A TIME-VARYING LINEAR STATE FEEDBACK TRACKING CONTROLLER FOR A BOOST-CONVERTER DRIVEN DC MOTOR

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Abstract: This article deals with the trajectory tracking problem for the angular velocity of a dc-motor shaft using a Boost-converter as the switch regulated electronic drive. The main result of our proposed control scheme is that measuring of the angular velocity is not really necessary and the control law is synthesized using only a linear time-varying combination of the converter current and voltage variables. The voltage reference trajectory for the converter is generated exploiting a partial differential flatness property of the combined system. The reference trajectories of the average control and the input current are calculated via stored energy considerations and planning for the initial and final stationary regimes. The discrete switching control realization of the designed continuous feedback control law is accomplished by means of a traditional PWM-modulation scheme. Experimental results are provided.

Keywords: Tracking control, dc-drives, power electronics, passivity-based control, flatness-based trajectory planning

1. INTRODUCTION

Frequently, dc-to-dc power converters are used as dc motor drives, which apply the required voltage to the dc motor in accordance with the demanded task represented by the desired angular velocity profile or angular position reference trajectory. Customarily, the proposed feedback controllers are devised under the restrictive assumption of ramps and constant values acting as reference trajectories for the specification of the desired angular velocity profile. This practise results in discontinuities and, sometimes, in impulse-like functions for the controller output. These impulses are invariable absorbed by the mechanical part of the system with important consequences related to the durability of dc machine (Linares-Flores and Sira-Ramirez, 2004).

Nonlinear average models are frequently used to describe dc-to-dc power converters in the design of feedback control laws (Lehman and Bass, 1996), (Kugi and Schlacher, 1999), (D. Maksimović and Erickson, 1996). The main idea behind the design methods is to devise a continuous feedback control expression, based on the average system description, in terms of bounded average control inputs and then implement this feedback control law in a *switched* fashion taking the continuous control signal as a *duty ratio* function in a PWM implementation scheme. There are countless applications where the pulse-width-modulation (PWM) acts as the "electronic actuator" for the commanded switch position function which is, strictly speaking, the actual control input (Sira-Ramirez, 1989), (D. Maksimović and Erickson, 1996), (Rashid, 2004).

In this paper, we presents a smooth "starter" for a dc motor constituted by a Boost power converter. The main task is to achieve angular velocity regulation of dc motor shaft. A simple linear-time-varying state feedback controller, based on exact tracking error dynamics passive output feedback, is shown to semi-globally stabilize the state trajectory tracking error to zero while requiring only converter current and voltages. The dynamic average model of the "Boost" converter-dc motor combination is shown to conform to a special energy managing structure which is suitable for passivity based feedback techniques. The required converter output voltage reference signal is generated exploiting a partial differential flatness property. The corresponding reference, or nominal, signals for the average control and the converter input current are calculated via energy consideration in the vicinity of stationary regimes. We use a spline interpolation time polynomial of the Bézier type for the calculation of the smooth energy interpolation trajectory corresponding to the initial and final desired velocities. This trajectory, in turn, completely defines, in an off-line manner, the reference trajectories of the converter input current and the average control.

Section 2 deals with the average modelling of the "Boost"-dc motor converter cascaded arrangement. Section 3 develops the linear time-varying feedback controller based on elementary passivity considerations on the Exact Tracking Error Dynamics Passive Output (ETEDPO). The proposed controller, called hereafter: Exact Tracking Error Dynamics Passive Output Feedback (ETEDPOF) results in a rather appealing linear state feedback controller. Section 4 deals with the off-line trajectory generation required by the proposed linear time-varying feedback controller. Section 5 briefly describes the experimental setting used to test the effectiveness of the proposed control method and Section 6 shows the experimental results. The conclusions are relegated to the last section.

2. MODEL OF THE COMBINATION DC-MOTOR—BOOST-CONVERTER

We consider a cascaded combination of a dc-to-dc "Boost" power converter and a dc-motor, as depicted in Figure 1. Using Kirchhoff's current

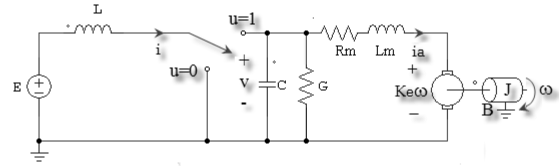

Fig. 1. The combination "*dc Motor–Boost-power-converter*" model

and voltage laws and Newton's second law of mechanics, we obtain the following average model of the system:

$$L\frac{di}{dt} = -v\,u + E \tag{1}$$

$$C\frac{dv}{dt} = i\,u - G\,v - i_a \tag{2}$$

$$L_m\frac{di_a}{dt} = v - R_m\,i_a - K_e\,w \tag{3}$$

$$J\frac{dw}{dt} = -B\,w + K_m\,i_a \tag{4}$$

We may represent this system in the following manner using matrix notation:

$$\dot{x} = (J(u) - R)\,\frac{\partial H(x)}{\partial x} + \varepsilon \tag{5}$$

where,

$$x^{\mathrm{T}} = (i, v, i_a, w) \tag{6}$$

and

$$\begin{aligned} H(x) &= \frac{1}{2}\left(L\,i^2 + C\,v^2 + L_m\,i_a^2 + J\,w^2\right) \\ &= \frac{1}{2}x^{\mathrm{T}}Mx \end{aligned} \tag{7}$$

which is taken as an implicit definition of the positive definite, symmetric, constant matrix M. The vector ε^{T} is simply given by:

$$\varepsilon^{\mathrm{T}} = (\frac{E}{L}, 0, 0, 0) \tag{8}$$

$$J(u) = \begin{pmatrix} 0 & -\frac{1}{CL}u & 0 & 0 \\ \frac{1}{CL}u & 0 & -\frac{1}{CL_m} & 0 \\ 0 & \frac{1}{CL_m} & 0 & -\frac{K_e}{JL_m} \\ 0 & 0 & \frac{K_e}{JL_m} & 0 \end{pmatrix} \tag{9}$$

$$R = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{G}{C^2} & 0 & 0 \\ 0 & 0 & \frac{R_m}{L_m^2} & 0 \\ 0 & 0 & 0 & \frac{B}{J^2} \end{pmatrix} \tag{10}$$

Note that the matrix $J(u)$ is an anti-symmetric and R is a symmetric positive semi-definite matrix, i.e., for all u $J^{\mathrm{T}}(u) = -J(u)$ and $R^{\mathrm{T}} = R$.

3. PASSIVITY-BASED AVERAGE CONTROLLER DESIGN

It is desired to have the motor shaft angular motion to track a certain reference angular velocity profile, denoted by $w^\star$. We assume that a state reference trajectory $x^*(t)$ is known which satisfies the following open loop dynamics:

$$\dot{x}^\star(t) = (J(u^\star(t)) - R)\,\frac{\partial H(x^\star(t))}{\partial x^\star} + \varepsilon \qquad (11)$$

where $u^\star(t)$ is the reference control input corresponding to the desired state reference $x^\star(t)$. A feasible off-line computation of the system state trajectory $x^\star(t)$ and the control input reference trajectory $u^\star(t)$, from the knowledge of the desired angular velocity trajectory $w^\star$, will be explained in Section 4.

Define the state error with respect to such an off-line computed state reference trajectory as $e = x - x^\star$. Also define the control input error as: $e_u = u - u^\star$. Let

$$H(e) = \frac{1}{2} e^{\mathrm{T}} M e$$

be a quadratic Hamiltonian for the system constituted by the total stored energy. Clearly,

$$\frac{\partial H}{\partial e} = M\,e = M(x - x^\star) = \frac{\partial H}{\partial x} - \frac{\partial H}{\partial x^\star}$$

We may conclude, after some algebraic manipulations, that

$$\begin{aligned}
\dot{e} &= J(u)\frac{\partial H}{\partial x} - J(u^\star)\frac{\partial H}{\partial x^\star} - R\left(\frac{\partial H}{\partial x} - \frac{\partial H}{\partial x^\star}\right) = \\
&= J(u)\left(\frac{\partial H}{\partial x} - \frac{\partial H}{\partial x^\star}\right) + (J(u) - J(u^\star))\,\frac{\partial H}{\partial x^\star} \\
&\quad -R\left(\frac{\partial H}{\partial x} - \frac{\partial H}{\partial x^\star}\right) = \\
&\quad J(u)\frac{\partial H}{\partial e} + \frac{\partial J(u)}{\partial u}(u - u^\star)\,\frac{\partial H}{\partial x^\star} - R\frac{\partial H}{\partial e} \\
&= J(u)\frac{\partial H}{\partial e} + \frac{\partial J(u)}{\partial u}\frac{\partial H}{\partial x^\star}\,e_u - R\,\frac{\partial H}{\partial e}
\end{aligned}$$

A natural feedback control input law, defined in terms of the control input error variable e_u, which achieves asymptotic stability of the origin of the error space for the average system may be chosen to be of the form:

$$e_u = -\gamma\left(\frac{\partial J(u)}{\partial u}\frac{\partial H}{\partial x^\star}\right)^{\mathrm{T}}\frac{\partial H}{\partial e}\,.$$

where the constant γ is chosen to be a strictly positive parameter.

The closed loop exact tracking error dynamics evolves according to

$$\dot{e} = J(u)\frac{\partial H}{\partial e} - \underbrace{\left(R + \gamma\left(\frac{\partial J(u)}{\partial u}\frac{\partial H}{\partial x^\star}\right)\left(\frac{\partial J(u)}{\partial u}\frac{\partial H}{\partial x^\star}\right)^{\mathrm{T}}\right)}_{=:\,\tilde{R}}\frac{\partial H}{\partial e}$$

It is not difficult to realize that, in the case under consideration, the matrix $\tilde{R}$ is positive definite.

Along the trajectories of the closed loop system, one has that the total stored energy time derivative satisfies the following relations

$$\begin{aligned}
\dot{H}(e) &= \left(\frac{\partial H}{\partial e}\right)^{\mathrm{T}}\dot{e} &(12)\\
&= \left(\frac{\partial H}{\partial e}\right)^{\mathrm{T}} J(u)\,\frac{\partial H}{\partial e} - \left(\frac{\partial H}{\partial e}\right)^{\mathrm{T}}\tilde{R}\,\frac{\partial H}{\partial e} \\
&= -\left(\frac{\partial H}{\partial e}\right)^{\mathrm{T}}\tilde{R}\,\frac{\partial H}{\partial e} < 0 &(13)
\end{aligned}$$

Clearly, the Hamiltonian function:

$$H(e) = \frac{1}{2} e^{\mathrm{T}} M e$$

is, indeed, a Lyapunov function and, therefore, the origin of the error space results in an asymptotically stable equilibrium point. Since the control input is bounded and subject to possible saturations, generally speaking the result cannot be global. In fact, the stability of the origin of the error space will be semi-global due to the fact that control input saturations will depend on the values of the initial states. We have, as a consequence that, semi-globally,

$$\lim_{t\to\infty} e = 0 \quad \Longleftrightarrow \quad \lim_{t\to\infty} x = x^\star$$

Specifically, in terms of the converters currents and voltages, we have that the following linear, time-varying, state feedback control law forces the motion of the "Boost" converter-dc motor combination to track the reference state trajectory $x^\star(t)$ with corresponding control input reference trajectory $u^\star(t)$,

$$\begin{aligned}
u &= u^\star(t) + \gamma\,[v^\star(t)(i - i^\star(t)) - i^\star(t)(v - v^\star(t))] \\
&= u^\star(t) + \gamma\,[v^\star(t)i - i^\star(t)v] &(14)
\end{aligned}$$

Notice that when $i \to i^\star(t)$ and $v \to v^\star(t)$ then $u \to u^\star(t)$.

4. REFERENCE TRAJECTORY GENERATION

In view of the nature of the derived feedback control law (14), we need to generate the voltage and current references for the boost converter circuit, i.e. $v^\star(t)$ and $i^\star(t)$. For the generation of the "Boost" converter nominal average output voltage $v^\star(t)$, which clearly coincides with the armature circuit input voltage, and for the corresponding armature circuit current $i_a^*(t)$, we may use their differential parameterizations in terms of the desired angular velocity $w^\star(t)$. Indeed, from the system model (1)-(4), we immediately have,

$$v^\star(t) = \left(\frac{JL_m}{K_m}\right)\ddot{w}^\star(t) + \left(\frac{BL_m}{K_m} + \frac{JR_m}{K_m}\right)\dot{w}^\star(t) + \left(\frac{BR_m}{K_m} + K_e\right)w^\star(t) \tag{15}$$

$$i_a^\star(t) = \frac{J}{K_m}\dot{w}^\star(t) + \frac{B}{K_m}w^\star(t) \tag{16}$$

Under equilibrium conditions, the above parametrization, establishes that for constant angular velocities (i.e., $w^\star(t) = \bar{w}$), it follows that the corresponding constant values of v and i_a, denoted respectively by $\bar{v}$ and $\bar{\imath}_a$, are given by

$$\bar{v} = \left(\frac{BR_m}{K_m} + K_e\right)\bar{w} \tag{17}$$

$$\bar{\imath}_a = \frac{B}{K_m}\bar{w} \tag{18}$$

It turns out that the inductor current i in the "Boost" converter cannot be differentially parameterized in the same simple manner as the dc motor armature circuit variables, due to a lack of *differential flatness* in the cascaded system. To achieve an indirect parametrization of this variable, we consider the stored energy of the Boost-converter, denoted by H_B, given by:

$$H_B = \frac{1}{2}Li^2 + \frac{1}{2}Cv^2$$

At an equilibrium point, we have $dH_B/dt = 0$ and, hence:

$$\begin{aligned}\bar{\imath} &= \frac{1}{E}\left[G\bar{v}^2 + \bar{\imath}_a\bar{v}\right] \\ &= \frac{1}{E}\left[G\left(\frac{BR_m}{K_m} + K_e\right)^2 \right.\\ &\qquad \left. + \frac{B}{K_m}\left(\frac{BR_m}{K_m} + K_e\right)\right]\bar{w}^2 = \alpha\bar{w}^2\end{aligned}$$

We then have, in equilibrium conditions parameterized by $\bar{w}$ that the stationary stored energy in the converter is expressed as:

$$\bar{H}_B = \frac{1}{2}L\bar{\imath}^2 + \frac{1}{2}C\bar{v}^2 = \frac{1}{2}L\left(\alpha\bar{w}^2\right)^2 + \frac{1}{2}C\left(\beta\bar{w}\right)^2 \tag{19}$$

where

$$\alpha = \frac{(BR_m + K_eK_m)(GR_mB + GK_eK_m + B)}{EK_m^2}$$
$$\beta = \frac{BR_m + K_eK_m}{K_m} \tag{20}$$

A rest-to-rest nominal reference trajectory may be specified for the total stored energy H_B corresponding to the initial and final angular velocity equilibrium points: $\bar{w}_{ini}$ and $\bar{w}_{fin}$. Once the trajectory $H_B^\star(t)$ is specified for the desired rest-to-rest maneuver one may finally compute the nominal inductor current trajectory and the nominal control input as:

$$\begin{aligned}i^\star(t) &= \sqrt{\frac{1}{L}\left[2H_B^\star(t) - C(v^\star(t))^2\right]} \\ u^\star(t) &= \frac{1}{v^\star(t)}\left[E - L\frac{di^\star(t)}{dt}\right] \\ &= \frac{1}{v^\star(t)}\left[E - \left(\frac{\dot{H}_B^\star(t) - Cv^\star(t)\dot{v}^\star(t)}{i^\star(t)}\right)\right]\end{aligned}$$

5. EXPERIMENTAL SET-UP FOR THE "BOOST" CONVERTER-DC MOTOR SYSTEM

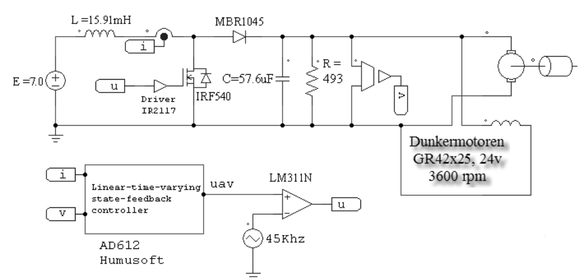

Fig. 2. Experimental set-up for the "Boost" converter-dc Motor with linear-time-varying state feedback controller

The design parameters used for the "Boost"-power converter are given by: $E = 7\,V$, $C = 57.6\,\mu F$, $G = 2.03 \times 10^{-3}\,\Omega^{-1}$, $L = 15.91\,mH$, while the switching frequency was taken to be $45\,kHz$. The parameters identified on a real dc motor (Dunkermotoren GR42x25) are given by: $R_m = 6.14\,\Omega$, $L_m = 8.9\,mH$, $K_e = 49.13\,mV$ sec/rad, $K_m = 49.13\,mV$ sec rad, $J = 7.95 \times 10^{-6}\,kgm^2$, $B = 41\,\mu Nm$ sec. The sampling time of the program controller was set to be of $220\,\mu$sec. We have chosen a controller gain equal to $\gamma = 0.150$. An acquisition card of the type AD612 was used for the implementation of the linear-time-varying feedback controller (see Figure 2). For the implementation by means of a PWM modulator commanding the switch position function of the converter, the average continuous time-varying, linear, feedback control law signal was taken to be the corresponding acting duty ratio function.

6. EXPERIMENTAL RESULTS

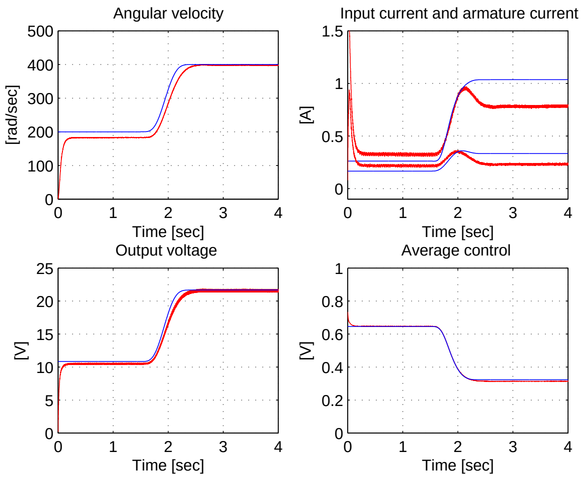

Fig. 3. Angular velocity, input current, output voltage and average control input response for a desired angular velocity trajectory tracking task

A nominal desired angular velocity profile exhibiting a rather smooth start for the dc machine was specified using an interpolating Bézier polynomial of tenth order, where the initial angular velocity was set to be $\bar{w}^*_{ini}(1.5 \text{ sec}) = 200 \,\text{rad/sec}$ and the final desired value of the angular velocity was specified as $\bar{w}^*_{fin}(3.5 \text{ sec}) = 400 \,\text{rad/sec}$. See Figure 3 for the resulting armature current and output voltage. The initial equilibrium stored energy in the converter, see equation (19), was found to be $\bar{H}_B(1.5 \text{ sec}) = 3.5 \, mJ$ and its final value $\bar{H}_B(3.5 \text{ sec}) = 13.7 \, mJ$. With this values, the energy reference $H^*_B(t)$ was specified by interpolation with an other Bézier polynomial of tenth order. The resulting input current and average control are depicted in Figure 3. The initial average output equilibrium voltage, or armature voltage, varies between the initial and final equilibrium values: $\bar{v}^*_{ini} = (1.5 \text{ sec}) = 10.5 \, V$ and $\bar{v}^*_{fin} = (3.5 \text{ sec}) = 21.5 \, V$. The corresponding average control input signal generated by the linear feedback controller of the "Boost" converter-dc motor system varies between the initial and final values $\bar{u}^{ini}_{av}(1.5 \text{ sec}) = 0.65 \, V$ and $\bar{u}^{fin}_{av}(3.5 \text{ sec}) = 0.31 \, V$. The input inductor current and the armature current show significant differences to the corresponding reference currents. Those differences are due to unmodelled losses, as for example resistances in the input inductor, diod, and transistor. However, the angular velocity tracking performance is quite satisfactory.

7. CONCLUSIONS

In this article, an exact tracking error dynamics (ETED) has been derived for a the smooth control of a "Boost"-dc motor cascade combination in which a desired angular velocity trajectory tracking is demanded on the motion of the motor shaft. In particular, using a "Boost" dc to dc power converter as the electronic actuator for the motor allows for a smooth start of the motor as well as a smooth angular velocity profile tracking. The ETED was shown to exhibit a convenient energy managing decomposition from where a static ETED passive output feedback controller is shown to achieve semi-global asymptotic stability for the origin of the state tracking error space. This ETED passive output feedback results in a linear, time-varying, state tracking error feedback control law which demands only electrical converter variables.

The nonlinear "Boost"- dc motor combination happens to be a non exactly feedback linearizable system, i.e., its lack of flatness makes it a challenging interesting problem where a complete parametrization of the system variables is not possible and off-line trajectory planning becomes interesting. In this instance, however, for a smooth rest-to-rest trajectory tracking task, it was shown that an indirect complete parametrization of all the system variables is still possible by resorting to an equilibrium-to-equilibrium trajectory planning of the total stored energy in the converter circuit.

Experimental results were attempted that achieve the demanded rest to rest task in an acceptable tracking performance.

Further work is needed regarding the enhancement of the robustness features of our proposed control scheme, specially to external load torques. In this regard new algebraic techniques for fast perturbation identification are being developed along with more classical adaptive control approaches.

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