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Lyapunov-design for a super-twisting sliding-mode controller using the certainty-equivalence principle

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Abstract: In this article, a Lyapunov-based control concept is presented combining variable structure and adaptive control. The considered system class comprises single-input systems which are affected by structured and unstructured uncertainties. The design is based on a Lyapunov function for the super-twisting algorithm. This Lyapunov function is exploited to derive the adaptive part of the proposed controller using the certainty equivalence principle. It is demonstrated that this combination of sliding-mode and adaptive control methodology allows to relax the boundedness condition known from the super-twisting algorithm while reducing the sliding-mode gain significantly. The effectiveness of the presented concept is demonstrated in detail, using results from numerical simulations and experimental data of a laboratory testbed.

Keywords: sliding-mode, uncertain systems, robust control, adaptive control, Lyapunov theory

1. INTRODUCTION

A major objective in control system design is to achieve stability in the presence of uncertainties, as for example external disturbances or model inaccuracies. As documented by numerous applications and articles, see e.g. Shtessel et al. (2012); Brégeault et al. (2010); Reichhartinger and Horn (2012); Pisano et al. (2008), sliding-mode control (SMC) may be an effective design method towards this goal. In addition, the implementation of control laws based on sliding-mode ideas is straight-forward and requires just a few controller parameters. However, tuning of these controller parameters may be rather time-consuming or rely on sophisticated methods (Oza et al., 2012; Pilloni et al., 2012; Rosales et al., 2010). Generally, the choice of controller parameters resorts to worst case assumptions on the uncertainties which may result in large controller gains. This however generates undesired effects like chattering (Boiko et al., 2007; Slotine and Li, 1991).

In this paper, an approach for combining sliding-mode and adaptive control is proposed. The overall rationale is to exploit as much information as possible about the structure of the uncertainties. We design an adaptive control law based on the certainty equivalence principle to compensate for this structured part of the uncertainty, leaving only the unstructured part to the sliding-mode controller. It turns out that this approach allows to reduce the gains of the sliding-mode controller significantly while maintaining its robustness. Several approaches dealing with combinations of adaptive and variable structure control have been published. A special case is represented by systems in *parametric strict-feedback-form* or *parametric pure-feedback-form* which admits to apply backstepping controller design procedures (Kanellakopoulos et al., 1991). For counteracting matched uncertainties, Sira-Ramírez and Llanes-Santiago (1993) suggest to employ SMC in the last step of the backstepping procedure. In further extensions, proposed in Bartolini et al. (1996, 2000), the adaptive backstepping design is combined with a second-order SMC by replacing the last two steps of the of the adaptive backstepping procedure. This approach also admits the compensation of nonparametric uncertainties. The super-twisting algorithm (STA), see e.g. Levant (1993), is able to stabilize a system subject to matched uncertainties in finite time by using a continuous control signal. In this approach, the choice of controller parameters is mainly affected by an upper bound on the uncertainties. However, these bounds are often unknown or may even vary with time. Therefore, the gains of the STA controller have to be chosen for a worst case scenario and an overestimation of the controller gains may consequently lead to unnecessarily large control effort (Brégeault et al., 2010).

This drawback is addressed in the remarkable work of Shtessel et al. (2010) where an *adaptive* STA is presented. The authors introduce adaptive controller gains which are increased whenever the sliding variable is nonzero. This bears two major advantages: Firstly, no upper bound on the uncertainty has to be estimated prior to the design, and secondly, the gain initially may be chosen small and shall only increase as much as the process demands. It is shown that stability can be achieved even if the bounds of the uncertainties are unknown. Yet, it may not grow faster than some square-root of the sliding variable. Recent results improve the approach by allowing also for a decrease of the controller gains (Kochalummoottil et al., 2011) and therefore lead to smaller controller gains when compared with the constant gain STA. However, the structure of the uncertainty is not addressed which may turn out unnecessarily restrictive with regard to the class of tractable disturbances.

The aim of the approach presented in this article is to demonstrate that sliding-mode controllers can be extended with an adaptation law in order to systematically compensate for the structured and unstructured uncertainties. Conventionally designed sliding-mode algorithms do not distinguish between structured and unstructured uncertainties, and the effect of both types of uncertainties is dominated by choosing sufficiently high controller gains. The advocated approach of this article takes into account this structural information and, consequently, helps reduce the gains of the sliding-mode based part of the controller. The design is motivated by the idea that only unstructured uncertainties should be compensated by the variable structure control law. Complementarily, uncertainties whose structure is at least partially known should be rejected by an appropriate adaptation-based control law. The entire control action then takes the form of an additive combination of both the sliding-mode and the adaptive control law. The proposed method is assessed by means of simulation examples and experimental results on a laboratory testbed, underscoring the practicability of the approach.

The contribution is organized as follows: Section 2 gives a formal problem definition. In Section 3 we present the novel controller design method and address the stability for a class of uncertainties. Section 4 discusses the applicability of related sliding-mode control methods which will then serve as benchmark designs for the simulation case study in Section 5 as well as for the experimental testbed in Section 6. We close with some final remarks in Section 7.

2. PROBLEM DESCRIPTION

We consider the single-input nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\big(\Delta(\boldsymbol{x},t) + u\big) \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ represents the state vector and $u(t) \in \mathbb{R}$ denotes the scalar input. The vector fields $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ shall be differentiable and assumed to be known.

The system is affected by the matched uncertainty

$$\Delta(\boldsymbol{x},t) = \Delta_{\mathrm{s}}(\boldsymbol{x}) + \Delta_{\mathrm{u}}(\boldsymbol{x},t) \tag{2}$$

where $\Delta_{\mathrm{s}}(\boldsymbol{x})$ denotes the structured and $\Delta_{\mathrm{u}}(\boldsymbol{x},t)$ the unstructured part. We assume that the structured uncertainty may be written as a product of an unknown linear parameter vector $\boldsymbol{\Theta} \in \mathbb{R}^p$ and a known base function $\boldsymbol{\phi} : \mathbb{R}^n \to \mathbb{R}^p$, i.e. $\Delta_{\mathrm{s}}(\boldsymbol{x}) = \boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{x})$.

A sliding variable $\sigma = \sigma(\boldsymbol{x})$ with $\sigma : \mathbb{R}^n \to \mathbb{R}$ is chosen such that the desired dynamics are achieved for $\sigma \equiv 0$. We assume relative degree one of the sliding variable σ with respect to the input u. The associated internal dynamics are assumed to be stable.

Consequently, the dynamics of σ read as follows

$$\begin{aligned}\dot{\sigma} =& \frac{\partial\sigma}{\partial\boldsymbol{x}}\Big(\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\big(\Delta_{\mathrm{s}}(\boldsymbol{x}) + \Delta_{\mathrm{u}}(\boldsymbol{x},t) + u\big)\Big) \\ =& \underbrace{\frac{\partial\sigma}{\partial\boldsymbol{x}}\boldsymbol{f}(\boldsymbol{x})}_{=:a_0(\boldsymbol{x})} + \boldsymbol{\Theta}^{\mathrm{T}}\underbrace{\boldsymbol{\phi}(\boldsymbol{x})\frac{\partial\sigma}{\partial\boldsymbol{x}}\boldsymbol{g}(\boldsymbol{x})}_{=:\boldsymbol{a}_1(\boldsymbol{x})} + \underbrace{\frac{\partial\sigma}{\partial\boldsymbol{x}}\boldsymbol{g}(\boldsymbol{x})\Delta_{\mathrm{u}}}_{=:a_2(\boldsymbol{x},t)} + \underbrace{\frac{\partial\sigma}{\partial\boldsymbol{x}}\boldsymbol{g}(\boldsymbol{x})}_{=:b(\boldsymbol{x})}u\end{aligned} \tag{3}$$

With these abbreviations (3) may be written as

$$\dot{\sigma} = \boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{a}_1(\boldsymbol{x}) + a_2(\boldsymbol{x},t) + \omega \tag{4}$$

where $\omega := a_0(\boldsymbol{x}) + b(\boldsymbol{x})\,u$. Note, that $b(\boldsymbol{x})$ is nonzero for all $\boldsymbol{x}$ since we assume σ to show relative degree one.

The objective in this contribution is to design a controller that stabilizes the origin of (4) subject to the uncertainty

$$\varphi(\boldsymbol{x},\boldsymbol{\Theta},t) := \boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{a}_1(\boldsymbol{x}) + a_2(\boldsymbol{x},t). \tag{5}$$

In particular, we are interested in the case where $\varphi(\boldsymbol{x},\boldsymbol{\Theta},t)$ grows faster than some square-root of the sliding variable.

3. CONTROLLER DESIGN

The main rationale of our contribution is to beneficially incorporate the structural information on the uncertainty into the design of a sliding-mode controller. To this end, a certainty-equivalence super-twisting algorithm (CESTA) is developed that resorts to a conventional super-twisting controller as a nominal controller. Exemplified on the super-twisting controller we demonstrate the large potential of combining variable structure controllers with the certainty-equivalence principle.

Consider system (4). In a first step, we design a controller that stabilizes the system under the uncertainty $a_2(\boldsymbol{x},t)$. The uncertainty shall be bounded as per

$$|a_2(\boldsymbol{x},t)| \leq \Omega\,|\sigma(\boldsymbol{x})|^{\frac{1}{2}} \tag{6}$$

with a known constant $\Omega > 0$. For better readability, in the following we shall drop the argument of $\sigma(\boldsymbol{x})$.

Based on the super-twisting algorithm let us choose ω as

$$\begin{aligned}\omega &= -k_1\,|\sigma|^{\frac{1}{2}}\,\mathrm{sign}\,(\sigma) + \nu - \hat{\boldsymbol{\Theta}}^{\mathrm{T}}\,\boldsymbol{a}_1(\boldsymbol{x}) \\ \dot{\nu} &= -k_2\,\mathrm{sign}\,(\sigma)\,, \quad \nu(0) = 0\end{aligned} \tag{7}$$

where $\hat{\boldsymbol{\Theta}}$ denotes some estimate of the unknown parameter vector $\boldsymbol{\Theta}$, $k_1, k_2 > 0$ are controller parameters and ν is a controller state. The solutions of the feedback-system (4) and (7) shall be understood in the sense of Filippov (Filippov and Arscott, 1988).

Remark: In what follows, employed Lyapunov functions V and V_{nom} will be continuous but not locally Lipschitz. Thus a non-smooth version of Lyapunov theory is required, as pointed out in Remark 1 of Moreno and Osorio (2008). Here, this amounts to exempting points where the Lyapunov function is not differentiable and analyzing them accordingly; see (Clarke et al., 1998) for further details.

Consider the weak Lyapunov function

$$V_{\mathrm{nom}} = k_2|\sigma| + \frac{1}{2}\nu^2 \tag{8}$$

proposed by Orlov (2005).

Taking the derivative with respect to time yields

$$\begin{aligned}\dot{V}_{\mathrm{nom}} =& k_2\,\mathrm{sign}\,(\sigma)\times \\ &\Big(-k_1|\sigma|^{\frac{1}{2}}\mathrm{sign}\,(\sigma) + a_2(\boldsymbol{x},t) - \underbrace{\big(\hat{\boldsymbol{\Theta}} - \boldsymbol{\Theta}\big)^{\mathrm{T}}}_{=:\tilde{\boldsymbol{\Theta}}^{\mathrm{T}}}\boldsymbol{a}_1(\boldsymbol{x})\Big)\end{aligned} \tag{9}$$

which is now devoid of ν.

Suppose we have a perfect estimate of the uncertainty such that $\tilde{\boldsymbol{\Theta}}^{\mathrm{T}}\boldsymbol{a}_1(\boldsymbol{x}) = 0$. Using (6) we obtain

$$\dot{V}_{\mathrm{nom}} \leq k_2 \operatorname{sign}(\sigma)\left(-k_1|\sigma|^{\frac{1}{2}}\operatorname{sign}(\sigma) + \Omega\,|\sigma|^{\frac{1}{2}}\right). \quad (10)$$

Choosing $k_1 > \Omega$ results in a negative semidefinite right-hand side. Then, stability can be shown using the invariance principle of Krasovskii-LaSalle since the only solution for which $\dot{V}_{\mathrm{nom}} = 0$ is $\sigma \equiv 0$. Taking into account (4)–(7) the only remaining solution is $\nu \equiv 0$. Hence, asymptotic stability of the closed loop system is ensured.

In the next step of our proposed design procedure we drop the assumption that the parameter vector $\boldsymbol{\Theta}$ is perfectly known. In order to obtain an adaptation law for $\hat{\boldsymbol{\Theta}}$ we augment the Lyapunov function (8) with a quadratic form in terms of the parameter error $\tilde{\boldsymbol{\Theta}} = \hat{\boldsymbol{\Theta}} - \boldsymbol{\Theta}$, that is

$$V_{\mathrm{ext}} = V_{\mathrm{nom}} + \frac{1}{2\gamma}\tilde{\boldsymbol{\Theta}}^{\mathrm{T}}\tilde{\boldsymbol{\Theta}} \quad (11)$$

where $\gamma > 0$.

The adaptation law for $\hat{\boldsymbol{\Theta}}$ is then obtained by taking the derivative of V_{ext}:

$$\dot{V}_{\mathrm{ext}} = \dot{V}_{\mathrm{nom}} + \frac{1}{\gamma}\tilde{\boldsymbol{\Theta}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{\Theta}}}\,.$$

For an unknown, constant parameter $\boldsymbol{\Theta}$ we have $\dot{\tilde{\boldsymbol{\Theta}}} = \dot{\hat{\boldsymbol{\Theta}}}$. Using (6) the latter equation may be written as

$$\begin{aligned}\dot{V}_{\mathrm{ext}} \leq{} & k_2 \operatorname{sign}(\sigma)\left(-k_1|\sigma|^{\frac{1}{2}}\operatorname{sign}(\sigma) + \Omega|\sigma|^{\frac{1}{2}}\right)\\ & + \tilde{\boldsymbol{\Theta}}^{\mathrm{T}}\left(-\boldsymbol{a}_1(\boldsymbol{x})\,k_2\operatorname{sign}(\sigma) + \frac{1}{\gamma}\dot{\hat{\boldsymbol{\Theta}}}\right). \quad (12)\end{aligned}$$

In order to render the stability invariant of the unknown parameter error, the adaption law is chosen as

$$\dot{\hat{\boldsymbol{\Theta}}} = \gamma\,k_2\operatorname{sign}(\sigma)\ \boldsymbol{a}_1(\boldsymbol{x})\,. \quad (13)$$

With (13) the inequality (12) formally turns into (10). Note that $\dot{V}_{\mathrm{ext}}$ again is only negative semidefinite since it does neither contain $\tilde{\boldsymbol{\Theta}}$ nor ν. Hence, we cannot prove asymptotic stability of the origin of the overall system given by (4),(7) and (13). Nevertheless, the system will converge to $\sigma \equiv 0$, possibly with non-vanishing controller states $\hat{\boldsymbol{\Theta}}$ and ν. In view of the assumptions in Section 2, the internal dynamics wrt. the sliding surface are stable, thus, all states of the closed loop system will be bounded.

Remark: There are some noteworthy characteristics. First of all, the proposed method is particularly powerful if the constant Ω in (6) that is bounding uncertainty $a_1(\boldsymbol{x}, t)$ is significantly lower than it would be for bounding the overall uncertainty $\varphi(\boldsymbol{x}, \boldsymbol{\Theta}, t)$. In this case, the structured uncertainty in (1) shows a decisively larger influence than the unstructured one.

Furthermore, the proposed method relies on some knowledge about the constant Ω in order to find a suitable controller parameter k_1. It is clear that adaptive-gain sliding-mode control may overcome this problem. But this goes along with the need to bound the overall uncertainty $\varphi(\boldsymbol{x}, \boldsymbol{\Theta}, t)$. The presented method, however, does not entail additional bounds on function $\boldsymbol{a}_1(x)$, besides to be known. This may help improve robustness as will be shown in the following sections.

The potential of this modus operandi suggests that it may be applied also to other higher order sliding-mode controllers for which continuously differentiable Lyapunov-functions have been presented recently, see Moreno and Sánchez (2014).

4. LOCAL PERFORMANCE USING AGSTA

The reduction of the sliding-mode controller gain is a very promising effect of our proposed method. This important goal has also been achieved by some of the most appealing extensions of the super-twisting algorithm (STA): the adaptive-gain STA, see (Shtessel et al., 2010, 2012). The algorithms, however, require a strong bound on the uncertainty:

$$|\varphi(\boldsymbol{x}, \boldsymbol{\Theta}, t)| \leq \delta|\sigma(\boldsymbol{x})|^{\frac{1}{2}}, \quad \forall \boldsymbol{x} \in \mathbb{R}^n, \forall t \in \mathbb{R}^+\,. \quad (14)$$

Unfortunately, this requirement is too restrictive for many problem classes. Consider for example a system where both the disturbance and the sliding surface are a linear combination of the system states, i.e.

$$\varphi(\boldsymbol{x}, \boldsymbol{\Theta}, t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}, \quad \sigma(\boldsymbol{x}) = \boldsymbol{d}^{\mathrm{T}}\,\boldsymbol{x} \quad (15)$$

where $\boldsymbol{c}, \boldsymbol{d}$ constant vectors of appropriate dimensions. In fact, there is no constant δ such that (14) holds for all $\boldsymbol{x} \in \mathbb{R}^n$ so as to render the algorithms in (Shtessel et al., 2010, 2012) applicable.

In this section we shall investigate whether it is possible to apply the adaptive-gain super-twisting algorithm (AGSTA) if (14) holds locally, meaning that we find some δ^* such that

$$\mathcal{U}_{\delta^*} = \left\{\boldsymbol{x} \in \mathbb{R}^n \,\middle|\, |\varphi(\boldsymbol{x}, \boldsymbol{\Theta}, t)| \leq \delta^*|\sigma(\boldsymbol{x})|^{\frac{1}{2}}\right\} \subset \mathbb{R}^n.$$

In the original work (Shtessel et al., 2010) the control law is given by

$$\omega = -\alpha|\sigma|^{\frac{1}{2}}\operatorname{sign}(\sigma) + \nu \ \text{ and } \ \dot{\nu} = -\beta\operatorname{sign}(\sigma)\,, \quad (16)$$

with respective adaptation of the controller gain α:

$$\dot{\alpha} = \begin{cases}\omega_1\sqrt{\frac{1}{2}\gamma_1}, & \text{if} \quad \sigma \neq 0,\\ 0, & \text{if} \quad \sigma = 0\end{cases} \quad (17)$$
$$\beta = 2\varepsilon\alpha + \lambda + 4\varepsilon^2$$

where $\omega_1, \gamma_1, \varepsilon, \lambda > 0$. If α is large enough satisfying

$$\alpha > \frac{\varepsilon\delta^2 + (\lambda + 4\varepsilon^2)(2\varepsilon + \delta) + \varepsilon}{\lambda} \quad (18)$$

the existence of a quadratic Lyapunov function V is shown (Shtessel et al., 2010). Since α grows with adaptation whenever $\sigma \neq 0$, the existence of the Lyapunov function is eventually guaranteed and thus convergence to $\sigma = 0$.

We can preserve this reasoning for our local considerations if we make sure that the domain $\mathcal{U}_{\delta^*}$ is not left. Assume that the local domain $\mathcal{U}_{\delta^*}$ is invariant on $\sigma = 0$. Then we can choose the initial value of the adapted controller gain $\alpha(0)$ large enough such that (18) holds for $\delta = \delta^*$. This ensures the existence of a local, quadratic Lyapunov function. Each level-set given by $V(x) \equiv$ const. provides an invariant set for the solutions of (4). Thus, there exists a region of attraction defined by some level-set of V that is fully contained in $\mathcal{U}_{\delta^*}$.

The analysis above shows that AGSTA indeed may be successfully applied even if (14) does not hold, globally.

However, there are some drawbacks. We need to choose δ^* *a priori* for fixing the local domain $\mathcal{U}_{\delta^*}$. The domain itself might be hard to compute if little is known about the uncertainty which in turn makes it difficult to determine a level-set of the Lyapunov-function fully contained in $\mathcal{U}_{\delta^*}$ for guaranteeing attractivity. Furthermore, we have to make sure that the initial state $\boldsymbol{x}_0$ is within that region of attraction. Ensuring that for all possibly unknown initial states, we might need to choose δ^* and also the initial value of α large. In doing so we loose one of the most intriguing features of AGSTA, namely, that the gain may be chosen small and only grows whenever required.

In the following sections we shall study these effects on a simulation example and a laboratory experiment, and further place into perspective achieved performance in comparison with our proposed approach.

5. ILLUSTRATIVE SIMULATION EXAMPLE

In this section, the certainty-equivalence super-twisting algorithm (CESTA) that we proposed in Section 3 will be compared with an adaptive gain super-twisting sliding-mode algorithm (AGSTA), given in Shtessel et al. (2012).

	both	AGSTA				CESTA		
	c	ω_1	γ_1	μ	ϵ	k_1	k_2	γ
Simulation	1	0.1	2	0.05	0.5	1	2	1
Experiment	1	1	2	0.1	0.5	2	3	10

Tab. 1. Controller parameters (simulation/experiment)

As a process model we consider the second-order system

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = \Theta\,\phi(x_1,x_2) + u\,. \tag{19}$$

with input $u(t) \in \mathbb{R}$ and states $x_1(t), x_2(t) \in \mathbb{R}$. Let the uncertainty be represented by

$$\Theta\,\phi(x_1,x_2) = 2\sin(2\,x_2)(x_1+x_2) \tag{20}$$

with unknown parameter $\Theta = 2$. For the sliding variable we choose $\sigma = x_1 + x_2$ exhibiting the dynamics

$$\dot{\sigma} = x_2 + \Theta\,\phi(x_1,x_2) + u\,. \tag{21}$$

Note that the disturbance (20) may be rewritten as

$$\Theta\,\phi(x_1,x_2) = 2\sin(2x_2)\sigma$$

which obviously does not satisfy AGSTA condition (14).

Referring to the discussion in Section 4 we shall nonetheless investigate whether AGSTA may still be successfully applied. For the control law we obtain

$$\begin{aligned} u &= -\,x_2 - \alpha|\sigma|^{1/2}\mathrm{sign}\,(\sigma) + \nu \\ \dot{\nu} &= -\,\beta\,\mathrm{sign}\,(\sigma)\,. \end{aligned} \tag{22}$$

Inserting this controller into (21) the evolution of the sliding variable σ is governed by

$$\begin{aligned} \dot{\sigma} &= -\,\alpha|\sigma|^{1/2}\mathrm{sign}\,(\sigma) + \nu + \Theta\,\phi(x_1,x_2) \\ \dot{\nu} &= -\,\beta\,\mathrm{sign}\,(\sigma) \end{aligned} \tag{23}$$

which shows exactly the structure of the adaptive gain super-twisting sliding-mode controller from Shtessel et al. (2012). Accordingly, the controller gains α and β read

$$\begin{aligned} \dot{\alpha} &= \begin{cases} \omega_1\sqrt{\dfrac{1}{2}}\gamma_1\,\mathrm{sign}\,(|\sigma|-\mu)\,, & \text{if} \quad \sigma \neq 0 \\ 0, & \text{if} \quad \sigma = 0 \end{cases} \\ \beta &= 2\varepsilon\alpha \end{aligned} \tag{24}$$

where ω_1, γ_1, μ and ϵ are positive constants. Note that the adaptation law in (Shtessel et al., 2012) allows for an increase as well as a decrease of the gains α and β. However, the discussion about the local performance in Section 4 still applies.

On the other hand, following the CESTA controller design steps in Section 3, controller and adaptation law read

$$\begin{aligned} u &= -\,x_2 - k_1|\sigma|^{1/2}\mathrm{sign}\,(\sigma) + \nu - \hat{\Theta}\,\phi(x_1,x_2) \\ \dot{\nu} &= -\,k_2\,\mathrm{sign}\,(\sigma) \\ \dot{\hat{\Theta}} &= \gamma\,k_2\,\mathrm{sign}\,(\sigma)\;\phi(x_1,x_2)\,. \end{aligned} \tag{25}$$

We discuss the performance of the two adaptive controllers using different initial conditions of the process model. The system model (19), AGSTA (24) and CESTA (25) are implemented in Matlab/Simulink. The controller parameters are given in Table 1. For both simulations the controllers are initialized with the values

$$\hat{\Theta}(0) = 0.5\,, \quad \nu(0) = 0\,, \quad \alpha(0) = 1\,.$$

Fig. 1 shows the simulation results for the initial process states $x_1(0) = x_2(0) = 2$. The blue lines represent results

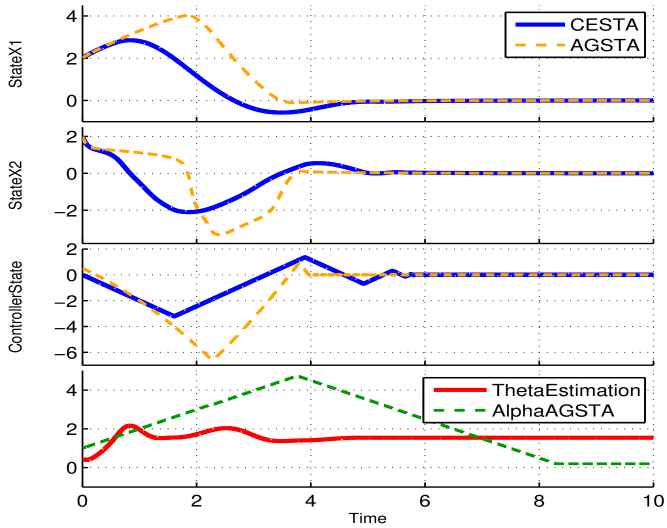

Fig. 1. Simulation results for $x_1(0) = x_2(0) = 2$

of the CESTA controller (25) and the yellow dashed line the results of the AGSTA controller (24). Clearly, both controllers are able to steer the system from the given initial point to the origin. The evolution of the state variables is similar. Even without matching the growth condition (14) the AGSTA controller stabilizes the system. However, this observation may in general be deceptive as we shall demonstrate below.

In the second simulation study we have modified the initial values which now are $x_1(0) = x_2(0) = 10$. The initial values of the controller gain α with respect to AGSTA and the adaption parameter γ in our advocated approach are left unchanged. Also the initial value of controller state ν is kept the same as in the previous simulation.

It is expected that the AGSTA controller cannot stabilize the system due to the modified initial conditions. Evaluating the disturbance (20) at the initial point shows that the uncertainty is significantly greater when compared to the first instance. For this reason, the AGSTA controller initially may not be able to dominate the disturbance and the states of the system will diverge.

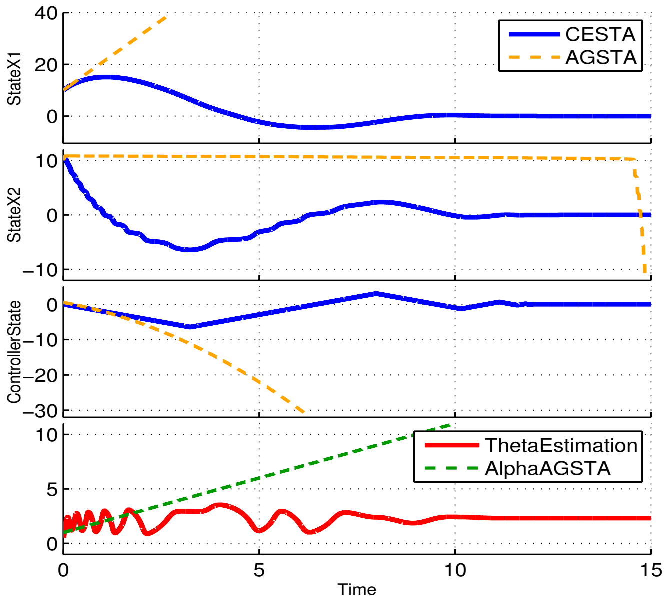

Fig. 2. Simulation results for $x_1(0) = x_2(0) = 10$ without modifying the controller parameters

The results of the second simulation are shown in Fig. 2. Similar to the first simulation run, our proposed adaptive super-twisting controller forces the state to the desired endpoint since there is no dependency on the initial state. The AGSTA controller, however, shows a completely different evolution of state when compared to the first simulation. A tentatively unknown, changed initial condition made the adaptive-gain controller fail to suppress the disturbance, hence, the states of the system diverge.

This observation may be an issue for applications where the system shows unstable dynamics and the disturbance growths locally faster than specified in condition (14). More precisely, a working AGSTA controller might suggest stability in a certain domain around the origin and a good overall performance. However, leaving this domain may suddenly lead to undesired behavior of the overall system.

In contrast, if some structural information on the disturbance is available, our algorithm is able to meet with the issue of varying initial conditions. In both cases, the CESTA controller handles the disturbance while driving the system to the origin. While achieving a similar performance the sliding-mode gain $k_1 = 1$ of the CESTA is much lower than the sliding-mode gain α of the AGSTA for most of the simulation. In fact, we only require $k_1 > 0$ as there is no unstructured uncertainty in this example.

In summary, both controllers comprise an adaptive part and, thus, are able to react on uncertain, varying or unknown system parameters. However, in our method there is no need to modify the gains of the discontinuous part of the controller. Instead, available additional information on the uncertainty is exploited which leads to a better robustness, as can be seen in the presented simulation.

6. LABORATORY EXPERIMENT

We carried out laboratory experiments in order to also assess the practical usability of the afore-examined approaches. The test-bench consists of a DC-motor with a motor driver and a specific load. The load of the motor is an aluminium block whose rotational axis does not cross the center of gravity, resulting in an imbalance, see Fig. 3. Furthermore, an extra eddy current brake may be attached to the system which introduces a friction term to the system that is approximately proportional to velocity.

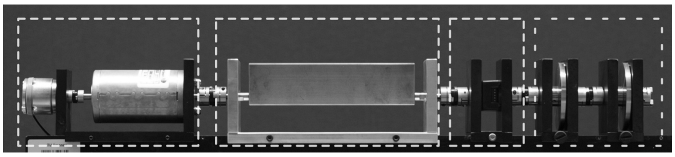

Fig. 3. DC-motor setup with an imbalance load and eddy current brake

Using the Lagrange formalism we obtain the model

$$J\,\ddot{\psi} + K_{\mathrm{fr}}\,\dot{\psi} - m\,g_0\,d\,\sin(\psi) = u \tag{26}$$

where J is the total moment of inertia of the motor, m the mass of the imbalance, and g_0 the gravitational constant. The parameter K_{fr} is a friction term which is mainly affected by the eddy current brake. The system consists of two states, the position ψ and the speed $\dot{\psi}$ of the motor with load, the latter obtained by a second-order unknown-input sliding-mode observer (Zhu, 2012). Furthermore, the torque generated by the DC motor is denoted by u and may be considered a control input. In this setup we assume that the motor driver is fast enough to provide the requested torques timely such that the motor dynamics is negligible. The control algorithms are implemented in a real-time DSpace environment with a sampling frequency of 1 kHz.

We devised two controllers for the nonlinear system (26): The AGSTA controller is designed using the method exposed in Shtessel et al. (2012). The novel adaptive super-twisting approach (CESTA) was designed as shown in Section 3 of this paper. Both controller designs resort to the sliding surface as in Section 5.

Note that assumption (14) is not fulfilled. In order to see this effect, experiments with and without eddy current were carried out. The parameters of the implemented controllers are listed in Table 1. The controllers are not tuned to meet a specific transient behavior.

Fig. 4 shows the result of these experiments. The blue and green dashed line represent the AGSTA controller without and with the eddy current brake, resp. Results achieved by the CESTA-approach are displayed in red and cyan dash-dotted lines. From the figure we may conclude that both controllers are able to stabilize the system to the origin, with and without additional damping brake. The main result is the difference in the closed-loop behavior when attaching the eddy current brake. Whereas the CESTA controller shows no noticeable difference between the two experiments, the convergence time of the AGSTA controller increases significantly. All experiments are performed using the same controller parameters.

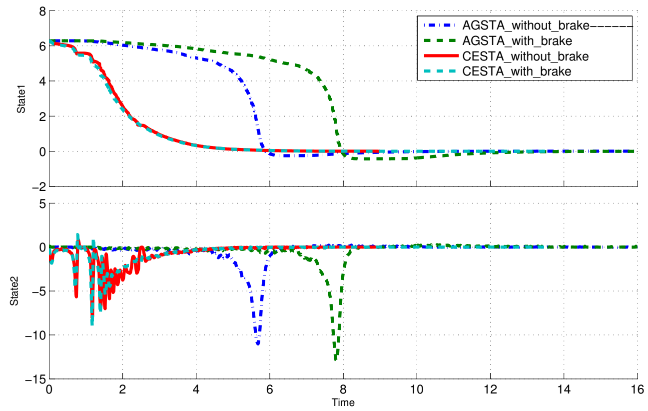

Fig. 4. Measured Position and Velocity of the Motor with and without a viscous load

7. CONCLUSION

An approach combining sliding-mode and adaptive control is proposed. The idea is to improve a sliding-mode controller by using available information on the structure of the uncertainty. This is achieved by a two step controller design: First a nominal sliding-mode controller is designed. This control law is then extended by an additive adaptive component referring to the Lyapunov function of the nominal closed-loop system. The adaptation law is obtained using the certainty-equivalence principle. The effectiveness of the proposed concept is demonstrated by means of simulation and experimental results. The novel approach (CESTA) is compared with the adaptive-gain super-twisting algorithm (AGSTA). It turns out that the proposed controller shows two main advantages: structured and unstructured uncertainties are considered in the controller design. Therefore, CESTA may also cope with uncertainties that grow faster than the square-root of the sliding variable. As shown by simulation and experiment, the controller gains may be reduced significantly when compared to other sliding-mode techniques.

Future activities will aim at applying the proposed method also on other higher-order sliding-mode algorithms for which differentiable Lyapunov functions have been obtained.

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