accomodated control signal $u'_{\text{acc}}(t)$.

the future. Thanks to the flatness of our system, the evolution of control signals can be calculated at any moment, under the assumption that the saturation is not hit. It is important to see that the presence of a constant actuator fault F_a, that we would like to accomodate the control law by eq. (25) to, is equivalent to a shift of of the non-accomodating control signal by by $-F_a$. In fact, it is sufficient always to consider control law eq.(25), since we have no knowledge of the presence of a fault. Only, in the fault free case, $\hat{F}_a$ will be very little. Let T_{est} be the duration that is needed to get a good fault estimation signal and T_{diff} be the duration to get a good estimation of the first derivative with the derivative estimation method. We propose the following strategy, which makes the assumption of being fault-free in the time interval $t \in [0, T_{diff}]$.

(1) **Offline**. Choose $y^{\star}(t)$ such that $u^{\star}(t) \in [-S+\epsilon, S-\epsilon]$.
(2) $\mathbf{t = T_{diff}}$. Now that a good estimate for $\dot{y}$ is available, calculate $u'(t)$ with equations (22) and (20). Check whether $u'(t) \in [-S+\epsilon, S-\epsilon]$. If not, modify the trajectory.
(3) **Fault occurence**, $t = T_a$. If a fault occurs and $\hat{F}_a$ has been properly estimated after the duration of T_{est}, check numerically whether the accomodating control signal u_{acc}, which now has be calculated with eq. (23), where $t_0 = T_a + T_{est}$ and eq.(25), lies within $[-S+\epsilon, S-\epsilon]$. If this is not the case, modify the trajectory.

Principally, the trajectory can be modified in two ways. Firstly, the arrival time t_f can be enlarged, which will require a smaller negative amplitude of the control signal, and should be applied when $u'(t) < -S$ or $u_{\text{acc}} < -S$ for some $t > T_a + T_{\text{est}}$. Secondly, the stationary regime y_f can be reduced. The latter means leads to a smaller maximum of the control signal, which is equal to y_f in the stationary case. This adaptation is adequate, whenever $u'(t) < -S$ or $u_{\text{acc}}(t) < -S$ for some $t > T_{fa}$. Of course, it might as well arise the case that a fault is so huge that instability cannot be prevented. For example, if $|F_a| > S$, then it won't be possible to attain any steady state, since the saturation of $u'(t)$ leads to $|u(t) - F_a| > 0$.

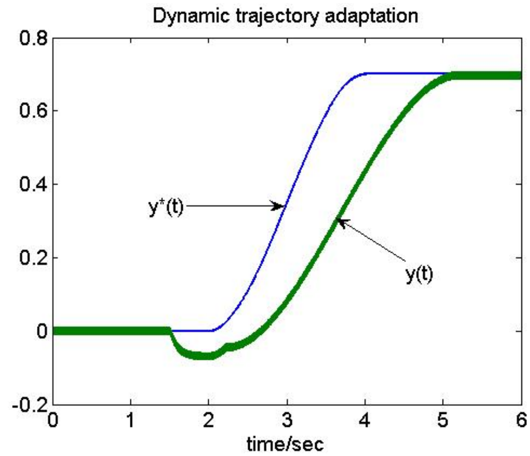

Fig. 4. Output signal y(t) after dynamic trajectory adaption to avoid saturation of control signal.

In figure 4, the extention of the arrival time is demonstrated. This was done in order to accomodate the control law to an actuator fault $f_a(t) = 0.7\sigma(t-1.5\,\text{sec})$, which would have let to a saturation ($S = 1$) hit without accomodation. The effect of postponing the arrival onto the nominal control signal is shown in figure 5, where it can be seen how the minimum of the nominal control signal grows through adaptation. Measurement noise was set to a maximum amplitude of 0.01. In figure 6, the stationary regime of the reference trajectory was changed from 0.7 to 0.5 because the actuator fault $f_a(t) = -0.2\sigma(t-2)$ was injected. In this case, the control signal would have crossed the security margin of 0.1 to the upper saturation

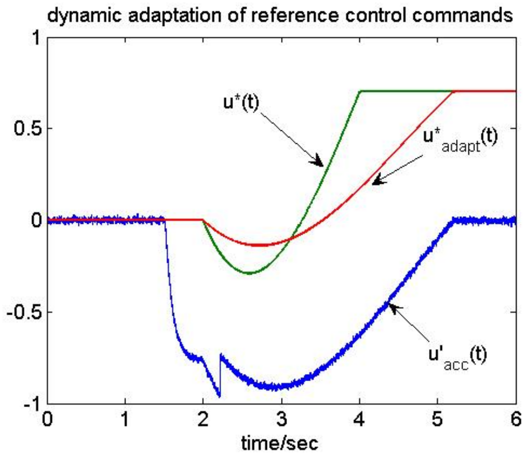

Fig. 5. $u^\star(t)$: original nominal control signal; $u^\star_{\text{adapt}}(t)$: nominal control signal after dynamic trajectory adaptation; $u'_{\text{acc}}(t)$: fault accomodating control signal.

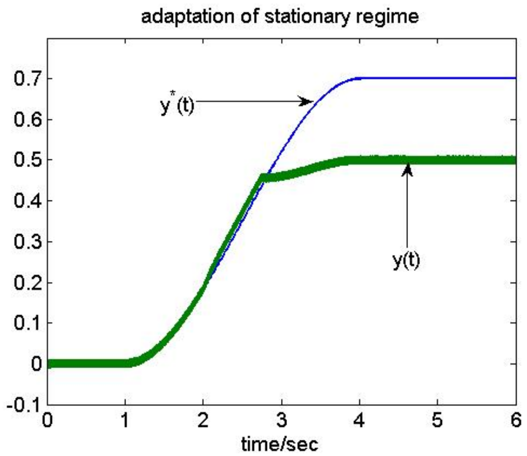

Fig. 6. Original reference trajectory $y^\star(t)$ and output signal y(t), which was tracked to a lower stationary regime.

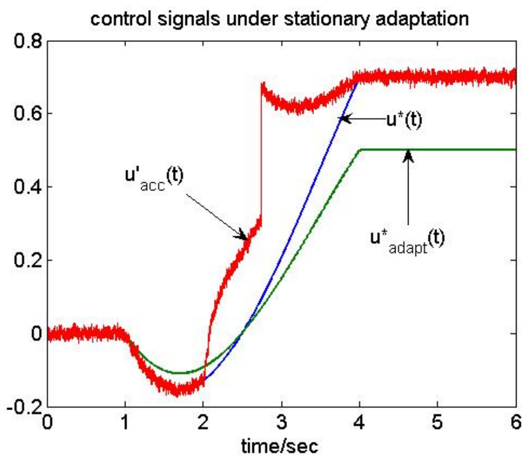

Fig. 7. $u^\star(t)$: original nominal control signal; $u^\star_{\text{adapt}}(t)$: nominal control signal after dynamic trajectory adaptation; $u'_{\text{acc}}(t)$: fault accomodating control signal.

limit ($S = 1$), which was considered here. The effect on the reference control signal is shown in figure 7, where it should be noted that the equivalence of $u'_{\text{acc}}(t)$ and the original reference trajectory $u^\star(t)$ in the stationary regime is due to the fact that $u'_{\text{acc}}(t)$ is shifted by the amplitude of the actuator fault above its adapted nominal control signal $u^\star_{\text{adapt}}(t)$. Finally, in fig. 8 an example of unstable output dynamics is shown, when no trajectory adaptation is applied. The actuator fault was equal to $f_a(t) = -0.3\sigma(t - 2\,\text{sec})$. An adaptation of the stationary value clearly would have saved the system stability in this case.

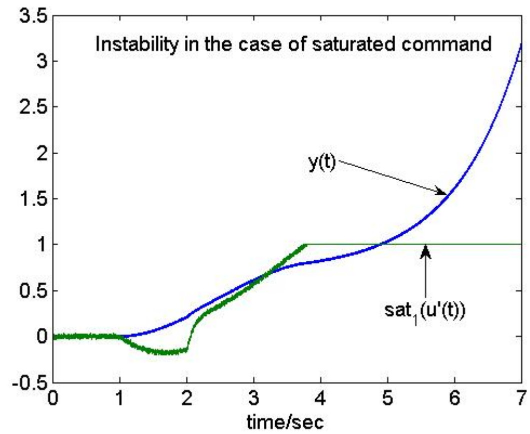

Fig. 8. Example of unstable output dynamics ($y(t)$) when no trajectory adaptation is applied and the free control command $u'(t)$ hits the saturation.

8. CONCLUSIONS

This works shows that new concepts of Fault Tolerant Control can efficiently be based on the concept of flatness. Using the algebraic derivative estimation to identify the actuator fault, it is possible to predict, perhaps for the first time, whether the original reference trajectory can still be tracked under given control saturations, and an online replanning of the reference trajectory, if needed, was discussed. The extension of the diagnosis methods proposed in this paper, which dealt with an unstable linear SISO system, to the nonlinear case presents no difficulty. Therefore, nonlinear systems will be our main focus in the future.

REFERENCES

M. Fliess, C. Join, M. Mboup, H. Sira-Ramírez, 2004, Compression différentielle de transitoires bruités, *C.R. Acad. Sci. Paris Ser. I*, **339**, 821-826.

M. Fliess, C. Join, M. Mboup, H. Sira-Ramírez, 2005a, Analyse et représentation de signaux transitoires: application à la compression, au débruitage et à la détection de ruptures, *Actes Coll. GRETSI*, Louvain-la-Neuve.

M. Fliess, C. Join, H. Sira-Ramírez, 2005, Closed-loop fault-tolerant control for uncertain nonlinear systems, in *Control and Observer Design for Nonlinear Finite and Infinite Dimensional Systems*, T. Meurer, K. Graichen, E.D. Gilles (Eds), Lect. Notes Control Informat. Sci. **322**, Springer, Berlin, pp. 217-233.

M. Fliess, C. Join, H. Sira-Ramírez, 2004, Robust residual generation for linear fault diagnosis: an algebraic setting with examples, *Internat. J. Control*, **77**, 1223-1242.

M. Fliess, J. Lévine, P. Martin, P. Rouchon, 1995, Flatness and defect of non-linear systems: introductory theory and examples, *Internat. J. Control*, **61**, 1327 – 1361.

P.M. Frank, 1990, Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy a survey and some new results, *Automatica*, **26**, 459-474.

J. Chen, R. Patton, 1999, *Robust Model-Based Fault Diagnosis for Dynamic Systems*, Kluwer, Boston.

C. De Persis, A. Isidori, 2001, A geometric approach to nonlinear fault detection and isolation, *IEEE Trans. Automatic Control*, **46**, 853-865.

J. Gertler, 1998, *Fault Detection and Diagnosis in Engineering Systems*, Marcel Dekker, New York.

C. Join, 2002, *Diagnotic des systèmes non linéaires - Contribution aux méthodes de découplage*, Thèse, Université Henri Poincaré, Nancy.

C. Join, J.-C. Ponsart, D. Sauter, 2003, Diagnotic des systèmes non linéaires - Contribution aux méthodes de découplage, *J. Europ. Systèmes Automatisés*, **37**, 1323-1328.

H. Jones, 1973, *Failure detection in linear systems*, PhD thesis, Massachusette Institute of Technologiy. Departement of Aeronautics and Astronautics, Cambridge, MA.

V. Kapila and K.M. Grigoriadis, 2002, Actuator Saturation Control, (Eds), Marcel Dekker, Inc., New York (USA).

J. Lunze, J. Askari-Marnani, A. Cela, P.M. Frank, A.L. Gehin, T. Marku, L. Rato, M. Staroswiecki, Three-tank control reconfiguration, in *Control of Complex Systems*, K.J. Aström, D. Blanke, A. Isidori, W. Schaufelberger, R. Sanz (Eds), Springer, Berlin, 2001, pp. 241-283.

M. Massoumnia, G. Verghese, A. Willsky, 1989, Failure detection and identification, *IEEE Trans. Automatic Control*, **34**, 316-321.

M. Staroswiecki, G. Comtet-Varga, 2001, Analytic redundancy for fault detection and isolation in algebraic dynamic system, *Automatica*, **37**, 687-699.

S. Tarbouriech and G. Garcia, 1997, Control of Uncertain Systems with Bounded Inputs, (Eds), Lect. Notes Control Informat. Sci. **227**, Springer, Berlin.

D. Theilliol, H. Noura, J.-C. Ponsart, 2002, Fault diagnosis and accommodation of a three-tank system based on analytical redundancy, *ISA Transactions*, **41**.

J.E. White, J.L. Speyer, 1987, Detection filters design : spectral theory and algorithms, *IEEE Trans. Signal Process.*, **32**, 593-603.

A. S. Willsky, 1976, A survey of design methods for failure detection in dynamic systems, *Automatica*, **12**, 601-611.